Problems

Key

- √ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ⋆ A difficult problem.

1 If one stereo system is capable of producing 20 watts of sound power and another can put out 50 watts, how many times greater is the amplitude of the sound wave that can be created by the more powerful system? (Assume they are playing the same music.)

2 Many fish have an organ known as a swim bladder, an air-filled cavity whose main purpose is to control the fish's buoyancy an allow it to keep from rising or sinking without having to use its muscles. In some fish, however, the swim bladder (or a small extension of it) is linked to the ear and serves the additional purpose of amplifying sound waves. For a typical fish having such an anatomy, the bladder has a resonant frequency of 300 Hz, the bladder's Q is 3, and the maximum amplification is about a factor of 100 in energy. Over what range of frequencies would the amplification be at least a factor of 50?

3 As noted in section 17.4, it is only approximately true that the amplitude has its maximum at $f = (1/2\pi)\sqrt{k/m}$. Being more careful, we should actually define two different symbols, $f_0 = (1/2\pi)\sqrt{k/m}$ and f_o for the slightly different frequency at which the amplitude is a maximum, i.e., the actual resonant frequency. In this notation, the amplitude as a function of frequency is

$$A = \frac{F}{2\pi\sqrt{4\pi^2 m^2\left(f^2 - f_0^2\right)^2 + b^2 f^2}} \qquad .$$

Show that the maximum occurs not at f_o but rather at the frequency

$$f_\text{o} = \sqrt{f_0^2 - \frac{b^2}{8\pi^2 m^2}} = \sqrt{f_0^2 - \frac{1}{2}\text{FWHM}^2}$$

Hint: Finding the frequency that minimizes the quantity inside the square root is equivalent to, but much easier than, finding the frequency that maximizes the amplitude.

4 (a) Let W be the amount of work done by friction in the first cycle of oscillation, i.e., the amount of energy lost to heat. Find the fraction of the original energy E that remains in the oscillations after n cycles of motion.

(b) From this, prove the equation

$$\left(1-\frac{W}{E}\right)^Q = e^{-2\pi}$$

(recalling that the number 535 in the definition of Q is $e^{2\pi}$).

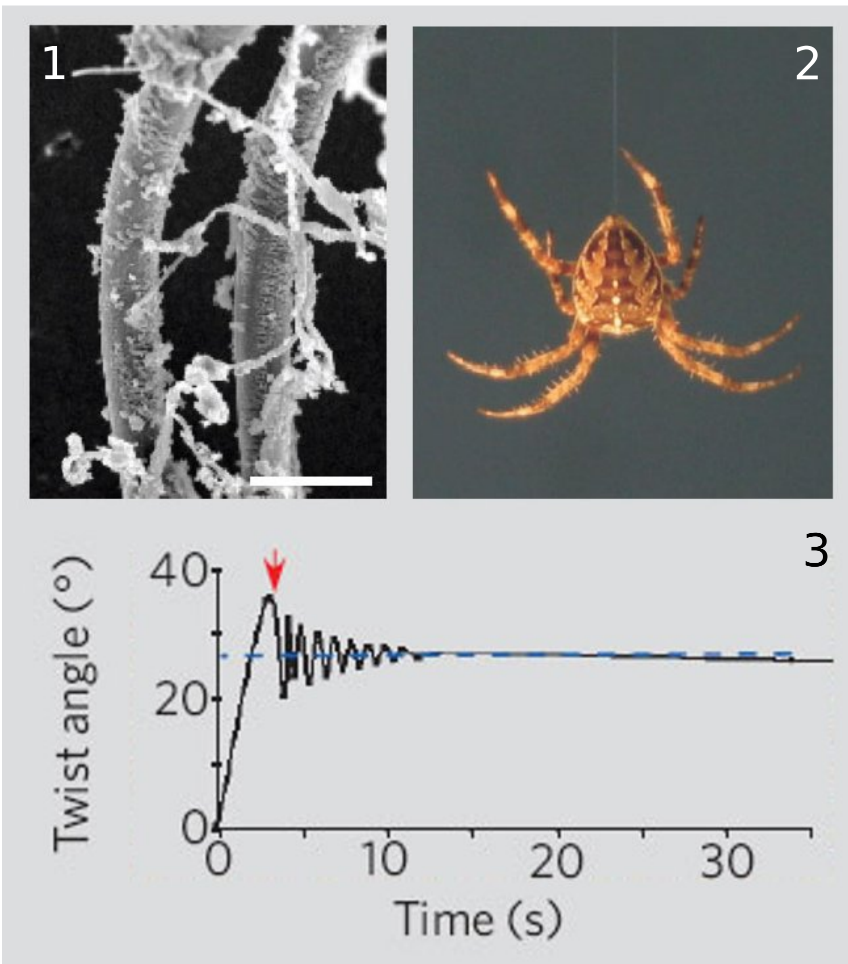
(c) Use this to prove the approximation $1/Q \approx (1/2\pi)W/E$. (Hint: Use the approximation $\ln(1+x) \approx x$, which is valid for small values of x.)

5 (a) We observe that the amplitude of a certain free oscillation decreases from A_o to A_o/Z after n oscillations. Find its Q. ✓
(b) The figure is from *Shape memory in Spider draglines*, Emile, Le Floch, and Vollrath, *Nature* 440:621 (2006). Panel 1 shows an electron microscope's image of a thread of spider silk. In 2, a spider is hanging from such a thread. From an evolutionary point of view, it's probably a bad thing for the spider if it twists back and forth while hanging like this. (We're referring to a back-and-forth rotation about the axis of the thread, not a swinging motion like a pendulum.) The authors speculate that such a vibration could make the spider easier for predators to see, and it also seems to me that it would be a bad thing just because the spider wouldn't be able to control its orientation and do what it was trying to do. Panel 3 shows a graph of such an oscillation, which the authors measured using a video camera and a computer, with a 0.1 g mass hung from it in place of a spider. Compared to human-made fibers such as kevlar or copper wire, the spider thread has an unusual set of properties:

1. It has a low Q, so the vibrations damp out quickly.
2. It doesn't become brittle with repeated twisting as a copper wire would.
3. When twisted, it tends to settle in to a new equilibrium angle, rather than insisting on returning to its original angle. You can see this in panel 2, because although the experimenters initially twisted the wire by 35 degrees, the thread only performed oscillations with an amplitude much smaller than ±35 degrees, settling down to a new equilibrium at 27 degrees.
4. Over much longer time scales (hours), the thread eventually resets itself to its original equilbrium angle (shown as zero degrees on the graph). (The graph reproduced here only shows the motion over a much shorter time scale.) Some human-made materials have this "memory" property as well, but they

typically need to be heated in order to make them go back to their original shapes.

Focusing on property number 1, estimate the Q of spider silk from the graph. $\checkmark$



Problem 5.

6 An oscillator with sufficiently strong damping has its maximum response at $\omega = 0$. Using equation [4] on p. 497 , find the value of Q at which this behavior sets in.

▷ Hint, p. 511 ▷ Answer, p. 530

7 The goal of this problem is to refine the proportionality FWHM $\propto f_{res}/Q$ into the equation FWHM $= f_{res}/Q$, i.e., to prove that the constant of proportionality equals 1.

(a) Show that the work done by a damping force $F = -bv$ over one cycle of steady-state motion equals $W_{damp} = -2\pi^2 bfA^2$. Hint: It is less confusing to calculate the work done over half a cycle, from $x = -A$ to $x = +A$, and then double it.

(b) Show that the fraction of the undriven oscillator's energy lost to damping over one cycle is $|W_{damp}|/E = 4\pi^2 bf/k$.

(c) Use the previous result, combined with the result of problem 4, to prove that Q equals $k/2\pi bf$.

(d) Combine the preceding result for Q with the equation FWHM $= b/2\pi m$ from section 17.4 to prove the equation FWHM $= f_{res}/Q$.

⋆

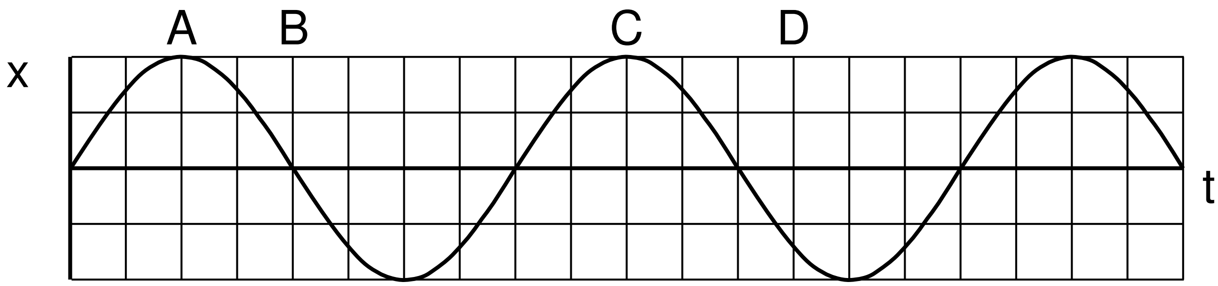
8 An oscillator has Q=6.00, and, for convenience, let's assume $F_m = 1.00$, $\omega_o = 1.00$, and $m = 1.00$. The usual approximations would give

$$\omega_{res} = \omega_o \quad ,$$
$$A_{res} = 6.00 \quad , \quad \text{and}$$
$$\Delta\omega = 1/6.00 \quad .$$

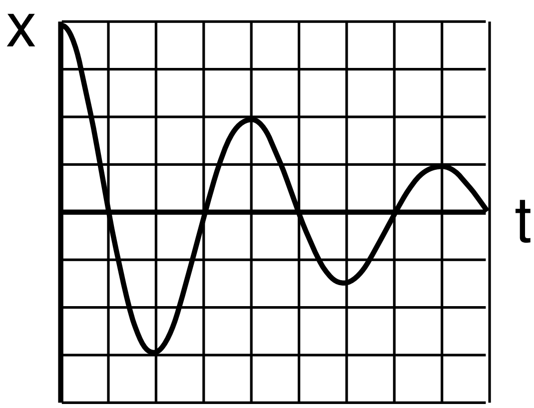
Determine these three quantities numerically using equation [4] on p. 497 , and compare with the approximations.

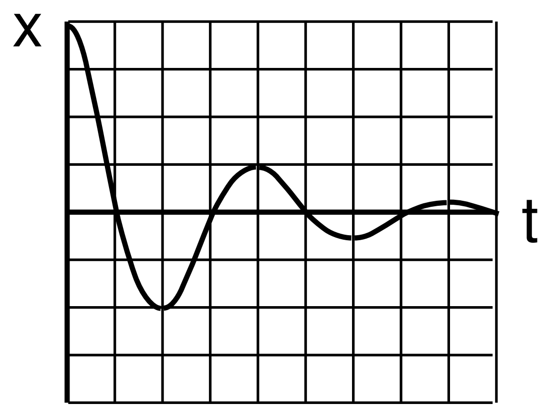
Exercise 17: Resonance

1. Compare the oscillator's energies at A, B, C, and D.

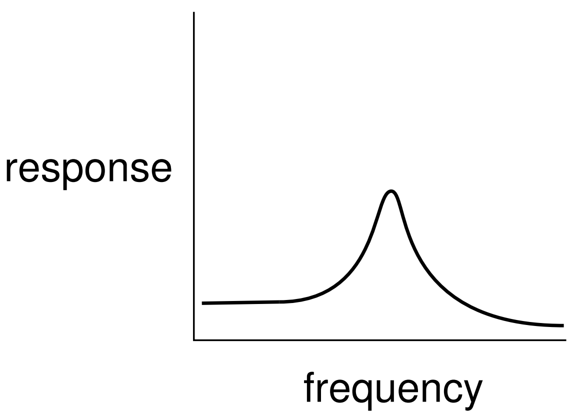


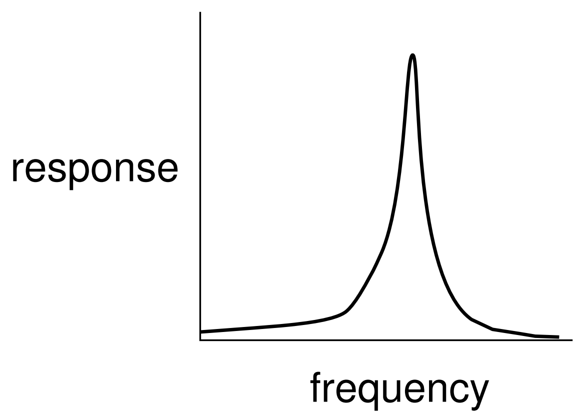
2. Compare the Q values of the two oscillators.





3. Match the x-t graphs in #2 with the amplitude-frequency graphs below.





Three essential mathematical skills

More often than not when a search-and-rescue team finds a hiker dead in the wilderness, it turns out that the person was not carrying some item from a short list of essentials, such as water and a map. There are three mathematical essentials in this course.

1. Converting units

basic technique: section 0.9, p. 26; conversion of area, volume, etc.: section 1.1, p. 37

Examples:

$$0.7\ \cancel{\text{kg}} \times \frac{10^3\ \text{g}}{1\ \cancel{\text{kg}}} = 700\ \text{g} \qquad .$$

To check that we have the conversion factor the right way up (10^3 rather then $1/10^3$), we note that the smaller unit of grams has been *compensated* for by making the number larger.

For units like m^2, kg/m^3, etc., we have to raise the conversion factor to the appropriate power:

$$4\ \text{m}^3 \times \left(\frac{10^3\ \text{mm}}{1\ \text{m}}\right)^3 = 4 \times 10^9\ \cancel{\text{m}^3} \times \frac{\text{mm}^3}{\cancel{\text{m}^3}} = 4 \times 10^9\ \text{mm}^3$$

Examples with solutions — p. 33, #1; p. 52, #2

Problems you can check at `lightandmatter.com/area1checker.html` — p. 33, #3; p. 33, #2; p. 33, #4; p. 52, #5; p. 52, #1

2. Reasoning about ratios and proportionalities

The technique is introduced in section 1.2, p. 39, in the context of area and volume, but it applies more generally to any relationship in which one variable depends on another raised to some power.

Example: When a car or truck travels over a road, there is wear and tear on the road surface, which incurs a cost. Studies show that the cost per kilometer of travel C is given by

$$C = kw^4 \qquad ,$$

where w is the weight per axle and k is a constant. The weight per axle is abot 13 times higher for a semi-trailer than for my Honda Fit. How many times greater is the cost imposed on the federal government when the semi travels a given distance on an interstate freeway?

▷ First we convert the equation into a proportionality by throwing out k, which is the same for both vehicles:

$$C \propto w^4$$

Next we convert this proportionality to a statement about ratios:

$$\frac{C_1}{C_2} = \left(\frac{w_1}{w_2}\right)^4 \approx 29,000$$

Since the gas taxes I pay to drive my Fit are nowhere near 29,000 times more than those paid to drive the truck the same distance, the federal government is effectively awarding a massive subsidy to the trucking company. Plus my Fit is cuter.

Examples with solutions — p. 52, #3; p. 52, #8; p. 52, #4; p. 112, #13; p. 112, #15; p. 247, #3; p. 274, #1; p. 275, #6; p. 274, #3; p. 309, #7; p. 309, #8

Problems you can check at `lightandmatter.com/area1checker.html` — p. 54, #13; p. 53, #9; p. 53, #10; p. 53, #11; p. 53, #12; p. 198, #4; p. 248, #6; p. 275, #7; p. 275, #8; p. 274, #4; p. 308, #3; p. 452, #10

3. Vector addition

section 7.3, p. 206

Example: The $\Delta\mathbf{r}$ vector from San Diego to Los Angeles has magnitude 190 km and direction 129°counterclockwise from east. The one from LA to Las Vegas is 370 km at 38°counterclockwise from east. Find the distance and direction from San Diego to Las Vegas.

▷ Graphical addition is discussed on p. 206. Here we concentrate on analytic addition, which involves adding the x components to find the total x component, and similarly for y. The trig needed in order to find the components of the second leg (LA to Vegas) is laid out in figure d on p. 204 and explained in detail in example 3 on p. 205:

$$\Delta x_2 = (370\text{ km})\cos 38° = 292\text{ km}$$
$$\Delta y_2 = (370\text{ km})\sin 38° = 228\text{ km}$$

(Since these are intermediate results, we keep an extra sig fig to avoid accumulating too much rounding error.) Once we understand the trig for one example, we don't need to reinvent the wheel every time. The pattern is completely universal, provided that we first make sure to get the angle expressed according to the usual trig convention, counterclockwise from the x axis. Applying the pattern to the first leg, we have:

$$\Delta x_1 = (190\text{ km})\cos 129° = -120\text{ km}$$
$$\Delta y_1 = (190\text{ km})\sin 129° = 148\text{ km}$$

For the vector directly from San Diego to Las Vegas, we have

$$\Delta x = \Delta x_1 + \Delta x_2 = 172\text{ km}$$
$$\Delta y = \Delta y_1 + \Delta y_2 = 376\text{ km} \qquad .$$

The distance from San Diego to Las Vegas is found using the Pythagorean theorem,

$$\sqrt{(172\text{ km})^2 + (376\text{ km})^2} = 410\text{ km}$$

(rounded to two sig figs because it's one of our final results). The direction is one of the two possible values of the inverse tangent

$$\tan^{-1}(\Delta y/\Delta x) = \{65°, 245°\} \qquad .$$

Consulting a sketch shows that the first of these values is the correct one.

Examples with solutions — p. 226, #3; p. 228, #9; p. 393, #8

Problems you can check at `lightandmatter.com/area1checker.html` — p. 213, #3; p. 213, #4; p. 226, #4; p. 226, #5; p. 229, #16; p. 275, #13; p. 276, #14; p. 393, #9

Programming with python

The purpose of this tutorial is to help you get familiar with a computer programming language called Python, which I've chosen because (a) it's free, and (b) it's easy to use interactively. I won't assume you have any previous experience with computer programming; you won't need to learn very much Python, and what little you do need to learn I'll explain explicitly. If you really want to learn Python more thoroughly, there are a couple of excellent books that you can download for free on the Web:

> **How to Think Like a Computer Scientist (Python Version)**, Allen B. Downey, Jeffrey Elkner, Moshe Zadka, http://www.ibiblio.org/obp/
>
> **Dive Into Python**, Mark Pilgrim, http://diveintopython.net/

The first book is meant for people who have never programmed before, while the second is a more complete introduction aimed at veteran programmers who know a different language already.

Using Python as a calculator

The easiest way to get Python going is to go to the web site `ideone.com`. Under "choose a language," select Python. Inside the window where it says "paste your source code or insert template or sample," type `print(2+2)`. Click on the "submit" button. The result, 4, is shown under "output." In other words, you can use Python just like a calculator.

For compactness, I'll show examples in the following style:

```
>>> print(2+2)
4
```

Here the `>>>` is not something you would type yourself; it's just a marker to distinguish your input from the program's output. (In some other versions of Python, the computer will actually print out `>>>` as a prompt to tell you it's ready to type something.)

There are only a couple of things to watch out for. First, Python distinguishes between integers and real numbers, so the following gives an unexpected result:

```
>>>  print(2/3)
0
```

To get it to treat these values as real numbers, you have to use decimal points:

```
>>> print(2./3.)
0.66666666666666666663
```

Multiplication is represented by "*":

```
>>> print(2.*3.)
6.0
```

Also, Python doesn't know about its own library of math functions unless you tell it explicitly to load them in:

```
>>> print (sqrt(2.))
Traceback (most recent call last):
File ``<stdin>'', line 1, in ?
NameError: There is no variable named `sqrt'
```

Here are the steps you have to go through to calculate the square root of 2 successfully:

```
>>> import math
>>> print(math.sqrt(2.))
1.4142135623730951
```

The first line is just something you can make a habit of doing every time you start up Python. In the second line, the name of the square root function had to be prefixed with "`math.`" to tell Python where you wanted to get this "`sqrt`" function from. (All of this may seem like a nuisance if you're just using Python as a calculator, but it's a good way to design a programming language so that names of functions never conflict.)

Try it. Experiment and figure out whether Python's trig functions assume radians or degrees.

Variables

Python lets you define variables and assign values to them using an equals sign:

```
>>> dwarfs=7
>>> print(dwarfs)
>>> print(dwarfs+3)
7
10
```

Note that a variable in computer programming isn't quite like a variable in algebra. In algebra, if a=7 then a=7 always, throughout a particular calculation. But in a programming language, the variable name really represents a place in memory where a number can be stored, so you can change its value:

```
>>> dwarfs=7
>>> dwarfs=37
>>> print(dwarfs)
37
```

You can even do stuff like this,

```
>>> dwarfs=37
>>> dwarfs=dwarfs+1
>>> print(dwarfs)
38
```

In algebra it would be nonsense to have a variable equal to itself plus one, but in a computer program, it's not an assertion that the two things are equal, its a command to calculate the value of the expression on the right side of the equals, and then put that number into the memory location referred to by the variable name on the left.

Try it. What happens if you do dwarfs+1 = dwarfs? Do you understand why?

Functions

Somebody had to teach Python how to do functions like `sqrt`, and it's handy to be able to define your own functions in the same way. Here's how to do it:

```
>>> def double(x):
>>>   return 2.*x
>>> print(double(5.))
10.0
```

Note that the indentation is mandatory. The first and second lines define a function called `double`. The final line evaluates that function with an input of 5.

Loops

Suppose we want to add up all the numbers from 0 to 99.

Automating this kind of thing is exactly what computers are best at, and Python provides a mechanism for this called a loop:

```
>>> sum=0
>>> for j in range(100):
>>>   sum=sum+j
>>> print(sum)
4950
```

The stuff that gets repeated — the inside of the loop — has to be indented, just like in a function definition. Python always counts loops starting from 0, so for `j in range(100)` actually causes `j` to range from 0 to 99, not from 1 to 100.

Hints

Hints for chapter 8

Page 230, problem 18:

The easiest way to do this problem is to use two different coordinate systems: one that's tilted to coincide with the upper slope, and one that's tilted to coincide with the lower one.

Page 230, problem 19:

Consider a section of the rope subtending a very small angle, and find an approximate equation relating the normal force to the tension. Apply small-angle approximations to any trig functions occurring in your result. Eliminate all variables except for the tension and the angle, and separate these variables.

Hints for chapter 10

Page 278, problem 22:

If you try to calculate the two forces and subtract, your calculator will probably give a result of zero due to rounding. Instead, reason about the fractional amount by which the quantity $1/r^2$ will change. As a warm-up, you may wish to observe the percentage change in $1/r^2$ that results from changing r from 1 to 1.01.

Hints for chapter 13

Page 359, problem 13:

What does the total energy have to be if the projectile's velocity is exactly escape velocity? Write down conservation of energy, change v to $\mathrm{d}r/\mathrm{d}t$, separate the variables, and integrate.

Page 361, problem 20:

You can use the geometric interpretation of the dot product.

Page 362, problem 25:

The analytic approach is a little cumbersome, although it can be done by using approximations like $1/\sqrt{1+\epsilon} \approx 1-(1/2)\epsilon$. A more straightforward, brute-force method is simply to write a computer program that calculates U/m for a given point in spherical coordinates. By trial and error, you can fairly rapidly find the r that gives a desired value of U/m.

Hints for chapter 15

Page 455, problem 28:

The choice of axis theorem only applies to a closed system, or to a system acted on by a total force of zero. Even if the box is not going to rotate, its center of mass is going to accelerate, and this can still cause a change in its angular momentum, unless the right axis is chosen. For example, if the axis is chosen at the bottom right corner, then the box will start accumulating clockwise angular momentum, even if it is just accelerating to the right without rotating. Only by choosing the axis at the center of mass (or at some other point on the same horizontal line) do we get a constant, zero angular momentum.

Page 457, problem 41:

You'll need the result of problem 26 in order to relate the energy and angular momentum of a rigidly rotating body. Since this relationship involves a variable raised to a power, you can't just graph the data and get the moment of inertia directly. One way to get around this is to manipulate one of the variables to make the graph linear. Here is an example of this technique from another context. Suppose you were given a table of the masses, m, of cubical pieces of wood, whose sides had various lengths, b. You want to find a best-fit value for the density of the wood. The relationship is $m = \rho b^3$. The graph of m versus b would be a curve, and you would not have any easy way to get the density from such a graph. But by graphing m versus b^3, you can produce a graph that is linear, and whose slope equals the density.

Hints for chapter 16

Page 477, problem 13:

The spring constant of this spring, k, is *not* the quantity you need in the equation for the period. What you need in that equation is the second derivative of the spring's energy with respect to the position of the thing that's oscillating. You need to start by finding the energy stored in the spring as a function of the vertical position, y, of the mass. This is similar to example 5 on page 470.

Hints for chapter 17

Page 503, problem 6:

The whole expression for the amplitude has maxima where the stuff inside the square root is at a minimum, and vice versa, so you can save yourself a lot of work by just working on the stuff inside the square root. For normal, large values of Q, the there are two extrema, one at $\omega = 0$ and one at resonance; one of these is a maximum and one is a minimum. You want to find out at what value of Q the zero-frequency extremum switches over from being a maximum to being a minimum.

Solutions to selected problems

Solutions for chapter 0

Page 33, problem 1:

$$134 \text{ mg} \times \frac{10^{-3} \text{ g}}{1 \text{ mg}} \times \frac{10^{-3} \text{ kg}}{1 \text{ g}} = 1.34 \times 10^{-4} \text{ kg}$$

Page 33, problem 7:

(a) Let's do 10.0 g and 1000 g. The arithmetic mean is 505 grams. It comes out to be 0.505 kg, which is consistent. (b) The geometric mean comes out to be 100 g or 0.1 kg, which is consistent. (c) If we multiply meters by meters, we get square meters. Multiplying grams by grams should give square grams! This sounds strange, but it makes sense. Taking the square root of square grams (g^2) gives grams again. (d) No. The superduper mean of two quantities with units of grams wouldn't even be something with units of grams! Related to this shortcoming is the fact that the superduper mean would fail the kind of consistency test carried out in the first two parts of the problem.

Page 34, problem 10:

(a) They're all defined in terms of the ratio of side of a triangle to another. For instance, the tangent is the length of the opposite side over the length of the adjacent side. Dividing meters by meters gives a unitless result, so the tangent, as well as the other trig functions, is unitless. (b) The tangent function gives a unitless result, so the units on the right-hand side had better cancel out. They do, because the top of the fraction has units of meters squared, and so does the bottom.

Solutions for chapter 1

Page 52, problem 1:

The proportionality $V \propto L^3$ can be restated in terms of ratios as $V_1/V_2 = (L_1/L_2)^3 = (1/10)^3 = 1/1000$, so scaling down the linear dimensions by a factor of 1/10 reduces the volume by 1/1000, to a milliliter.

Page 52, problem 2:

$$1 \text{ mm}^2 \times \left(\frac{1 \text{ cm}}{10 \text{ mm}}\right)^2 = 10^{-2} \text{ cm}^2$$

Page 52, problem 3:

The bigger scope has a diameter that's ten times greater. Area scales as the square of the linear dimensions, so $A \propto d^2$, or in the language of ratios $A_1/A_2 = (d_1/d_2)^2 = 100$. Its light-gathering power is a hundred times greater.

Page 52, problem 4:

The cone of mixed gin and vermouth is the same shape as the cone of vermouth, but its linear dimensions are doubled. Translating the proportionality $V \propto L^3$ into an equation about ratios, we have $V_1/V_2 = (L_1/L_2)^3 = 8$. Since the ratio of the whole thing to the vermouth is 8, the ratio of gin to vermouth is 7.

Page 52, problem 8:

Since they differ by two steps on the Richter scale, the energy of the bigger quake is 10^4 times greater. The wave forms a hemisphere, and the surface area of the hemisphere over which the energy is spread is proportional to the square of its radius, $A \propto r^2$, or $r \propto \sqrt{A}$, which means $r_1/r_2 = \sqrt{A_1/A_2}$. If the amount of vibration was the same, then the surface areas must be in the ratio $A_1/A_2 = 10^4$, which means that the ratio of the radii is 10^2.

Page 55, problem 22:

Let's estimate the Great Wall's volume, and then figure out how many bricks that would represent. The wall is famous because it covers pretty much all of China's northern border, so let's say it's 1000 km long. From pictures, it looks like it's about 10 m high and 10 m wide, so the total volume would be $10^6 \text{ m} \times 10 \text{ m} \times 10 \text{ m} = 10^8 \text{ m}^3$. If a single brick has a volume of 1 liter, or 10^{-3} m^3, then this represents about 10^{11} bricks. If one person can lay 10 bricks in an hour (taking into account all the preparation, etc.), then this would be 10^{10} man-hours.

Page 55, problem 24:

Directly guessing the number of jelly beans would be like guessing volume directly. That would be a mistake. Instead, we start by estimating the linear dimensions, in units of beans. The contents of the jar look like they're about 10 beans deep. Although the jar is a cylinder, its exact geometrical shape doesn't really matter for the purposes of our order-of-magnitude estimate. Let's pretend it's a rectangular jar. The horizontal dimensions are also something like 10 beans, so it looks like the jar has about $10 \times 10 \times 10$ or $\sim 10^3$ beans inside.

Solutions for chapter 2

Page 92, problem 1:

Since the lines are at intervals of one m/s and one second, each box represents one meter. From $t = 0$ to $t = 2$ s, the area under the curve represents a positive Δx of 6 m. (The triangle has half the area of the 2×6 rectangle it fits inside.) After $t = 2$ s, the area above the curve represents negative Δx. To get -6 m worth of area, we need to go out to $t = 6$ s, at which point the triangle under the axis has a width of 4 s and a height of 3 m/s, for an area of 6 m (half of 3×4).

Page 93, problem 8:
(a) Let f and g be functions. Then the chain rule states that if we construct the function $f(g(x))$, its derivative is

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}g} \cdot \frac{\mathrm{d}g}{\mathrm{d}x} \qquad .$$

On the right-hand side, the units of $\mathrm{d}g$ on the top cancel with the units of $\mathrm{d}g$ on the bottom, so the units do match up with those of $\mathrm{d}f/\mathrm{d}x$ on the left.
(b) The cosine function requires a unitless input and produces a unitless output. Therefore A must have units of meters, and b must have units of s^{-1} (inverse seconds, or "per second"). A is the distance the object moves on either side of the origin, and b is a measure of how fast it vibrates back and forth (how many radians it passes through per second).
(b) The derivative is $v = \mathrm{d}x/\mathrm{d}t = -Ab\sin(bt)$, where the factor of b in front comes from the chain rule. The product Ab does have units of m/s. If we hadn't put in the factor of b as required by the chain rule, the units would have been wrong. Physically, it also makes sense that a larger b, indicating a more rapid vibration, produces a greater v.

Page 93, problem 10:
In one second, the ship moves v meters to the east, and the person moves v meters north relative to the deck. Relative to the water, he traces the diagonal of a triangle whose length is given by the Pythagorean theorem, $(v^2 + v^2)^{1/2} = \sqrt{2}v$. Relative to the water, he is moving at a 45-degree angle between north and east.

Page 93, problem 11:
Velocity is relative, so having to lean tells you nothing about the train's velocity. Fullerton is moving at a huge speed relative to Beijing, but that doesn't produce any noticeable effect in either city. The fact that you have to lean tells you that the train is *changing* its speed, but it doesn't tell you what the train's current speed is.

Page 93, problem 13:
To the person riding the moving bike, bug A is simply going in circles. The only difference between the motions of the two wheels is that one is traveling through space, but motion is relative, so this doesn't have any effect on the bugs. It's equally hard for each of them.

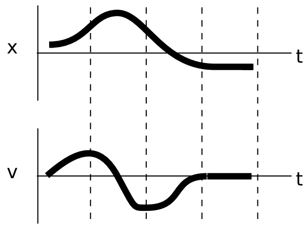
Solutions for chapter 3

Page 111, problem 1:
Taking g to be 10 $\mathrm{m/s^2}$, the bullet loses 10 m/s of speed every second, so it will take 10 s to come to a stop, and then another 10 s to come back down, for a total of 20 s.

Page 111, problem 4:

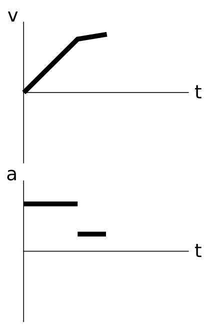
$$\begin{aligned} v &= \frac{\mathrm{d}x}{\mathrm{d}t} \\ &= 10 - 3t^2 \\ a &= \frac{\mathrm{d}v}{\mathrm{d}t} \\ &= -6t \\ &= -18 \ \mathrm{m/s^2} \end{aligned}$$

Page 111, problem 6:

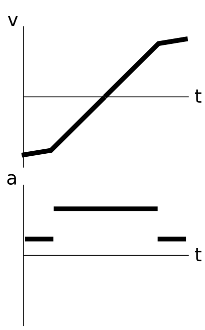


Page 111, problem 7:

(a) We choose a coordinate system with positive pointing to the right. Some people might expect that the ball would slow down once it was on the more gentle ramp. This may be true if there is significant friction, but Galileo's experiments with inclined planes showed that when friction is negligible, a ball rolling on a ramp has constant acceleration, not constant speed. The speed stops increasing as quickly once the ball is on the more gentle slope, but it still keeps on increasing. The a-t graph can be drawn by inspecting the slope of the v-t graph.

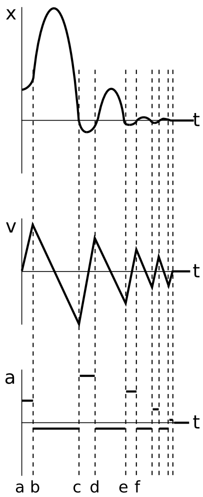


(b) The ball will roll back down, so the second half of the motion is the same as in part a. In the first (rising) half of the motion, the velocity is negative, since the motion is in the opposite direction compared to the positive x axis. The acceleration is again found by inspecting the slope of the v-t graph.



Page 111, problem 8:

This is a case where it's probably easiest to draw the acceleration graph first. While the ball is in the air (bc, de, etc.), the only force acting on it is gravity, so it must have the same, constant acceleration during each hop. Choosing a coordinate system where the positive x axis points up, this becomes a negative acceleration (force in the opposite direction compared to the axis). During the short times between hops when the ball is in contact with the ground (cd, ef, etc.), it experiences a large acceleration, which turns around its velocity very rapidly. These short positive accelerations probably aren't constant, but it's hard to know how they'd really look. We just idealize them as constant accelerations. Similarly, the hand's force on the ball during the time ab is probably not constant, but we can draw it that way, since we don't know how to draw it more realistically. Since our acceleration graph consists of constant-acceleration segments, the velocity graph must consist of line segments, and the position graph must consist of parabolas. On the x graph, I chose zero to be the height of the center of the ball above the floor when the ball is just lying on the floor. When the ball is touching the floor and compressed, as in interval cd, its center is below this level, so its x is negative.



Page 112, problem 11:
(a) Solving for $\Delta x = \frac{1}{2}at^2$ for a, we find $a = 2\Delta x/t^2 = 5.51$ m/s^2. (b) $v = \sqrt{2a\Delta x} = 66.6$ m/s. (c) The actual car's final velocity is less than that of the idealized constant-acceleration car. If the real car and the idealized car covered the quarter mile in the same time but the real car was moving more slowly at the end than the idealized one, the real car must have been going faster than the idealized car at the beginning of the race. The real car apparently has a greater acceleration at the beginning, and less acceleration at the end. This make sense, because every car has some maximum speed, which is the speed beyond which it cannot accelerate.

Page 112, problem 13:
$\Delta x = \frac{1}{2}at^2$, so for a fixed value of Δx, we have $t \propto 1/\sqrt{a}$. Translating this into the language of ratios gives $t_M/t_E = \sqrt{a_E/a_M} = \sqrt{3} = 1.7$.

Page 112, problem 15:
We have $v_f^2 = 2a\Delta x$, so the distance is proportional to the square of the velocity. To get up to half the speed, the ball needs 1/4 the distance, i.e., $L/4$.

Solutions for chapter 4

Page 148, problem 1:
$a = \Delta v/\Delta t$, and also $a = F/m$, so

$$\begin{aligned}\Delta t &= \frac{\Delta v}{a}\\ &= \frac{m\Delta v}{F}\\ &= \frac{(1000\text{ kg})(50\text{ m/s} - 20\text{ m/s})}{3000\text{ N}}\\ &= 10\text{ s}\end{aligned}$$

Page 148, problem 4:

(a) This is a measure of the box's resistance to a change in its state of motion, so it measures the box's mass. The experiment would come out the same in lunar gravity.
(b) This is a measure of how much gravitational force it feels, so it's a measure of weight. In lunar gravity, the box would make a softer sound when it hit.
(c) As in part a, this is a measure of its resistance to a change in its state of motion: its mass. Gravity isn't involved at all.

Solutions for chapter 5

Page 180, problem 1:
(a) The swimmer's acceleration is caused by the water's force on the swimmer, and the swimmer makes a backward force on the water, which accelerates the water backward. (b) The club's normal force on the ball accelerates the ball, and the ball makes a backward normal force on the club, which decelerates the club. (c) The bowstring's normal force accelerates the arrow, and the arrow also makes a backward normal force on the string. This force on the string causes the string to accelerate less rapidly than it would if the bow's force was the only one acting on it. (d) The tracks' backward frictional force slows the locomotive down. The locomotive's forward frictional force causes the whole planet earth to accelerate by a tiny amount, which is too small to measure because the earth's mass is so great.

Page 180, problem 2:
The person's normal force on the box is paired with the box's normal force on the person. The dirt's frictional force on the box pairs with the box's frictional force on the dirt. The earth's gravitational force on the box matches the box's gravitational force on the earth.

Page 180, problem 3:
(a) A liter of water has a mass of 1.0 kg. The mass is the same in all three locations. Mass indicates how much an object resists a change in its motion. It has nothing to do with gravity. (b) The term "weight" refers to the force of gravity on an object. The bottle's weight on earth is $F_W = mg = 9.8$ N. Its weight on the moon is about one sixth that value, and its weight in interstellar space is zero.

Page 185, problem 26:

(a)

top spring's rightward force on connector
...connector's leftward force on top spring
bottom spring's rightward force on connector
...connector's leftward force on bottom spring
hand's leftward force on connector
...connector's rightward force on hand

Looking at the three forces on the connector, we see that the hand's force must be double the force of either spring. The value of $x - x_o$ is the same for both springs and for the arrangement as a whole, so the spring constant must be $2k$. This corresponds to a stiffer spring (more force to produce the same extension).

(b) Forces in which the left spring participates:

hand's leftward force on left spring
...left spring's rightward force on hand
right spring's rightward force on left spring
...left spring's leftward force on right spring

Forces in which the right spring participates:

left spring's leftward force on right spring
...right spring's rightward force on left spring
wall's rightward force on right spring
...right spring's leftward force on wall

Since the left spring isn't accelerating, the total force on it must be zero, so the two forces acting on it must be equal in magnitude. The same applies to the two forces acting on the right spring. The forces between the two springs are connected by Newton's third law, so all eight of these forces must be equal in magnitude. Since the value of $x - x_o$ for the whole setup is double what it is for either spring individually, the spring constant of the whole setup must be $k/2$, which corresponds to a less stiff spring.

Page 185, problem 28:
(a) Spring constants in parallel add, so the spring constant has to be proportional to the cross-sectional area. Two springs in series give half the spring constant, three springs in series give 1/3, and so on, so the spring constant has to be inversely proportional to the length. Summarizing, we have $k \propto A/L$. (b) With the Young's modulus, we have $k = (A/L)E$.The spring constant has units of N/m, so the units of E would have to be N/m^2.

Solutions for chapter 7

Page 214, problem 7:
We'll use the same approach as in the example in section 7.5, which is to find an example such that when the calculation is carried out in a rotated frame of reference, the result is clearly not the same vector expressed in the new frame. Let $\mathbf{A} = \pi\hat{\mathbf{x}}$ in the original coordinate system. Then in this coordinate system $\mathbf{B} = 0$.

But now suppose we choose a new coordinate system, rotated by 10 degrees relative to the first one. In this new coordinate system, A_x is a little less than π. Since A_x is no longer a multiple of π, B_x is no longer zero, and $\mathbf{B}$ is no longer zero. The nonzero $\mathbf{B}$ computed in the new coordinate system is clearly not the same as the old $\mathbf{B}$ expressed in a new way, since rotating our coordinate system should not change the magnitudes of vectors.

Solutions for chapter 8

Page 226, problem 3:
We want to find out about the velocity vector v_{BG} of the bullet relative to the ground, so we need to add Annie's velocity relative to the ground v_{AG} to the bullet's velocity vector v_{BA} relative to her. Letting the positive x axis be east and y north, we have

$$\begin{aligned} v_{BA,x} &= (140 \text{ mi/hr}) \cos 45^\circ \\ &= 100 \text{ mi/hr} \\ v_{BA,y} &= (140 \text{ mi/hr}) \sin 45^\circ \\ &= 100 \text{ mi/hr} \end{aligned}$$

and

$$v_{AG,x} = 0$$
$$v_{AG,y} = 30 \text{ mi/hr} \qquad .$$

The bullet's velocity relative to the ground therefore has components

$$v_{BG,x} = 100 \text{ mi/hr} and$$
$$v_{BG,y} = 130 \text{ mi/hr} \qquad .$$

Its speed on impact with the animal is the magnitude of this vector

$$|v_{BG}| = \sqrt{(100 \text{ mi/hr})^2 + (130 \text{ mi/hr})^2}$$
$$= 160 \text{ mi/hr}$$

(rounded off to 2 significant figures).

Page 228, problem 9:
Since its velocity vector is constant, it has zero acceleration, and the sum of the force vectors acting on it must be zero. There are three forces acting on the plane: thrust, lift, and gravity. We are given the first two, and if we can find the third we can infer its mass. The sum of the y components of the forces is zero, so

$$0 = F_{thrust,y} + F_{lift,y} + F_{W,y}$$
$$= |\mathbf{F}_{thrust}| \sin\theta + |\mathbf{F}_{lift}| \cos\theta - mg \qquad .$$

The mass is

$$m = (|\mathbf{F}_{thrust}| \sin\theta + |\mathbf{F}_{lift}| \cos\theta)/g$$
$$= 6.9 \times 10^4 \text{ kg}$$

Page 229, problem 13:
(a) If there was no friction, the angle of repose would be zero, so the coefficient of static friction, μ_s, will definitely matter. We also make up symbols θ, m and g for the angle of the slope, the mass of the object, and the acceleration of gravity. The forces form a triangle just like the one in section 8.3, but instead of a force applied by an external object, we have static friction, which is less than $\mu_s|\mathbf{F}_N|$. As in that example, $|\mathbf{F}_s| = mg\sin\theta$, and $|\mathbf{F}_s| < \mu_s|\mathbf{F}_N|$, so

$$mg\sin\theta < \mu_s|\mathbf{F}_N| \qquad .$$

From the same triangle, we have $|\mathbf{F}_N| = mg\cos\theta$, so

$$mg\sin\theta < \mu_s mg\cos\theta \qquad .$$

Rearranging,

$$\theta < \tan^{-1}\mu_s \qquad .$$

(b) Both m and g canceled out, so the angle of repose would be the same on an asteroid.

Page 229, problem 14:
(a) Since the wagon has no acceleration, the total forces in both the x and y directions must be zero. There are three forces acting on the wagon: $\mathbf{F}_T$, $\mathbf{F}_W$, and the normal force from the

ground, $\mathbf{F}_N$. If we pick a coordinate system with x being horizontal and y vertical, then the angles of these forces measured counterclockwise from the x axis are $90° - \phi$, $270°$, and $90° + \theta$, respectively. We have

$$\begin{aligned} F_{x,total} &= |\mathbf{F}_T|\cos(90° - \phi) + |\mathbf{F}_W|\cos(270°) + |\mathbf{F}_N|\cos(90° + \theta) \\ F_{y,total} &= |\mathbf{F}_T|\sin(90° - \phi) + |\mathbf{F}_W|\sin(270°) + |\mathbf{F}_N|\sin(90° + \theta) \end{aligned} \qquad ,$$

which simplifies to

$$\begin{aligned} 0 &= |\mathbf{F}_T|\sin\phi - |\mathbf{F}_N|\sin\theta \\ 0 &= |\mathbf{F}_T|\cos\phi - |\mathbf{F}_W| + |\mathbf{F}_N|\cos\theta. \end{aligned}$$

The normal force is a quantity that we are not given and do not with to find, so we should choose it to eliminate. Solving the first equation for $|\mathbf{F}_N| = (\sin\phi/\sin\theta)|\mathbf{F}_T|$, we eliminate $|\mathbf{F}_N|$ from the second equation,

$$0 = |\mathbf{F}_T|\cos\phi - |\mathbf{F}_W| + |\mathbf{F}_T|\sin\phi\cos\theta/\sin\theta$$

and solve for $|\mathbf{F}_T|$, finding

$$|\mathbf{F}_T| = \frac{|\mathbf{F}_W|}{\cos\phi + \sin\phi\cos\theta/\sin\theta} \qquad .$$

Multiplying both the top and the bottom of the fraction by $\sin\theta$, and using the trig identity for $\sin(\theta + \phi)$ gives the desired result,

$$|\mathbf{F}_T| = \frac{\sin\theta}{\sin(\theta + \phi)}|\mathbf{F}_W| \qquad .$$

(b) The case of $\phi = 0$, i.e., pulling straight up on the wagon, results in $|\mathbf{F}_T| = |\mathbf{F}_W|$: we simply support the wagon and it glides up the slope like a chair-lift on a ski slope. In the case of $\phi = 180° - \theta$, $|\mathbf{F}_T|$ becomes infinite. Physically this is because we are pulling directly into the ground, so no amount of force will suffice.

Solutions for chapter 9

Page 247, problem 3:

(a) The inward normal force must be sufficient to produce circular motion, so

$$|\mathbf{F}_N| = mv^2/r \qquad .$$

We are searching for the minimum speed, which is the speed at which the static friction force is just barely able to cancel out the downward gravitational force. The maximum force of static friction is

$$|\mathbf{F}_s| = \mu_s|\mathbf{F}_N| \qquad ,$$

and this cancels the gravitational force, so

$$|\mathbf{F}_s| = mg \qquad .$$

Solving these three equations for v gives

$$v = \sqrt{\frac{gr}{\mu_s}} \qquad .$$

(b) Greater by a factor of $\sqrt{3}$.

Page 248, problem 5:
The inward force must be supplied by the inward component of the normal force,

$$|\mathbf{F}_N| \sin\theta = mv^2/r \qquad .$$

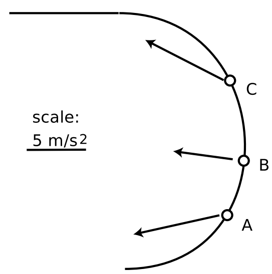
The upward component of the normal force must cancel the downward force of gravity,

$$|\mathbf{F}_N| \cos\theta = mg.$$

Eliminating $|\mathbf{F}_N|$ and solving for θ, we find

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) \qquad .$$

Page 249, problem 10:
Each cyclist has a radial acceleration of $v^2/r = 5\ \text{m/s}^2$. The tangential accelerations of cyclists A and B are 375 N/75 kg = 5 m/s^2.



Solutions for chapter 10

Page 274, problem 1:
Newton's law of gravity tells us that her weight will be 6000 times smaller because of the asteroid's smaller mass, but $13^2 = 169$ times greater because of its smaller radius. Putting these two factors together gives a reduction in weight by a factor of 6000/169, so her weight will be (400 N)(169)/(6000) = 11 N.

Page 274, problem 3:
(a) The asteroid's mass depends on the cube of its radius, and for a given mass the surface gravity depends on r^{-2}. The result is that surface gravity is directly proportional to radius. Half the gravity means half the radius, or one eighth the mass. (b) To agree with a, Earth's mass would have to be 1/8 Jupiter's. We assumed spherical shapes and equal density. Both planets are at least roughly spherical, so the only way out of the contradiction is if Jupiter's density is significantly less than Earth's.

Page 275, problem 6:
Newton's law of gravity depends on the inverse square of the distance, so if the two planets' masses had been equal, then the factor of $0.83/0.059 = 14$ in distance would have caused the force on planet c to be $14^2 = 2.0 \times 10^2$ times weaker. However, planet c's mass is 3.0 times greater, so the force on it is only smaller by a factor of $2.0 \times 10^2/3.0 = 65$.

Page 278, problem 20:
Newton's law of gravity says $F = Gm_1m_2/r^2$, and Newton's second law says $F = m_2a$, so $Gm_1m_2/r^2 = m_2a$. Since m_2 cancels, a is independent of m_2.

Page 278, problem 21:
Newton's second law gives

$$F = m_D a_D \qquad ,$$

where F is Ida's force on Dactyl. Using Newton's universal law of gravity, F= $Gm_I m_D/r^2$,and the equation $a = v^2/r$ for circular motion, we find

$$Gm_I m_D/r^2 = m_D v^2/r.$$

Dactyl's mass cancels out, giving

$$Gm_I/r^2 = v^2/r.$$

Dactyl's velocity equals the circumference of its orbit divided by the time for one orbit: $v = 2\pi r/T$. Inserting this in the above equation and solving for m_I, we find

$$m_I = \frac{4\pi^2 r^3}{GT^2} \qquad ,$$

so Ida's density is

$$\begin{aligned} \rho &= m_I/V \\ &= \frac{4\pi^2 r^3}{GVT^2} \qquad . \end{aligned}$$

Page 278, problem 22:
Any fractional change in r results in double that amount of fractional change in $1/r^2$. For example, raising r by 1% causes $1/r^2$ to go down by very nearly 2%. A 27-day orbit is 1/13.5 of a year, so the fractional change in $1/r^2$ is

$$2 \times \frac{(1/13.5)\ \text{cm}}{3.84 \times 10^5\ \text{km}} \times \frac{1\ \text{km}}{10^5\ \text{cm}} = 4 \times 10^{-12}$$

Solutions for chapter 11

Page 309, problem 6:
A force is an interaction between two objects, so while the bullet is in the air, there is no force. There is only a force while the bullet is in contact with the book. There is energy the whole time, and the total amount doesn't change. The bullet has some kinetic energy, and transfers some of it to the book as heat, sound, and the energy required to tear a hole through the book.

Page 309, problem 7:
(a) The energy stored in the gasoline is being changed into heat via frictional heating, and also probably into sound and into energy of water waves. Note that the kinetic energy of the propeller and the boat are not changing, so they are not involved in the energy transformation. (b) The crusing speed would be greater by a factor of the cube root of 2, or about a 26% increase.

Page 309, problem 8:
We don't have actual masses and velocities to plug in to the equation, but that's OK. We just have to reason in terms of ratios and proportionalities. Kinetic energy is proportional to mass and to the square of velocity, so B's kinetic energy equals

$$(13.4\ \text{J})(3.77)/(2.34)^2 = 9.23\ \text{J}$$

Page 309, problem 11:
Room temperature is about 20°C. The fraction of the energy that actually goes into heating the water is

$$\frac{(250\text{ g})/(0.24\text{ g}\cdot{}^\circ\text{C/J})\times(100^\circ\text{C}-20^\circ\text{C})}{(1.25\times10^3\text{ J/s})\,(126\text{ s})}=0.53$$

So roughly half of the energy is wasted. The wasted energy might be in several forms: heating of the cup, heating of the oven itself, or leakage of microwaves from the oven.

Solutions for chapter 12

Page 327, problem 6:

$$\begin{aligned}
E_{total,i} &= E_{total,f}\\
PE_i+\text{heat}_i &= PE_f+KE_f+\text{heat}_f\\
\frac{1}{2}mv^2 &= PE_i-PE_f+\text{heat}_i-\text{heat}_f\\
&= -\Delta PE-\Delta\text{heat}\\
v &= \sqrt{2\left(\frac{-\Delta PE-\Delta\text{heat}}{m}\right)}\\
&= 6.4\text{ m/s}
\end{aligned}$$

Page 328, problem 10:
(a) Example: As one child goes up on one side of a see-saw, another child on the other side comes down. (b) Example: A pool ball hits another pool ball, and transfers some KE.

Page 328, problem 12:
Suppose the river is 1 m deep, 100 m wide, and flows at a speed of 10 m/s, and that the falls are 100 m tall. In 1 second, the volume of water flowing over the falls is 10^3 m^3, with a mass of 10^6 kg. The potential energy released in one second is $(10^6\text{ kg})(g)(100\text{ m})=10^9$ J, so the power is 10^9 W. A typical household might have 10 hundred-watt applicances turned on at any given time, so it consumes about 10^3 watts on the average. The plant could supply a about million households with electricity.

Page 329, problem 16:
Let θ be the angle by which he has progressed around the pipe. Conservation of energy gives

$$\begin{aligned}
E_{total,i} &= E_{total,f}\\
PE_i &= PE_f+KE_f
\end{aligned}$$

Let's make $PE=0$ at the top, so

$$0=mgr(\cos\theta-1)+\frac{1}{2}mv^2 \qquad .$$

While he is still in contact with the pipe, the radial component of his acceleration is

$$a_r=\frac{v^2}{r} \qquad ,$$

and making use of the previous equation we find

$$a_r=2g(1-\cos\theta) \qquad .$$

There are two forces on him, a normal force from the pipe and a downward gravitation force from the earth. At the moment when he loses contact with the pipe, the normal force is zero, so the radial component, $mg\cos\theta$, of the gravitational force must equal ma_r,

$$mg\cos\theta = 2mg(1-\cos\theta) \qquad ,$$

which gives

$$\cos\theta = \frac{2}{3} \qquad .$$

The amount by which he has dropped is $r(1-\cos\theta)$, which equals $r/3$ at this moment.

Solutions for chapter 13

Page 356, problem 4:

No. Work describes how energy was transferred by some process. It isn't a measurable property of a system.

Solutions for chapter 14

Page 392, problem 3:

By conservation of momentum, the total momenta of the pieces after the explosion is the same as the momentum of the firework before the explosion. However, there is no law of conservation of kinetic energy, only a law of conservation of energy. The chemical energy in the gunpowder is converted into heat and kinetic energy when it explodes. All we can say about the kinetic energy of the pieces is that their total is greater than the kinetic energy before the explosion.

Page 393, problem 8:

Let m be the mass of the little puck and $M = 2.3m$ be the mass of the big one. All we need to do is find the direction of the total momentum vector before the collision, because the total momentum vector is the same after the collision. Given the two components of the momentum vector $p_x = Mv$ and $p_y = mv$, the direction of the vector is $\tan^{-1}(p_y/p_x) = 23°$ counterclockwise from the big puck's original direction of motion.

Page 394, problem 12:

Momentum is a vector. The total momentum of the molecules is always zero, since the momenta in different directions cancal out on the average. Cooling changes individual molecular momenta, but not the total.

Page 394, problem 15:

(a) Particle i had velocity v_i in the center-of-mass frame, and has velocity $v_i + u$ in the new frame. The total kinetic energy is

$$\frac{1}{2}m_1\left(\mathbf{v}_1+\mathbf{u}\right)^2+\ldots \qquad ,$$

where "..." indicates that the sum continues for all the particles. Rewriting this in terms of the vector dot product, we have

$$\frac{1}{2}m_1\left(\mathbf{v}_1+\mathbf{u}\right)\cdot\left(\mathbf{v}_1+\mathbf{u}\right)+\ldots=\frac{1}{2}m_1\left(\mathbf{v}_1\cdot\mathbf{v}_1+2\mathbf{u}\cdot\mathbf{v}_1+\mathbf{u}\cdot\mathbf{u}\right)+\ldots \qquad .$$

When we add up all the terms like the first one, we get K_{cm}. Adding up all the terms like the third one, we get $M|\mathbf{u}|^2/2$. The terms like the second term cancel out:

$$m_1\mathbf{u}\cdot\mathbf{v}_1+\ldots=\mathbf{u}\cdot\left(m_1\mathbf{v}_1+\ldots\right) \qquad ,$$

where the sum in brackets equals the total momentum in the center-of-mass frame, which is zero by definition.
(b) Changing frames of reference doesn't change the distances between the particles, so the potential energies are all unaffected by the change of frames of reference. Suppose that in a given frame of reference, frame 1, energy is conserved in some process: the initial and final energies add up to be the same. First let's transform to the center-of-mass frame. The potential energies are unaffected by the transformation, and the total kinetic energy is simply reduced by the quantity $M|\mathbf{u}_1|^2/2$, where $\mathbf{u}_1$ is the velocity of frame 1 relative to the center of mass. Subtracting the same constant from the initial and final energies still leaves them equal. Now we transform to frame 2. Again, the effect is simply to change the initial and final energies by adding the same constant.

Page 394, problem 16:
A conservation law is about addition: it says that when you add up a certain thing, the total always stays the same. Funkosity would violate the additive nature of conservation laws, because a two-kilogram mass would have twice as much funkosity as a pair of one-kilogram masses moving at the same speed.

Solutions for chapter 15

Page 452, problem 8:
The pliers are not moving, so their angular momentum remains constant at zero, and the total torque on them must be zero. Not only that, but each half of the pliers must have zero total torque on it. This tells us that the magnitude of the torque at one end must be the same as that at the other end. The distance from the axis to the nut is about 2.5 cm, and the distance from the axis to the centers of the palm and fingers are about 8 cm. The angles are close enough to 90° that we can pretend they're 90 degrees, considering the rough nature of the other assumptions and measurements. The result is (300 N)(2.5 cm) = (F)(8 cm), or $F = 90$ N.

Page 458, problem 46:
The foot of the rod is moving in a circle relative to the center of the rod, with speed $v = \pi b/T$, and acceleration $v^2/(b/2) = (\pi^2/8)g$. This acceleration is initially upward, and is greater in magnitude than g, so the foot of the rod will lift off without dragging. We could also worry about whether the foot of the rod would make contact with the floor again before the rod finishes up flat on its back. This is a question that can be settled by graphing, or simply by inspection of figure aj on page 431. The key here is that the two parts of the acceleration are both independent of m and b, so the result is univeral, and it does suffice to check a graph in a single example. In practical terms, this tells us something about how difficult the trick is to do. Because $\pi^2/8 = 1.23$ isn't much greater than unity, a hit that is just a little too weak (by a factor of $1.23^{1/2} = 1.11$) will cause a fairly obvious qualitative change in the results. This is easily observed if you try it a few times with a pencil.

Answers to self-checks

Answers to self-checks for chapter 0

Page 13, self-check A:
If only he has the special powers, then his results can never be reproduced.

Page 15, self-check B:
They would have had to weigh the rays, or check for a loss of weight in the object from which

they were have emitted. (For technical reasons, this was not a measurement they could actually do, hence the opportunity for disagreement.)

Page 21, self-check C:
A dictionary might define "strong" as "possessing powerful muscles," but that's not an operational definition, because it doesn't say how to measure strength numerically. One possible operational definition would be the number of pounds a person can bench press.

Page 25, self-check D:
A microsecond is 1000 times longer than a nanosecond, so it would seem like 1000 seconds, or about 20 minutes.

Page 26, self-check E:
Exponents have to do with multiplication, not addition. The first line should be 100 times longer than the second, not just twice as long.

Page 29, self-check F:
The various estimates differ by 5 to 10 million. The CIA's estimate includes a ridiculous number of gratuitous significant figures. Does the CIA understand that every day, people in are born in, die in, immigrate to, and emigrate from Nigeria?

Page 29, self-check G:
(1) 4; (2) 2; (3) 2

Answers to self-checks for chapter 1

Page 38, self-check A:
$1\ \text{yd}^2 \times (3\ \text{ft}/1\ \text{yd})^2 = 9\ \text{ft}^2$
$1\ \text{yd}^3 \times (3\ \text{ft}/1\ \text{yd})^3 = 27\ \text{ft}^3$

Page 44, self-check B:
$C_1/C_2 = (w_1/w_2)^4$

Answers to self-checks for chapter 2

Page 63, self-check A:
Coasting on a bike and coasting on skates give one-dimensional center-of-mass motion, but running and pedaling require moving body parts up and down, which makes the center of mass move up and down. The only example of rigid-body motion is coasting on skates. (Coasting on a bike is not rigid-body motion, because the wheels twist.)

Page 63, self-check B:
By shifting his weight around, he can cause the center of mass not to coincide with the geometric center of the wheel.

Page 64, self-check C:
(1) a point in time; (2) time in the abstract sense; (3) a time interval

Page 66, self-check D:
Zero, because the "after" and "before" values of x are the same.

Page 71, self-check E:
(1) The effect only occurs during blastoff, when their velocity is changing. Once the rocket engines stop firing, their velocity stops changing, and they no longer feel any effect. (2) It is only an observable effect of your motion relative to the air.

Page 84, self-check F:

At $v = 0$, we get $\gamma = 1$, so $t = T$. There is no time distortion unless the two frames of reference are in relative motion.

Answers to self-checks for chapter 3

Page 99, self-check A:

Its speed increases at a steady rate, so in the next second it will travel 19 cm.

Answers to self-checks for chapter 4

Page 131, self-check A:

(1) The case of $\rho = 0$ represents an object falling in a vacuum, i.e., there is no density of air. The terminal velocity would be infinite. Physically, we know that an object falling in a vacuum would never stop speeding up, since there would be no force of air friction to cancel the force of gravity. (2) The 4-cm ball would have a mass that was greater by a factor of $4 \times 4 \times 4$, but its cross-sectional area would be greater by a factor of 4×4. Its terminal velocity would be greater by a factor of $\sqrt{4^3/4^2} = 2$. (3) It isn't of any general importance. It's just an example of one physical situation. You should not memorize it.

Page 134, self-check B:

(1) This is motion, not force. (2) This is a description of how the sub is able to get the water to produce a forward force on it. (3) The sub runs out of energy, not force.

Answers to self-checks for chapter 5

Page 155, self-check A:

The sprinter pushes backward against the ground, and by Newton's third law, the ground pushes forward on her. (Later in the race, she is no longer accelerating, but the ground's forward force is needed in order to cancel out the backward forces, such as air friction.)

Page 162, self-check B:

(1) It's kinetic friction, because her uniform is sliding over the dirt. (2) It's static friction, because even though the two surfaces are moving relative to the landscape, they're not slipping over each other. (3) Only kinetic friction creates heat, as when you rub your hands together. If you move your hands up and down together without sliding them across each other, no heat is produced by the static friction.

Page 163, self-check C:

By the POFOSTITO mnemonic, we know that each of the bird's forces on the trunk will be of the same type as the corresponding force of the tree on the bird, but in the opposite direction. The bird's feet make a normal force on the tree that is to the right and a static frictional force that is downward.

Page 163, self-check D:

Frictionless ice can certainly make a normal force, since otherwise a hockey puck would sink into the ice. Friction is not possible without a normal force, however: we can see this from the equation, or from common sense, e.g., while sliding down a rope you do not get any friction unless you grip the rope.

Page 164, self-check E:

(1) Normal forces are always perpendicular to the surface of contact, which means right or left in this figure. Normal forces are repulsive, so the cliff's force on the feet is to the right, i.e., away from the cliff. (2) Frictional forces are always parallel to the surface of contact, which means right or left in this figure. Static frictional forces are in the direction that would tend to keep

the surfaces from slipping over each other. If the wheel was going to slip, its surface would be moving to the left, so the static frictional force on the wheel must be in the direction that would prevent this, i.e., to the right. This makes sense, because it is the static frictional force that accelerates the dragster. (3) Normal forces are always perpendicular to the surface of contact. In this diagram, that means either up and to the left or down and to the right. Normal forces are repulsive, so the ball is pushing the bat away from itself. Therefore the ball's force is down and to the right on this diagram.

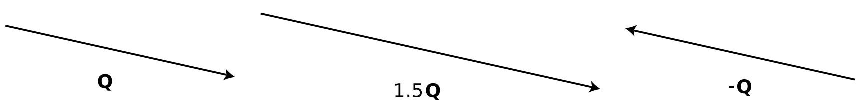
Answers to self-checks for chapter 6

Page 191, self-check A:
The wind increases the ball's overall speed. If you think about it in terms of overall speed, it's not so obvious that the increased speed is exactly sufficient to compensate for the greater distance. However, it becomes much simpler if you think about the forward motion and the sideways motion as two separate things. Suppose the ball is initially moving at one meter per second. Even if it picks up some sideways motion from the wind, it's still getting closer to the wall by one meter every second.

Answers to self-checks for chapter 7

Page 203, self-check A:
$\mathbf{v} = \Delta\mathbf{r}/\Delta t$

Page 203, self-check B:



Page 208, self-check C:
$\mathbf{A} - \mathbf{B}$ is equivalent to $\mathbf{A} + (-\mathbf{B})$, which can be calculated graphically by reversing $\mathbf{B}$ to form $-\mathbf{B}$, and then adding it to $\mathbf{A}$.

Answers to self-checks for chapter 8

Page 218, self-check A:
(1) It is speeding up, because the final velocity vector has the greater magnitude. (2) The result would be zero, which would make sense. (3) Speeding up produced a $\Delta\mathbf{v}$ vector in the same direction as the motion. Slowing down would have given a $\Delta\mathbf{v}$ that pointed backward.

Page 219, self-check B:
As we have already seen, the projectile has $a_x = 0$ and $a_y = -g$, so the acceleration vector is pointing straight down.

Answers to self-checks for chapter 9

Page 237, self-check A:
(1) Uniform. They have the same motion as the drum itself, which is rotating as one solid piece. No part of the drum can be rotating at a different speed from any other part. (2) Nonuniform. Gravity speeds it up on the way down and slows it down on the way up.

Answers to self-checks for chapter 10

Page 256, self-check A:
It would just stay where it was. Plugging $v = 0$ into eq. [1] would give $F = 0$, so it would not

accelerate from rest, and would never fall into the sun. No astronomer had ever observed an object that did that!

Page 257, self-check B:

$$F \propto mr/T^2 \propto mr/(r^{3/2})^2 \propto mr/r^3 = m/r^2$$

Page 260, self-check C:
The equal-area law makes equally good sense in the case of a hyperbolic orbit (and observations verify it). The elliptical orbit law had to be generalized by Newton to include hyperbolas. The law of periods doesn't make sense in the case of a hyperbolic orbit, because a hyperbola never closes back on itself, so the motion never repeats.

Page 265, self-check D:
Above you there is a small part of the shell, comprising only a tiny fraction of the earth's mass. This part pulls you up, while the whole remainder of the shell pulls you down. However, the part above you is extremely close, so it makes sense that its force on you would be far out of proportion to its small mass.

Answers to self-checks for chapter 11

Page 296, self-check A:
(1) A spring-loaded toy gun can cause a bullet to move, so the spring is capable of storing energy and then converting it into kinetic energy. (2) The amount of energy stored in the spring relates to the amount of compression, which can be measured with a ruler.

Answers to self-checks for chapter 12

Page 320, self-check A:
Both balls start from the same height and end at the same height, so they have the same Δy. This implies that their losses in potential energy are the same, so they must both have gained the same amount of kinetic energy.

Answers to self-checks for chapter 13

Page 334, self-check A:
Work is defined as the transfer of energy, so like energy it is a scalar with units of joules.

Page 337, self-check B:
Whenever energy is transferred out of the spring, the same amount has to be transferred into the ball, and vice versa. As the spring compresses, the ball is doing positive work on the spring (giving up its KE and transferring energy into the spring as PE), and as it decompresses the ball is doing negative work (extracting energy).

Page 340, self-check C:
(a) No. The pack is moving at constant velocity, so its kinetic energy is staying the same. It is only moving horizontally, so its gravitational potential energy is also staying the same. No energy transfer is occurring. (b) No. The horse's upward force on the pack forms a 90-degree angle with the direction of motion, so $\cos\theta = 0$, and no work is done.

Page 343, self-check D:
Only in (a) can we use Fd to calculate work. In (b) and (c), the force is changing as the distance changes.

Answers to self-checks for chapter 14

Page 387, self-check A:
When $m = 0$, we have $E = p$ (or $E = pc$, in units with $c \neq 1$), which is what we expect.

Answers to self-checks for chapter 15

Page 409, self-check A:
1, 2, and 4 all have the same sigm, because they are trying to twist the wrench clockwise. The sign of torque 3 is opposite to the signs of the others. The magnitude of torque 3 is the greatest, since it has a large r, and the force is nearly all perpendicular to the wrench. Torques 1 and 2 are the same because they have the same values of r and $F_\perp$. Torque 4 is the smallest, due to its small r.

Page 420, self-check B:
One person's θ-t graph would simply be shifted up or down relative to the others. The derivative equals the slope of the tangent line, and this slope isn't changed when you shift the graph, so both people would agree on the angular velocity.

Page 422, self-check C:
Reversing the direction of ω also reverses the direction of motion, and this is reflected by the relationship between the plus and minus signs. In the equation for the radial acceleration, ω is squared, so even if ω is negative, the result is positive. This makes sense because the acceleration is always inward in circular motion, never outward.

Page 434, self-check D:
All the rotations around the x axis give ω vectors along the positive x axis (thumb pointing along positive x), and all the rotations about the y axis have ω vectors with positive y components.

Page 437, self-check E:
For example, if we take $(\mathbf{A}\times\mathbf{B})_x = A_yB_z - B_yA_z$ and reverse the A's and B's, we get $(\mathbf{B}\times\mathbf{A})_x = B_yA_z - A_yB_z$, which is just the negative of the original expression.

Answers to self-checks for chapter 17

Page 486, self-check A:
The two graphs start off with the same amplitude, but the solid curve loses amplitude more rapidly. For a given time, t, the quantity e^{-ct} is apparently smaller for the solid curve, meaning that ct is greater. The solid curve has the higher value of c.

Page 492, self-check B:
She should tap the wine glasses she finds in the store and look for one with a high Q, i.e., one whose vibrations die out very slowly. The one with the highest Q will have the highest-amplitude response to her driving force, making it more likely to break.

Answers

Answers for chapter 1

Page 53, problem 10:
Check: The actual number of species of lupine occurring in the San Gabriels is 22. You should find that your answer comes out in the same ballpark as this figure, but not exactly the same, of course, because the scaling rule is only a generalization.

Answers for chapter 6

Page 199, problem 5:

(a) $R = (2v^2/g)\sin\theta\cos\theta$ (c) 45 °

Page 199, problem 5:
(a) $R = (2v^2/g)\sin\theta\cos\theta$ (c) 45 °

Answers for chapter 7

Page 214, problem 6:
(a) The optimal angle is about 40°, and the resulting range is about 124 meters, which is about the length of a home run. (b) It goes about 9 meters farther. For comparison with reality, the stadium's web site claims a home run goes about 11 meters farther there than in a sea-level stadium.

Page 214, problem 6:
(a) The optimal angle is about 40°, and the resulting range is about 124 meters, which is about the length of a home run. (b) It goes about 9 meters farther. For comparison with reality, the stadium's web site claims a home run goes about 11 meters farther there than in a sea-level stadium.

Answers for chapter 17

Page 503, problem 6:
$Q = 1/\sqrt{2}$

Photo credits

Except as specifically noted below or in a parenthetical credit in the caption of a figure, all the illustrations in this book are under my own copyright, and are copyleft licensed under the same license as the rest of the book.

In some cases it's clear from the date that the figure is public domain, but I don't know the name of the artist or photographer; I would be grateful to anyone who could help me to give proper credit. I have assumed that images that come from U.S. government web pages are copyright-free, since products of federal agencies fall into the public domain. I've included some public-domain paintings; photographic reproductions of them are not copyrightable in the U.S. (Bridgeman Art Library, Ltd. v. Corel Corp., 36 F. Supp. 2d 191, S.D.N.Y. 1999).

When "PSSC Physics" is given as a credit, it indicates that the figure is from the first edition of the textbook entitled Physics, by the Physical Science Study Committee. The early editions of these books never had their copyrights renewed, and are now therefore in the public domain. There is also a blanket permission given in the later PSSC College Physics edition, which states on the copyright page that "The materials taken from the original and second editions and the Advanced Topics of PSSC PHYSICS included in this text will be available to all publishers for use in English after December 31, 1970, and in translations after December 31, 1975."

Credits to Millikan and Gale refer to the textbooks Practical Physics (1920) and Elements of Physics (1927). Both are public domain. (The 1927 version did not have its copyright renewed.) Since it is possible that some of the illustrations in the 1927 version had their copyrights renewed and are still under copyright, I have only used them when it was clear that they were originally taken from public domain sources.

In a few cases, I have made use of images under the fair use doctrine. However, I am not a lawyer, and the laws on fair use are vague, so you should not assume that it's legal for you to use these images. In particular, fair use law may give you less leeway than it gives me, because I'm using the images for educational purposes, and giving the book away for free. Likewise, if the photo credit says "courtesy of ...," that means the copyright owner gave me permission to use it, but that doesn't mean you have permission to use it.

Cover Shukhov Tower: Wikipedia user Arssenev, CC-BY-SA licensed.

Contents See photo credits below, referring to the places where the images appear in the main text.

11 *Mars Climate Orbiter:* NASA/JPL/CIT. **22** *Standard kilogram:* Bo Bengtsen, GFDL licensed. Further retouching by Wikipedia user Greg L and by B. Crowell. **37** *Bee:* Wikipedia user Fir0002, CC-BY-SA licensed. **48** *Jar of jellybeans:* Flickr user cbgrfx123, CC-BY-SA licensed. **49** *Amphicoelias:* Wikimedia commons users Dinoguy2, Niczar, ArthurWeasley, Steveoc 86, Dropzink, and Piotr Jaworski, CC-BY-SA licensed. **53** *Galaxy:* ESO, CC-BY license. **54** *E. Coli bacteria:* Eric Erbe, digital colorization by Christopher Pooley, both of USDA, ARS, EMU. A public-domain product of the Agricultural Research Service.. **55** *Stacked oranges:* Wikimedia Commons user J.J. Harrison, CC-BY-SA license. **60** *Trapeze:* Calvert Litho. Co., Detroit, ca. 1890. **63** *High jumper:* Dunia Young. **63** *Gymnastics wheel:* Copyright Hans Genten, Aachen, Germany. "The copyright holder of this file allows anyone to use it for any purpose, provided that this remark is referenced or copied.". **70** *Rocket sled:* U.S. Air Force, public domain work of the U.S. Government. **70** *Shanghai:* Agnieszka Bojczuk, CC-BY-SA. **70** *Angel Stadium:* U.S. Marine Corps, Staff Sgt. Chad McMeen, public domain work of the U.S. Government. **70** *Aristotle:* Francesco Hayez, 1811. **70** *Jets over New York:* U.S. Air Force, Tech. Sgt. Sean Mateo White, public domain work of the U.S. Government. **74** *Flight path:* Wikimedia Commons user Mysid, public domain. **74** *Wreckage:* Agencia Brasil, CC-BY-2.5 Brasil. **74** *Pitot tube:* Wikimedia Commons user Kolossos, CC-BY-SA. **74** *Aircraft on runway:* Pawel Kierzkowski, CC-BY-SA. **75** *Atomic clock on plane:* Copyright 1971, Associated press, used under U.S. fair use exception to copyright law. **97** *Galileo's trial:* Cristiano Banti (1857). **103** *Gravity map:* US Navy, European Space Agency, D. Sandwell, and W. Smith. **112** *Astronaut jumping:* NASA. **114** *Flea jumping:* Burrows and Sutton, "Biomechanics of jumping in the flea," J. Exp. Biology 214 (2011) 836. Used under the U.S. fair use exception to copyright. **119** *Newton:* Godfrey Kneller, 1702. **154** *Space shuttle launch:* NASA. **155** *Swimmer:* Karen Blaha, CC-BY-SA licensed. **162** *Partridge:* Redrawn from K.P. Dial, "Wing-Assisted Incline Running and the Evolution of Flight," Science 299 (2003) 402. **165** *Turbulence:* C. Fukushima and J. Westerweel, Technical University of Delft, The Netherlands, CC-BY license. **165** *Series of vortices:* Wikimedia Commons user Onera, CC-BY license. **165** *Locomotive:* Locomotive Cyclopedia of American Practice, 1922, public domain. **165** *Wind tunnel:* Jeff Caplan/NASA Langley, public domain. **165** *Crop duster:* NASA Langley Research Center, public domain. **166** *Prius:* Wikimedia commons user IFCAR, public domain. **166** *Shark:* Wikimedia Commons user Pterantula, CC-BY-SA license. **166** *Golf ball:* Wikimedia Commons user Paolo Neo, CC-BY-SA license. **166** *Hummer:* Wikimedia commons user Bull-Doser, public domain. **168** *Dog:* From a photo by Wikimedia

Commons user Ron Armstrong, CC-BY licensed. **170** *Golden Gate Bridge:* Wikipedia user Dschwen, CC-BY-SA licensed. **176** *Ernst Mach:* Public domain in the U.S. since Mach died in 1916. **177** *Simplified Cavendish experiment:* Wikimedia commons user Chris Burks, public domain.. **177** *Kreuzer experiment:* Redrawn from Kreuzer, 1966.. **180** *Football player and old lady:* Hazel Abaya. **180** *Biplane:* Open Clip Art Library, public domain. **189** *Ring toss:* Clarence White, 1899. **201** *Aerial photo of Mondavi vineyards:* NASA. **215** *Galloping horse:* Eadweard Muybridge, 1878. **221** *Sled:* Modified from Millikan and Gale, 1920. **226** *Hurricane track:* Public domain, NASA and Wikipedia user Nilfanion. **229** *Hanging boy:* Millikan and Gale, 1927. **236** *Crane fly:* Wikipedia user Pinzo, public domain. **239** *Motorcyclist:* Wikipedia user Fir0002, CC-BY-SA licensed. **244** *Space colony:* NASA. **253** *Saturn:* Voyager 2 team, NASA. **254** *Tycho Brahe:* public domain. **259** *Pluto and Charon:* Hubble Space Telescope, STSCi. **266** *Simplified Cavendish experiment:* Wikimedia commons user Chris Burks, public domain.. **270** *WMAP:* NASA. **274** *Earth:* Apollo 11, NASA. **274** *Uranus:* Voyager 2 team, NASA. **276** *New Horizons trajectory:* Wikimedia commons user Martinw89, CC-BY-SA. **276** *New Horizons spacecraft image:* Wikipedia user NFRANGA, CC-BY-SA. **287** *Jupiter:* Images from the Hubble Space Telescope, NASA, not copyrighted. **292** *Hoover Dam:* U.S. Department of the Interior, Bureau of Reclamation, Lower Colorado Region, not copyrighted. **311** *Hydraulic ram:* Millikan and Gale, 1920. **313** *Bonfire, grapes:* CC-BY-SA licensed, by Wikipedia user Fir0002. **316** *Skater in pool:* Courtesy of J.D. Rogge. **321** *Plutonium pellet:* U.S. Department of Energy, public domain.. **324** *Ring of detectors in PET scanner:* Wikipedia user Damato, public domain. **324** *PET body scan:* Jens Langner, public domain. **324** *Photo of PET scanner:* Wikipedia user Hg6996, public domain. **325** *Eclipse:* 1919, public domain. **325** *Newspaper headline:* 1919, public domain. **329** *Skateboarder on top of pipe:* Oula Lehtinen, Wikimedia Commons, CC-BY-SA. **335** *Baseball pitch:* Wikipedia user Rick Dikeman, CC-BY-SA. **338** *Woman doing pull-ups:* Sergio Savarese, CC-BY. **338** *Male gymnast:* Wikipedia user Gonzo-wiki, public domain. **340** *Breaking Trail:* Art by Walter E. Bohl. Image courtesy of the University of Michigan Museum of Art/School of Information and Library Studies. **367** *Deep Space 1 engine:* NASA. **368** *Nucleus of Halley's comet:* NASA, not copyrighted. **374** *Chadwick's apparatus:* Redrawn from the public-domain figure in Chadwick's original paper. **375** *Wrench:* PSSC Physics. **377** *Jupiter:* Uncopyrighted image from the Voyager probe. Line art by the author. **379** *Air bag:* DaimlerChrysler AG, CC-BY-SA licensed.. **397** *Tornado:* NOAA Photo Library, NOAA Central Library; OAR/ERL/National Severe Storms Laboratory (NSSL); public-domain product of the U.S. government. **398** *Longjump:* Thomas Eakins, public domain. **405** *Pendulum:* PSSC Physics. **406** *Diver:* PSSC Physics. **411** *Cow:* Drawn by the author, from a CC-BY-SA-licensed photo on commons.wikimedia.org by user B.navez.. **414** *Modern windmill farm, Tehachapi, CA:* U.S. Department of Energy, not copyrighted. **414** *Old-fashioned windmill:* Photo by the author. **417** *Ballerina:* Rick Dikeman, 1981, CC-BY-SA license, www.gnu.org/copyleft/fdl.html, from the Wikipedia article on ballet (retouched by B. Crowell). **451** *White dwarf:* Image of NGC 2440 from the Hubble Space Telescope, H. Bond and R. Ciardullo. **465** *Locomotive linkages:* David Ingham, CC-BY-SA licensed.. **469** *Jupiter:* Uncopyrighted image from the Voyager probe. Line art by the author. **481** *Tacoma Narrows Bridge:* Public domain, from Stillman Fires Collection: Tacoma Fire Dept, www.archive.org. **489** *Nimitz Freeway:* Unknown photographer, courtesy of the UC Berkeley Earth Sciences and Map Library. **494** *Three-dimensional brain:* R. Malladi, LBNL. **494** *Two-dimensional MRI:* Image of the author's wife. **503** *Spider oscillations:* Emile, Le Floch, and Vollrath, *Nature* 440 (2006) 621.

Index

Trig Table

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0.000	1.000	0.000	30°	0.500	0.866	0.577	60°	0.866	0.500	1.732
1°	0.017	1.000	0.017	31°	0.515	0.857	0.601	61°	0.875	0.485	1.804
2°	0.035	0.999	0.035	32°	0.530	0.848	0.625	62°	0.883	0.469	1.881
3°	0.052	0.999	0.052	33°	0.545	0.839	0.649	63°	0.891	0.454	1.963
4°	0.070	0.998	0.070	34°	0.559	0.829	0.675	64°	0.899	0.438	2.050
5°	0.087	0.996	0.087	35°	0.574	0.819	0.700	65°	0.906	0.423	2.145
6°	0.105	0.995	0.105	36°	0.588	0.809	0.727	66°	0.914	0.407	2.246
7°	0.122	0.993	0.123	37°	0.602	0.799	0.754	67°	0.921	0.391	2.356
8°	0.139	0.990	0.141	38°	0.616	0.788	0.781	68°	0.927	0.375	2.475
9°	0.156	0.988	0.158	39°	0.629	0.777	0.810	69°	0.934	0.358	2.605
10°	0.174	0.985	0.176	40°	0.643	0.766	0.839	70°	0.940	0.342	2.747
11°	0.191	0.982	0.194	41°	0.656	0.755	0.869	71°	0.946	0.326	2.904
12°	0.208	0.978	0.213	42°	0.669	0.743	0.900	72°	0.951	0.309	3.078
13°	0.225	0.974	0.231	43°	0.682	0.731	0.933	73°	0.956	0.292	3.271
14°	0.242	0.970	0.249	44°	0.695	0.719	0.966	74°	0.961	0.276	3.487
15°	0.259	0.966	0.268	45°	0.707	0.707	1.000	75°	0.966	0.259	3.732
16°	0.276	0.961	0.287	46°	0.719	0.695	1.036	76°	0.970	0.242	4.011
17°	0.292	0.956	0.306	47°	0.731	0.682	1.072	77°	0.974	0.225	4.331
18°	0.309	0.951	0.325	48°	0.743	0.669	1.111	78°	0.978	0.208	4.705
19°	0.326	0.946	0.344	49°	0.755	0.656	1.150	79°	0.982	0.191	5.145
20°	0.342	0.940	0.364	50°	0.766	0.643	1.192	80°	0.985	0.174	5.671
21°	0.358	0.934	0.384	51°	0.777	0.629	1.235	81°	0.988	0.156	6.314
22°	0.375	0.927	0.404	52°	0.788	0.616	1.280	82°	0.990	0.139	7.115
23°	0.391	0.921	0.424	53°	0.799	0.602	1.327	83°	0.993	0.122	8.144
24°	0.407	0.914	0.445	54°	0.809	0.588	1.376	84°	0.995	0.105	9.514
25°	0.423	0.906	0.466	55°	0.819	0.574	1.428	85°	0.996	0.087	11.430
26°	0.438	0.899	0.488	56°	0.829	0.559	1.483	86°	0.998	0.070	14.301
27°	0.454	0.891	0.510	57°	0.839	0.545	1.540	87°	0.999	0.052	19.081
28°	0.469	0.883	0.532	58°	0.848	0.530	1.600	88°	0.999	0.035	28.636
29°	0.485	0.875	0.554	59°	0.857	0.515	1.664	89°	1.000	0.017	57.290
								90°	1.000	0.000	∞

Mathematical Review

Algebra

Quadratic equation:

The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Logarithms and exponentials:

$$\ln(ab) = \ln a + \ln b$$

$$e^{a+b} = e^a e^b$$

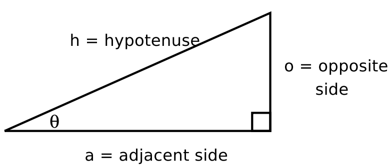
$$\ln e^x = e^{\ln x} = x$$

$$\ln(a^b) = b \ln a$$

Geometry, area, and volume

area of a triangle of base b and height h	$= \frac{1}{2}bh$
circumference of a circle of radius r	$= 2\pi r$
area of a circle of radius r	$= \pi r^2$
surface area of a sphere of radius r	$= 4\pi r^2$
volume of a sphere of radius r	$= \frac{4}{3}\pi r^3$

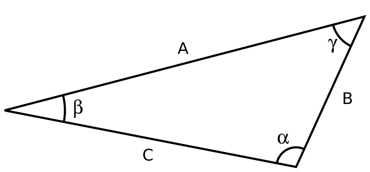
Trigonometry with a right triangle



$$\sin\theta = o/h \quad \cos\theta = a/h \quad \tan\theta = o/a$$

Pythagorean theorem: $h^2 = a^2 + o^2$

Trigonometry with any triangle



Law of Sines:

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Law of Cosines:

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

Properties of the derivative and integral (for students in calculus-based courses)

Let f and g be functions of x, and let c be a constant.

Linearity of the derivative:

$$\frac{\mathrm{d}}{\mathrm{d}x}(cf) = c\frac{\mathrm{d}f}{\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(f+g) = \frac{\mathrm{d}f}{\mathrm{d}x} + \frac{\mathrm{d}g}{\mathrm{d}x}$$

The chain rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$$

Derivatives of products and quotients:

$$\frac{\mathrm{d}}{\mathrm{d}x}(fg) = \frac{\mathrm{d}f}{\mathrm{d}x}g + \frac{\mathrm{d}g}{\mathrm{d}x}f$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f}{g}\right) = \frac{f'}{g} - \frac{fg'}{g^2}$$

Some derivatives:

$\frac{\mathrm{d}}{\mathrm{d}x}x^m = mx^{m-1}$, except for $m = 0$

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ $\quad$ $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

$\frac{\mathrm{d}}{\mathrm{d}x}e^x = e^x$ $\quad$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$

The fundamental theorem of calculus:

$$\int \frac{\mathrm{d}f}{\mathrm{d}x}\mathrm{d}x = f$$

Linearity of the integral:

$$\int cf(x)\mathrm{d}x = c\int f(x)\mathrm{d}x$$

$$\int [f(x) + g(x)] = \int f(x)\mathrm{d}x + \int g(x)\mathrm{d}x$$

Integration by parts:

$$\int f\mathrm{d}g = fg - \int g\mathrm{d}f$$

Useful Data

Metric Prefixes

M-	mega-	10^6
k-	kilo-	10^3
m-	milli-	10^{-3}
μ- (Greek mu)	micro-	10^{-6}
n-	nano-	10^{-9}
p-	pico-	10^{-12}
f-	femto-	10^{-15}

(Centi-, 10^{-2}, is used only in the centimeter.)

The Greek Alphabet

α	A	alpha	ν	N	nu
β	B	beta	ξ	Ξ	xi
γ	Γ	gamma	o	O	omicron
δ	Δ	delta	π	Π	pi
ϵ	E	epsilon	ρ	P	rho
ζ	Z	zeta	σ	Σ	sigma
η	H	eta	τ	T	tau
θ	Θ	theta	υ	Y	upsilon
ι	I	iota	ϕ	Φ	phi
κ	K	kappa	χ	X	chi
λ	Λ	lambda	ψ	Ψ	psi
μ	M	mu	ω	Ω	omega

Subatomic Particles

particle	**mass (kg)**	**radius (fm)**
electron	9.109×10^{-31}	$\lesssim 0.01$
proton	1.673×10^{-27}	~ 1.1
neutron	1.675×10^{-27}	~ 1.1

The radii of protons and neutrons can only be given approximately, since they have fuzzy surfaces. For comparison, a typical atom is about a million fm in radius.

Notation and Units

quantity	**unit**	**symbol**
distance	meter, m	$x, \Delta x$
time	second, s	$t, \Delta t$
mass	kilogram, kg	m
density	kg/m^3	ρ
velocity	m/s	$\mathbf{v}$
acceleration	m/s^2	$\mathbf{a}$
force	N = kg·m/s^2	$\mathbf{F}$
pressure	Pa=1 N/m^2	P
energy	J = kg·m^2/s^2	E
power	W = 1 J/s	P
momentum	kg·m/s	$\mathbf{p}$
angular momentum	kg·m^2/s or J·s	$\mathbf{L}$
period	s	T
wavelength	m	λ
frequency	s^{-1} or Hz	f
gamma factor	unitless	γ
probability	unitless	P
prob. distribution	various	D
electron wavefunction	m$^{-3/2}$	Ψ

Earth, Moon, and Sun

body	**mass (kg)**	**radius (km)**	**radius of orbit (km)**
earth	5.97×10^{24}	6.4×10^3	1.49×10^8
moon	7.35×10^{22}	1.7×10^3	3.84×10^5
sun	1.99×10^{30}	7.0×10^5	—

Fundamental Constants

gravitational constant	$G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$
Coulomb constant	$k = 8.99 \times 10^9\ \text{N·m}^2/\text{C}^2$
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s