16  (a) If the earth was of uniform density, would your weight be increased or decreased at the bottom of a mine shaft? Explain.
(b) In real life, objects weigh slightly more at the bottom of a mine shaft. What does that allow us to infer about the Earth?

17  (a) A geosynchronous orbit is one in which the satellite orbits above the equator, and has an orbital period of 24 hours, so that it is always above the same point on the spinning earth. Calculate the altitude of such a satellite.
(b) What is the gravitational field experienced by the satellite? Give your answer as a percentage in relation to the gravitational field at the earth’s surface.

18  If a bullet is shot straight up at a high enough velocity, it will never return to the earth. This is known as the escape velocity. We will discuss escape velocity using the concept of energy later in the course, but it can also be gotten at using straightforward calculus. In this problem, you will analyze the motion of an object of mass \(m\) whose initial velocity is exactly equal to escape velocity. We assume that it is starting from the surface of a spherically symmetric planet of mass \(M\) and radius \(b\). The trick is to guess at the general form of the solution, and then determine the solution in more detail. Assume (as is true) that the solution is of the form \(r = kt^p\), where \(r\) is the object’s distance from the center of the planet at time \(t\), and \(k\) and \(p\) are constants.
(a) Find the acceleration, and use Newton’s second law and Newton’s law of gravity to determine \(k\) and \(p\). You should find that the result is independent of \(m\).
(b) What happens to the velocity as \(t\) approaches infinity?
(c) Determine escape velocity from the Earth’s surface.

19  (a) Suppose a rotating spherical body such as a planet has a radius \(r\) and a uniform density \(\rho\), and the time required for one rotation is \(T\). At the surface of the planet, the apparent acceleration of a falling object is reduced by the acceleration of the ground out from under it. Derive an equation for the apparent acceleration of gravity, \(g\), at the equator in terms of \(r\), \(\rho\), \(T\), and \(G\).
(b) Applying your equation from a, by what fraction is your apparent weight reduced at the equator compared to the poles, due to the Earth’s rotation?
(c) Using your equation from a, derive an equation giving the value of \(T\) for which the apparent acceleration of gravity becomes zero, i.e., objects can spontaneously drift off the surface of the planet. Show that \(T\) only depends on \(\rho\), and not on \(r\).
(d) Applying your equation from c, how long would a day have to be in order to reduce the apparent weight of objects at the equator of the Earth to zero? [Answer: 1.4 hours]
(e) Astronomers have discovered objects they called pulsars, which emit bursts of radiation at regular intervals of less than a second.
If a pulsar is to be interpreted as a rotating sphere beaming out a natural “searchlight” that sweeps past the earth with each rotation, use your equation from c to show that its density would have to be much greater than that of ordinary matter.

(f) Astrophysicists predicted decades ago that certain stars that used up their sources of energy could collapse, forming a ball of neutrons with the fantastic density of \( \sim 10^{17} \text{ kg/m}^3 \). If this is what pulsars really are, use your equation from c to explain why no pulsar has ever been observed that flashes with a period of less than 1 ms or so.

20

Prove, based on Newton’s laws of motion and Newton’s law of gravity, that all falling objects have the same acceleration if they are dropped at the same location on the earth and if other forces such as friction are unimportant. Do not just say, “\( g = 9.8 \text{ m/s}^2 \) – it’s constant.” You are supposed to be proving that \( g \) should be the same number for all objects.

\( \triangleright \) Solution, p. 561

21

The figure shows an image from the Galileo space probe taken during its August 1993 flyby of the asteroid Ida. Astronomers were surprised when Galileo detected a smaller object orbiting Ida. This smaller object, the only known satellite of an asteroid in our solar system, was christened Dactyl, after the mythical creatures who lived on Mount Ida, and who protected the infant Zeus. For scale, Ida is about the size and shape of Orange County, and Dactyl the size of a college campus. Galileo was unfortunately unable to measure the time, \( T \), required for Dactyl to orbit Ida. If it had, astronomers would have been able to make the first accurate determination of the mass and density of an asteroid. Find an equation for the density, \( \rho \), of Ida in terms of Ida’s known volume, \( V \), the known radius, \( r \), of Dactyl’s orbit, and the lamentably unknown variable \( T \). (This is the same technique that was used successfully for determining the masses and densities of the planets that have moons.)

\( \triangleright \) Solution, p. 561

22

As discussed in more detail in example 3 on p. 436, tidal interactions with the earth are causing the moon’s orbit to grow gradually larger. Laser beams bounced off of a mirror left on the moon by astronauts have allowed a measurement of the moon’s rate of recession, which is about 4 cm per year. This means that the gravitational force acting between earth and moon is decreasing. By what fraction does the force decrease with each 27-day orbit?

[Based on a problem by Arnold Arons.]

\( \triangleright \) Hint, p. 550 \( \triangleright \) Solution, p. 562
Astronomers calculating orbits of planets often work in a nonmetric system of units, in which the unit of time is the year, the unit of mass is the sun’s mass, and the unit of distance is the astronomical unit (A.U.), defined as half the long axis of the earth’s orbit. In these units, find an exact expression for the gravitational constant, \( G \).

Suppose that we inhabited a universe in which, instead of Newton’s law of gravity, we had \( F = k\sqrt{m_1m_2}/r^2 \), where \( k \) is some constant with different units than \( G \). (The force is still attractive.) However, we assume that \( a = F/m \) and the rest of Newtonian physics remains true, and we use \( a = F/m \) to define our mass scale, so that, e.g., a mass of 2 kg is one which exhibits half the acceleration when the same force is applied to it as to a 1 kg mass.

(a) Is this new law of gravity consistent with Newton’s third law?
(b) Suppose you lived in such a universe, and you dropped two unequal masses side by side. What would happen?
(c) Numerically, suppose a 1.0-kg object falls with an acceleration of 10 m/s\(^2\). What would be the acceleration of a rain drop with a mass of 0.1 g? Would you want to go out in the rain?
(d) If a falling object broke into two unequal pieces while it fell, what would happen?
(e) Invent a law of gravity that results in behavior that is the opposite of what you found in part b. [Based on a problem by Arnold Arons.]

The structures that we see in the universe, such as solar systems, galaxies, and clusters of galaxies, are believed to have condensed from clumps that formed, due to gravitational attraction, in preexisting clouds of gas and dust. Observations of the cosmic microwave background radiation (p. 294) suggest that the mixture of hot hydrogen and helium that existed soon after the Big Bang was extremely uniform, but not perfectly so. We can imagine that any region that started out a little more dense would form a natural center for the collapse of a clump. Suppose that we have a spherical region with density \( \rho \) and radius \( r \), and for simplicity let’s just assume that it’s surrounded by vacuum. (a) Find the acceleration of the material at the edge of the cloud. To what power of \( r \) is it proportional?
(b) The cloud will take a time \( t \) to collapse to some fraction of its original size. Show that \( t \) is independent of \( r \).

Remark: This result suggests that structures would get a chance to form at all scales in the universe. That is, solar systems would not form before galaxies got to, or vice versa. It is therefore physically natural that when we look at the universe at essentially all scales less than a billion light-years, we see structure.
26 You have a fixed amount of material with a fixed density. If the material is formed into some shape $S$, then there will be some point in space at which the resulting gravitational field attains its maximum value $g_S$. What shape maximizes $g_S$? ∗

27 Complete the proof of the shell theorem in section 10.7 by filling in the case where $m$ is inside the shell.

28 The shell theorem was proved in section 10.7. Prove that the theorem fails if the exponent of $r$ in Newton’s law of gravity differs from $-2$.

29 The shell theorem describes two cases, inside and outside. Show that for an alternative law of gravity $F = GMmr$ (with $r^1$ rather than $r^{-2}$), the outside case still holds.

30 On an airless body such as the moon, there is no atmospheric friction, so it should be possible for a satellite to orbit at a very low altitude, just high enough to keep from hitting the mountains. (a) Suppose that such a body is a smooth sphere of uniform density $\rho$ and radius $r$. Find the velocity required for a ground-skimming orbit. √

(b) A typical asteroid has a density of about 2 g/cm³, i.e., twice that of water. (This is a lot lower than the density of the earth’s crust, probably indicating that the low gravity is not enough to compact the material very tightly, leaving lots of empty space inside.) Suppose that it is possible for an astronaut in a spacesuit to jump at 2 m/s. Find the radius of the largest asteroid on which it would be possible to jump into a ground-skimming orbit. √
31 The figure shows a region of outer space in which two stars have exploded, leaving behind two overlapping spherical shells of gas, which we assume to remain at rest. The figure is a cross-section in a plane containing the shells’ centers. A space probe is released with a very small initial speed at the point indicated by the arrow, initially moving in the direction indicated by the dashed line. Without any further information, predict as much as possible about the path followed by the probe and its changes in speed along that path.

32 Approximate the earth’s density as being constant. (a) Find the gravitational field at a point P inside the earth and half-way between the center and the surface. Express your result as a ratio \( gp/gs \) relative to the field we experience at the surface. (b) As a check on your answer, make sure that the same reasoning leads to a reasonable result when the fraction 1/2 is replaced by the value 0 (P being the earth’s center) or the value 1 (P being a point on the surface).
The earth is divided into solid inner core, a liquid outer core, and a plastic mantle. Physical properties such as density change discontinuously at the boundaries between one layer and the next. Although the density is not completely constant within each region, we will approximate it as being so for the purposes of this problem. (We neglect the crust as well.) Let $R$ be the radius of the earth as a whole and $M$ its mass. The following table gives a model of some properties of the three layers, as determined by methods such as the observation of earthquake waves that have propagated from one side of the planet to the other.

<table>
<thead>
<tr>
<th>region</th>
<th>outer radius/R</th>
<th>mass/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>mantle</td>
<td>1</td>
<td>0.69</td>
</tr>
<tr>
<td>outer core</td>
<td>0.55</td>
<td>0.29</td>
</tr>
<tr>
<td>inner core</td>
<td>0.19</td>
<td>0.017</td>
</tr>
</tbody>
</table>

The boundary between the mantle and the outer core is referred to as the Gutenberg discontinuity. Let $g_s$ be the strength of the earth’s gravitational field at its surface and $g_G$ its value at the Gutenberg discontinuity. Find $g_G/g_s$.

The figure shows the International Space Station (ISS). One of the purposes of the ISS is supposed to be to carry out experiments in microgravity. However, the following factor limits this application. The ISS orbits the earth once every 92.6 minutes. It is desirable to keep the same side of the station always oriented toward the earth, which means that the station has to rotate with the same period. In the photo, the direction of orbital motion is left or right on the page, so the rotation is about the axis shown as up and down on the page. The greatest distance of any pressurized compartment from the axis of rotation is 36.5 meters. Find the acceleration due to the rotation at this point, and the apparent weight of a 60 kg astronaut at that location.

Problems 35-37 all investigate the following idea. Cosmological surveys at the largest observable distance scales have detected structures like filaments. As an idealization of such a structure, consider a uniform mass distribution lying along the entire $x$ axis, with mass density $\lambda$ in units of kg/m. The purpose of this problem is to find the gravitational field created by this structure at a distance $y$.

(a) Determine as much as possible about the form of the solution, based on units.

(b) To evaluate the actual result, find the contribution $dg_y$ to the $y$ component of the field arising from the mass $dm$ lying between $x$ and $x + dx$, then integrate it.

$\triangleright$ Solution, p. 562
Let us slightly change the physical situation described in problem 35, letting the filament have a finite size, while retaining its symmetry under rotation about the $x$ axis. The details don’t actually matter very much for our purposes, but if we like, we can take the mass density to be constant within a cylinder of radius $b$ centered on the $x$ axis. Now consider the following two limits:

$$g_1 = \lim_{y \to 0} \lim_{b \to 0} g$$

and

$$g_2 = \lim_{b \to 0} \lim_{y \to 0} g.$$

Each of these is a limit inside another limit, the only difference being the order of the limits. Either of these could be used as a definition of the field at a point on an infinitely thin filament. Do they agree? ⋆

Suppose we have a mass filament like the one described in problems 35 and 36, but now rather than taking it to be straight, let it have the shape of an arbitrary smooth curve. Locally, “under a microscope,” this curve will look like an arc of a circle, i.e., we can describe its shape solely in terms of a radius of curvature. As in problem 36, consider a point $P$ lying on the filament itself, taking $g$ to be defined as in definition $g_1$. Investigate whether $g$ is finite, and also whether it points in a specific direction. To clarify the mathematical idea, consider the following two limits:

$$A = \lim_{x \to 0} \frac{1}{x}$$

and

$$B = \lim_{x \to 0} \frac{1}{x^2}.$$

We say that $A = \infty$, while $B = +\infty$, i.e., both diverge, but $B$ diverges with a definite sign. For a straight filament, as in problem 35, with an infinite radius of curvature, symmetry guarantees that the field at $P$ has no specific direction, in analogy with limit $A$. For a curved filament, a calculation is required in order to determine whether we get behavior $A$ or $B$. Based on your result, what is the expected dynamical behavior of such a filament? ⋆
Exercise 10: The shell theorem

This exercise is an approximate numerical test of the shell theorem. There are seven masses A-G, each being one kilogram. Masses A-F, each one meter from the center, form a shape like two Egyptian pyramids joined at their bases; this is a rough approximation to a six-kilogram spherical shell of mass. Mass G is five meters from the center of the main group. The class will divide into six groups and split up the work required in order to calculate the vector sum of the six gravitational forces exerted on mass G. Depending on the size of the class, more than one group may be assigned to deal with the contribution of the same mass to the total force, and the redundant groups can check each other’s results.

1. Discuss as a class what can be done to simplify the task of calculating the vector sum, and how to organize things so that each group can work in parallel with the others.

2. Each group should write its results on the board in units of piconewtons, retaining five significant figures of precision. Everyone will need to use the same value for the gravitational constant, \( G = 6.6743 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \).

3. The class will determine the vector sum and compare with the result that would be obtained with the shell theorem.
Conservation Laws
In July of 1994, Comet Shoemaker-Levy struck the planet Jupiter, depositing $7 \times 10^{22}$ joules of energy, and incidentally giving rise to a series of Hollywood movies in which our own planet is threatened by an impact by a comet or asteroid. There is evidence that such an impact caused the extinction of the dinosaurs. Left: Jupiter's gravitational force on the near side of the comet was greater than on the far side, and this difference in force tore up the comet into a string of fragments. Two separate telescope images have been combined to create the illusion of a point of view just behind the comet. (The colored fringes at the edges of Jupiter are artifacts of the imaging system.) Top: A series of images of the plume of superheated gas kicked up by the impact of one of the fragments. The plume is about the size of North America. Bottom: An image after all the impacts were over, showing the damage done.

Chapter 11

Conservation of Energy

11.1 The search for a perpetual motion machine

Don’t underestimate greed and laziness as forces for progress. Modern chemistry was born from the collision of lust for gold with distaste for the hard work of finding it and digging it up. Failed efforts by generations of alchemists to turn lead into gold led finally to the conclusion that it could not be done: certain substances, the chem-
ical elements, are fundamental, and chemical reactions can neither increase nor decrease the amount of an element such as gold.

Now flash forward to the early industrial age. Greed and laziness have created the factory, the train, and the ocean liner, but in each of these is a boiler room where someone gets sweaty shoveling the coal to fuel the steam engine. Generations of inventors have tried to create a machine, called a perpetual motion machine, that would run forever without fuel. Such a machine is not forbidden by Newton’s laws of motion, which are built around the concepts of force and inertia. Force is free, and can be multiplied indefinitely with pulleys, gears, or levers. The principle of inertia seems even to encourage the belief that a cleverly constructed machine might not ever run down.

Figures a and b show two of the innumerable perpetual motion machines that have been proposed. The reason these two examples don’t work is not much different from the reason all the others have failed. Consider machine a. Even if we assume that a properly shaped ramp would keep the ball rolling smoothly through each cycle, friction would always be at work. The designer imagined that the machine would repeat the same motion over and over again, so that every time it reached a given point its speed would be exactly the same as the last time. But because of friction, the speed would actually be reduced a little with each cycle, until finally the ball would no longer be able to make it over the top.

Friction has a way of creeping into all moving systems. The rotating earth might seem like a perfect perpetual motion machine, since it is isolated in the vacuum of outer space with nothing to exert frictional forces on it. But in fact our planet’s rotation has slowed drastically since it first formed, and the earth continues to slow its rotation, making today just a little longer than yesterday. The very subtle source of friction is the tides. The moon’s gravity raises bulges in the earth’s oceans, and as the earth rotates the bulges progress around the planet. Where the bulges encounter land, there is friction, which slows the earth’s rotation very gradually.

11.2 Energy

The analysis based on friction is somewhat superficial, however. One could understand friction perfectly well and yet imagine the following situation. Astronauts bring back a piece of magnetic ore from the moon which does not behave like ordinary magnets. A normal bar magnet, c/1, attracts a piece of iron essentially directly toward it, and has no left- or right-handedness. The moon rock, however, exerts forces that form a whirlpool pattern around it, 2. NASA goes to a machine shop and has the moon rock put in a lathe and machined down to a smooth cylinder, 3. If we now release a ball bearing on the surface of the cylinder, the magnetic force whips it
around and around at ever higher speeds. Of course there is some friction, but there is a net gain in speed with each revolution.

Physicists would lay long odds against the discovery of such a moon rock, not just because it breaks the rules that magnets normally obey but because, like the alchemists, they have discovered a very deep and fundamental principle of nature which forbids certain things from happening. The first alchemist who deserved to be called a chemist was the one who realized one day, “In all these attempts to create gold where there was none before, all I’ve been doing is shuffling the same atoms back and forth among different test tubes. The only way to increase the amount of gold in my laboratory is to bring some in through the door.” It was like having some of your money in a checking account and some in a savings account. Transferring money from one account into the other doesn’t change the total amount.

We say that the number of grams of gold is a conserved quantity. In this context, the word “conserve” does not have its usual meaning of trying not to waste something. In physics, a conserved quantity is something that you wouldn’t be able to get rid of even if you wanted to. Conservation laws in physics always refer to a closed system, meaning a region of space with boundaries through which the quantity in question is not passing. In our example, the alchemist’s laboratory is a closed system because no gold is coming in or out through the doors.

Conservation of mass example 1

In figure d, the stream of water is fatter near the mouth of the faucet, and skinnier lower down. This is because the water speeds up as it falls. If the cross-sectional area of the stream was equal all along its length, then the rate of flow through a lower cross-section would be greater than the rate of flow through a cross-section higher up. Since the flow is steady, the amount of water between the two cross-sections stays constant. The cross-sectional area of the stream must therefore shrink in inverse proportion to the increasing speed of the falling water. This is an example of conservation of mass.

In general, the amount of any particular substance is not conserved. Chemical reactions can change one substance into another, and nuclear reactions can even change one element into another. The total mass of all substances is however conserved:

the law of conservation of mass

The total mass of a closed system always remains constant. Mass cannot be created or destroyed, but only transferred from one system to another.

A similar lightbulb eventually lit up in the heads of the people
who had been frustrated trying to build a perpetual motion machine. In perpetual motion machine a, consider the motion of one of the balls. It performs a cycle of rising and falling. On the way down it gains speed, and coming up it slows back down. Having a greater speed is like having more money in your checking account, and being high up is like having more in your savings account. The device is simply shuffling funds back and forth between the two. Having more balls doesn’t change anything fundamentally. Not only that, but friction is always draining off money into a third “bank account:” heat. The reason we rub our hands together when we’re cold is that kinetic friction heats things up. The continual buildup in the “heat account” leaves less and less for the “motion account” and “height account,” causing the machine eventually to run down.

These insights can be distilled into the following basic principle of physics:

**the law of conservation of energy**

It is possible to give a numerical rating, called energy, to the state of a physical system. The total energy is found by adding up contributions from characteristics of the system such as motion of objects in it, heating of the objects, and the relative positions of objects that interact via forces. The total energy of a closed system always remains constant. Energy cannot be created or destroyed, but only transferred from one system to another.

The moon rock story violates conservation of energy because the rock-cylinder and the ball together constitute a closed system. Once the ball has made one revolution around the cylinder, its position relative to the cylinder is exactly the same as before, so the numerical energy rating associated with its position is the same as before. Since the total amount of energy must remain constant, it is impossible for the ball to have a greater speed after one revolution. If it had picked up speed, it would have more energy associated with motion, the same amount of energy associated with position, and a little more energy associated with heating through friction. There cannot be a net increase in energy.

*Converting one form of energy to another* example 2

**Dropping a rock:** The rock loses energy because of its changing position with respect to the earth. Nearly all that energy is transformed into energy of motion, except for a small amount lost to heat created by air friction.

**Sliding in to home base:** The runner’s energy of motion is nearly all converted into heat via friction with the ground.

**Accelerating a car:** The gasoline has energy stored in it, which is released as heat by burning it inside the engine. Perhaps 10%
of this heat energy is converted into the car’s energy of motion. The rest remains in the form of heat, which is carried away by the exhaust.

_Cruising in a car:_ As you cruise at constant speed in your car, all the energy of the burning gas is being converted into heat. The tires and engine get hot, and heat is also dissipated into the air through the radiator and the exhaust.

_Stepping on the brakes:_ All the energy of the car’s motion is converted into heat in the brake shoes.

_Stevin’s machine_ example 3

The Dutch mathematician and engineer Simon Stevin proposed the imaginary machine shown in figure e, which he had inscribed on his tombstone. This is an interesting example, because it shows a link between the force concept used earlier in this course, and the energy concept being developed now.

The point of the imaginary machine is to show the mechanical advantage of an inclined plane. In this example, the triangle has the proportions 3-4-5, but the argument works for any right triangle. We imagine that the chain of balls slides without friction, so that no energy is ever converted into heat. If we were to slide the chain clockwise by one step, then each ball would take the place of the one in front of it, and the overall configuration would be exactly the same. Since energy is something that only depends on the state of the system, the energy would have to be the same. Similarly for a counterclockwise rotation, no energy of position would be released by gravity. This means that if we place the chain on the triangle, and release it at rest, it can’t start moving, because there would be no way for it to convert energy of position into energy of motion. Thus the chain must be perfectly balanced. Now by symmetry, the arc of the chain hanging underneath the triangle has equal tension at both ends, so removing this arc wouldn’t affect the balance of the rest of the chain. This means that a weight of three units hanging vertically balances a weight of five units hanging diagonally along the hypotenuse.

The mechanical advantage of the inclined plane is therefore \( \frac{5}{3} \), which is exactly the same as the result, \( \frac{1}{\sin \theta} \), that we got on p. 238 by analyzing force vectors. What this shows is that Newton’s laws and conservation laws are not logically separate, but rather are very closely related descriptions of nature. In the cases where Newton’s laws are true, they give the same answers as the conservation laws. This is an example of a more general idea, called the correspondence principle, about how science progresses over time. When a newer, more general theory is proposed to replace an older theory, the new theory must agree with the old one in the realm of applicability of the old theory, since the old theory only became accepted as a valid theory by being ver-
Discussion question A. The water behind the Hoover Dam has energy because of its position relative to the planet earth, which is attracting it with a gravitational force. Letting water down to the bottom of the dam converts that energy into energy of motion. When the water reaches the bottom of the dam, it hits turbine blades that drive generators, and its energy of motion is converted into electrical energy.

Discussion question B. Hydroelectric power (water flowing over a dam to spin turbines) appears to be completely free. Does this violate conservation of energy? If not, then what is the ultimate source of the electrical energy produced by a hydroelectric plant?

B How does the proof in example 3 fail if the assumption of a frictionless surface doesn’t hold?

11.3 A numerical scale of energy

Energy comes in a variety of forms, and physicists didn’t discover all of them right away. They had to start somewhere, so they picked one form of energy to use as a standard for creating a numerical energy scale. (In fact the history is complicated, and several different energy units were defined before it was realized that there was a single general energy concept that deserved a single consistent unit of measurement.) One practical approach is to define an energy unit based on heating water. The SI unit of energy is the joule, J, (rhymes with “cool”), named after the British physicist James Joule. One Joule is the amount of energy required in order to heat 0.24 g of water by 1°C. The number 0.24 is not worth memorizing. A convenient way of restating this definition is that when heating water, heat = \( cm\Delta T \), where \( \Delta T \) is the change in temperature in °C, \( m \) is the mass, and we have defined the joule by defining the constant \( c \), called the specific heat capacity of water, to have the value \( 4.2 \times 10^3 \) J/kg·°C.

Note that heat, which is a form of energy, is completely different from temperature, which is not. Twice as much heat energy is required to prepare two cups of coffee as to make one, but two cups of coffee mixed together don’t have double the temperature. In other words, the temperature of an object tells us how hot it is, but the heat energy contained in an object also takes into account the object’s mass and what it is made of.\(^1\)

Later we will encounter other quantities that are conserved in physics, such as momentum and angular momentum, and the method for defining them will be similar to the one we have used for energy:

\(^1\)In standard, formal terminology, there is another, finer distinction. The word “heat” is used only to indicate an amount of energy that is transferred, whereas “thermal energy” indicates an amount of energy contained in an object. I’m informal on this point, and refer to both as heat, but you should be aware of the distinction.
pick some standard form of it, and then measure other forms by comparison with this standard. The flexible and adaptable nature of this procedure is part of what has made conservation laws such a durable basis for the evolution of physics.

Heating a swimming pool example 4
> If electricity costs 3.9 cents per MJ (1 MJ = 1 megajoule = $10^6$ J), how much does it cost to heat a 26000-gallon swimming pool from $10^\circ$C to $18^\circ$C?

> Converting gallons to cm

\[ 26000 \text{ gallons} \times \frac{3780 \text{ cm}^3}{1 \text{ gallon}} = 9.8 \times 10^7 \text{ cm}^3. \]

Water has a density of 1 gram per cubic centimeter, so the mass of the water is $9.8 \times 10^4$ kg. The energy needed to heat the swimming pool is

\[ mc\Delta T = 3.3 \times 10^3 \text{ MJ}. \]

The cost of the electricity is $(3.3 \times 10^3 \text{ MJ})(0.039/\text{MJ}) = \$130$.

Irish coffee example 5
> You make a cup of Irish coffee out of 300 g of coffee at $100^\circ$C and 30 g of pure ethyl alcohol at $20^\circ$C. The specific heat capacity of ethanol is $2.4 \times 10^3$ J/kg.$^\circ$C (i.e., alcohol is easier to heat than water). What temperature is the final mixture?

> Adding up all the energy after mixing has to give the same result as the total before mixing. We let the subscript $i$ stand for the initial situation, before mixing, and $f$ for the final situation, and use subscripts $c$ for the coffee and $a$ for the alcohol. In this notation, we have

\[
E_{ci} + E_{ai} = E_{cf} + E_{af}.
\]

We assume coffee has the same heat-carrying properties as water. Our information about the heat-carrying properties of the two substances is stated in terms of the change in energy required for a certain change in temperature, so we rearrange the equation to express everything in terms of energy differences:

\[ E_{af} - E_{ai} = E_{ci} - E_{cf}. \]

Using the heat capacities $c_c$ for coffee (water) and $c_a$ for alcohol, we have

\[
E_{ci} - E_{cf} = (T_{ci} - T_{cf})m_cc_c \quad \text{and} \quad E_{af} - E_{ai} = (T_{af} - T_{ai})m_aca_a.
\]
Setting these two quantities to be equal, we have
\[(T_{af} - T_{ai})m_ac_a = (T_{ci} - T_{cf})m_cc_c.\]

In the final mixture the two substances must be at the same temperature, so we can use a single symbol \(T_f = T_{cf} = T_{af}\) for the two quantities previously represented by two different symbols,
\[(T_f - T_{ai})m_ac_a = (T_{ci} - T_f)m_cc_c.\]

Solving for \(T_f\) gives
\[
T_f = \frac{T_{ci}m_cc_c + T_{ai}m_ac_a}{m_cc_c + m_ac_a} = 96^{\circ}C.
\]

Once a numerical scale of energy has been established for some form of energy such as heat, it can easily be extended to other types of energy. For instance, the energy stored in one gallon of gasoline can be determined by putting some gasoline and some water in an insulated chamber, igniting the gas, and measuring the rise in the water’s temperature. (The fact that the apparatus is known as a “bomb calorimeter” will give you some idea of how dangerous these experiments are if you don’t take the right safety precautions.) Here are some examples of other types of energy that can be measured using the same units of joules:

<table>
<thead>
<tr>
<th>type of energy</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>chemical energy released by burning</td>
<td>About 50 MJ are released by burning a kg of gasoline.</td>
</tr>
<tr>
<td>energy required to break an object</td>
<td>When a person suffers a spiral fracture of the thighbone (a common type in skiing accidents), about 2 J of energy go into breaking the bone.</td>
</tr>
<tr>
<td>energy required to melt a solid substance</td>
<td>7 MJ are required to melt 1 kg of tin.</td>
</tr>
<tr>
<td>chemical energy released by digesting food</td>
<td>A bowl of Cheerios with milk provides us with about 800 kJ of usable energy.</td>
</tr>
<tr>
<td>raising a mass against the force of gravity</td>
<td>Lifting 1.0 kg through a height of 1.0 m requires 9.8 J.</td>
</tr>
<tr>
<td>nuclear energy released in fission</td>
<td>1 kg of uranium oxide fuel consumed by a reactor releases (2 \times 10^{12}) J of stored nuclear energy.</td>
</tr>
</tbody>
</table>

It is interesting to note the disproportion between the megajoule energies we consume as food and the joule-sized energies we expend in physical activities. If we could perceive the flow of energy around us the way we perceive the flow of water, eating a bowl of cereal...
would be like swallowing a bathtub’s worth of energy, the continual loss of body heat to one’s environment would be like an energy-hose left on all day, and lifting a bag of cement would be like flicking it with a few tiny energy-drops. The human body is tremendously inefficient. The calories we “burn” in heavy exercise are almost all dissipated directly as body heat.

You take the high road and I’ll take the low road. example 6

Figure f shows two ramps which two balls will roll down. Compare their final speeds, when they reach point B. Assume friction is negligible.

Each ball loses some energy because of its decreasing height above the earth, and conservation of energy says that it must gain an equal amount of energy of motion (minus a little heat created by friction). The balls lose the same amount of height, so their final speeds must be equal.

It’s impressive to note the complete impossibility of solving this problem using only Newton’s laws. Even if the shape of the track had been given mathematically, it would have been a formidable task to compute the balls’ final speed based on vector addition of the normal force and gravitational force at each point along the way.

How new forms of energy are discovered

Textbooks often give the impression that a sophisticated physics concept was created by one person who had an inspiration one day, but in reality it is more in the nature of science to rough out an idea and then gradually refine it over many years. The idea of energy was tinkered with from the early 1800’s on, and new types of energy kept getting added to the list.

To establish the existence of a new form of energy, a physicist has to

1. show that it could be converted to and from other forms of energy; and

2. show that it related to some definite measurable property of the object, for example its temperature, motion, position relative to another object, or being in a solid or liquid state.

For example, energy is released when a piece of iron is soaked in water, so apparently there is some form of energy already stored in the iron. The release of this energy can also be related to a definite measurable property of the chunk of metal: it turns reddish-orange. There has been a chemical change in its physical state, which we call rusting.

Although the list of types of energy kept getting longer and longer, it was clear that many of the types were just variations on a theme. There is an obvious similarity between the energy needed
to melt ice and to melt butter, or between the rusting of iron and many other chemical reactions. The topic of the next chapter is how this process of simplification reduced all the types of energy to a very small number (four, according to the way I’ve chosen to count them).

It might seem that if the principle of conservation of energy ever appeared to be violated, we could fix it up simply by inventing some new type of energy to compensate for the discrepancy. This would be like balancing your checkbook by adding in an imaginary deposit or withdrawal to make your figures agree with the bank’s statements. Step (2) above guards against this kind of chicanery. In the 1920s there were experiments that suggested energy was not conserved in radioactive processes. Precise measurements of the energy released in the radioactive decay of a given type of atom showed inconsistent results. One atom might decay and release, say, $1.1 \times 10^{-10}$ J of energy, which had presumably been stored in some mysterious form in the nucleus. But in a later measurement, an atom of exactly the same type might release $1.2 \times 10^{-10}$ J. Atoms of the same type are supposed to be identical, so both atoms were thought to have started out with the same energy. If the amount released was random, then apparently the total amount of energy was not the same after the decay as before, i.e., energy was not conserved.

Only later was it found that a previously unknown particle, which is very hard to detect, was being spewed out in the decay. The particle, now called a neutrino, was carrying off some energy, and if this previously unsuspected form of energy was added in, energy was found to be conserved after all. The discovery of the energy discrepancies is seen with hindsight as being step (1) in the establishment of a new form of energy, and the discovery of the neutrino was step (2). But during the decade or so between step (1) and step (2) (the accumulation of evidence was gradual), physicists had the admirable honesty to admit that the cherished principle of conservation of energy might have to be discarded.

**self-check A**

How would you carry out the two steps given above in order to establish that some form of energy was stored in a stretched or compressed spring?

> Answer, p. 570

**Mass Into Energy**

Einstein showed that mass itself could be converted to and from energy, according to his celebrated equation $E = mc^2$, in which $c$ is the speed of light. We thus speak of mass as simply another form of energy, and it is valid to measure it in units of joules. The mass of a 15-gram pencil corresponds to about $1.3 \times 10^{15}$ J. The issue is largely academic in the case of the pencil, because very violent processes such as nuclear reactions are required in order to convert any significant fraction of an object’s mass into energy. Cosmic rays, however, are continually striking you and your surroundings and converting part of their energy of motion...
into the mass of newly created particles. A single high-energy cosmic ray can create a “shower” of millions of previously nonexistent particles when it strikes the atmosphere. Einstein’s theories are discussed later in this book.

Even today, when the energy concept is relatively mature and stable, a new form of energy has been proposed based on observations of distant galaxies whose light began its voyage to us billions of years ago. Astronomers have found that the universe’s continuing expansion, resulting from the Big Bang, has not been decelerating as rapidly in the last few billion years as would have been expected from gravitational forces. They suggest that a new form of energy may be at work.

Discussion question

A I’m not making this up. XS Energy Drink has ads that read like this: All the “Energy” ... Without the Sugar! Only 8 Calories! Comment on this.

11.4 Kinetic energy

The technical term for the energy associated with motion is kinetic energy, from the Greek word for motion. (The root is the same as the root of the word “cinema” for a motion picture, and in French the term for kinetic energy is “énergie cinétique.”) To find how much kinetic energy is possessed by a given moving object, we must convert all its kinetic energy into heat energy, which we have chosen as the standard reference type of energy. We could do this, for example, by firing projectiles into a tank of water and measuring the increase in temperature of the water as a function of the projectile’s mass and velocity. Consider the following data from a series of three such experiments:

<table>
<thead>
<tr>
<th>m (kg)</th>
<th>v (m/s)</th>
<th>energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>1.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>2.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Comparing the first experiment with the second, we see that doubling the object’s velocity doesn’t just double its energy; it quadruples it. If we compare the first and third lines, however, we find that doubling the mass only doubles the energy. This suggests that kinetic energy is proportional to mass and to the square of velocity, \( KE \propto mv^2 \), and further experiments of this type would indeed establish such a general rule. The proportionality factor equals 0.5 because of the design of the metric system, so the kinetic energy of a moving object is given by

\[ KE = \frac{1}{2}mv^2. \]

The metric system is based on the meter, kilogram, and second, with other units being derived from those. Comparing the units on
the left and right sides of the equation shows that the joule can be reexpressed in terms of the basic units as kg·m²/s².

<table>
<thead>
<tr>
<th>Energy released by a comet impact</th>
<th>example 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ Comet Shoemaker-Levy, which struck the planet Jupiter in 1994, had a mass of roughly $4 \times 10^{13}$ kg, and was moving at a speed of 60 km/s. Compare the kinetic energy released in the impact to the total energy in the world’s nuclear arsenals, which is $2 \times 10^{19}$ J. Assume for the sake of simplicity that Jupiter was at rest.</td>
<td></td>
</tr>
<tr>
<td>▶ Since we assume Jupiter was at rest, we can imagine that the comet stopped completely on impact, and 100% of its kinetic energy was converted to heat and sound. We first convert the speed to mks units, $v = 6 \times 10^4$ m/s, and then plug in to the equation to find that the comet’s kinetic energy was roughly $7 \times 10^{22}$ J, or about 3000 times the energy in the world’s nuclear arsenals.</td>
<td></td>
</tr>
</tbody>
</table>

**Energy and relative motion**

Galileo’s Aristotelian enemies (and it is no exaggeration to call them enemies!) would probably have objected to conservation of energy. Galilean got in trouble by claiming that an object in motion would continue in motion indefinitely in the absence of a force. This is not so different from the idea that an object’s kinetic energy stays the same unless there is a mechanism like frictional heating for converting that energy into some other form.

More subtly, however, it’s not immediately obvious that what we’ve learned so far about energy is strictly mathematically consistent with Galileo’s principle that motion is relative. Suppose we verify that a certain process, say the collision of two pool balls, conserves energy as measured in a certain frame of reference: the sum of the balls’ kinetic energies before the collision is equal to their sum after the collision. But what if we were to measure everything in a frame of reference that was in a different state of motion? It’s not immediately obvious that the total energy before the collision will still equal the total energy after the collision. It *does* still work out. Homework problem 13, p. 335, gives a simple numerical example, and the general proof is taken up in problem 15 on p. 428 (with the solution given in the back of the book).

**Why kinetic energy obeys the equation it does**

I’ve presented the magic expression for kinetic energy, $(1/2)mv^2$, as a purely empirical fact. Does it have any deeper reason that might be knowable to us mere mortals? Yes and no. It contains three factors, and we need to consider each separately.

The reason for the factor of 1/2 is understandable, but only as an arbitrary historical choice. The metric system was designed so that some of the equations relating to energy would come out looking simple, at the expense of some others, which had to have
inconvenient conversion factors in front. If we were using the old British Engineering System of units in this course, then we’d have the British Thermal Unit (BTU) as our unit of energy. In that system, the equation you’d learn for kinetic energy would have an inconvenient proportionality constant, \( KE = (1.29 \times 10^{-3}) \, mv^2 \), with \( KE \) measured in units of BTUs, \( v \) measured in feet per second, and so on. At the expense of this inconvenient equation for kinetic energy, the designers of the British Engineering System got a simple rule for calculating the energy required to heat water: one BTU per degree Fahrenheit per pound. The inventor of kinetic energy, Thomas Young, actually defined it as \( KE = mv^2 \), which meant that all his other equations had to be different from ours by a factor of two. All these systems of units work just fine as long as they are not combined with one another in an inconsistent way.

The proportionality to \( m \) is inevitable because the energy concept is based on the idea that we add up energy contributions from all the objects within a system. Therefore it is logically necessary that a 2 kg object moving at 1 m/s have the same kinetic energy as two 1 kg objects moving side-by-side at the same speed.

What about the proportionality to \( v^2 \)? Consider:

1. It’s surprisingly hard to tamper with this factor without breaking things: see discussion questions A and B on p. 324.

2. The proportionality to \( v^2 \) is not even correct, except as a low-velocity approximation. Experiments show deviations from the \( v^2 \) rule at high speeds (figure g), an effect that is related to Einstein’s theory of relativity.

3. As described on p. 322, we want conservation of energy to keep working when we switch frames of reference. The fact that this does work for \( KE \propto v^2 \) is intimately connected with the assumption that when we change frames, velocities add as described in section 2.5. This assumption turns out to be an approximation, which only works well at low velocities.

4. Conservation laws are of more general validity than Newton’s laws, which apply to material objects moving at low speeds. Under the conditions where Newton’s laws are accurate, they follow logically from the conservation laws. Therefore we need kinetic energy to have low-velocity behavior that ends up correctly reproducing Newton’s laws.

So under a certain set of low-velocity approximations, \( KE \propto v^2 \) is what works. We verify in problem 15, p. 428, that it satisfies criterion 3, and we show in section 13.6, p. 377, that it is the only such relation that satisfies criterion 4.
Discussion questions

A Suppose that, like Young or Einstein, you were trying out different equations for kinetic energy to see if they agreed with the experimental data. Based on the meaning of positive and negative signs of velocity, why would you suspect that a proportionality to \(mv\) would be less likely than \(mv^2\)?

B As in discussion question A, try to think of an argument showing that \(m(v^2 + v^4)\) is not a possible formula for kinetic energy.

C The figure shows a pendulum that is released at A and caught by a peg as it passes through the vertical, B. To what height will the bob rise on the right?

11.5 Power

A car may have plenty of energy in its gas tank, but still may not be able to increase its kinetic energy rapidly. A Porsche doesn’t necessarily have more energy in its gas tank than a Hyundai, it is just able to transfer it more quickly. The rate of transferring energy from one form to another is called power. The definition can be written as an equation,

\[
P = \frac{\Delta E}{\Delta t},
\]

where the use of the delta notation in the symbol \(\Delta E\) has the usual interpretation: the final amount of energy in a certain form minus the initial amount that was present in that form. Power has units of J/s, which are abbreviated as watts, W (rhymes with “lots”).

If the rate of energy transfer is not constant, the power at any instant can be defined as the derivative \(dE/dt\).

Converting kilowatt-hours to joules example 8

The electric company bills you for energy in units of kilowatt-hours (kilowatts multiplied by hours) rather than in SI units of joules. How many joules is a kilowatt-hour?

\(1 \text{ kilowatt-hour} = (1 \text{ kW})(1 \text{ hour}) = (1000 \text{ J/s})(3600 \text{ s}) = 3.6 \text{ MJ}.\)

Human wattage example 9

A typical person consumes 2000 kcal of food in a day, and converts nearly all of that directly to heat. Compare the person’s heat output to the rate of energy consumption of a 100-watt lightbulb.

Looking up the conversion factor from calories to joules, we find

\[
\Delta E = 2000 \text{ kcal} \times \frac{1000 \text{ cal}}{1 \text{ kcal}} \times \frac{4.18 \text{ J}}{1 \text{ cal}} = 8 \times 10^6 \text{ J}
\]

for our daily energy consumption. Converting the time interval likewise into mks,

\[
\Delta t = 1 \text{ day} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hour}} \times \frac{60 \text{ s}}{1 \text{ min}} = 9 \times 10^4 \text{ s}.
\]

Dividing, we find that our power dissipated as heat is 90 J/s = 90 W, about the same as a lightbulb.
Wind power and wind power density

Wind power is a renewable energy resource, but it is most practical in areas where the wind is both strong and reliably strong. When a horizontal-axis wind turbine faces directly into a wind flowing at speed \( v \), the air it intercepts in time \( \Delta t \) forms a cylinder whose length is \( v \Delta t \), and whose mass is proportional to the same factor. The kinetic energy of this cylinder represents the maximum energy that can theoretically be extracted in this time. Since the mass is proportional to \( v \), the kinetic energy is proportional to \( v \times v^2 = v^3 \). That is, the “wind power density” varies as the cube of the wind’s speed.

It is easy to confuse the concepts of force, energy, and power, especially since they are synonyms in ordinary speech. The table on the following page may help to clear this up:
### A force is an interaction between two objects that causes a push or a pull. A force can be defined as anything that is capable of changing an object’s state of motion.

### Heating an object, making it move faster, or increasing its distance from another object that is attracting it are all examples of things that would require fuel or physical effort. All these things can be quantified using a single scale of measurement, and we describe them all as forms of energy.

### Power is the rate at which energy is transformed from one form to another or transferred from one object to another.

### A spring scale can be used to measure force.

### If we define a unit of energy as the amount required to heat a certain amount of water by a $1^\circ \text{C}$, then we can measure any other quantity of energy by transferring it into heat in water and measuring the temperature increase.

### More power means you are paying money at a higher rate. A 100-W lightbulb costs a certain number of cents per hour.

### One of the main reasons for preferring conservation laws to Newton’s laws as a foundation for physics is that conservation laws are more general. For example, Newton’s laws apply only to matter, whereas conservation laws can handle light as well. No experiment in Newton’s day had ever shown anything but zero for the mass...
or weight of a ray of light, and substituting $m = 0$ into $a = F/m$ results in an infinite acceleration, which doesn’t make sense. With hindsight, this is to be expected because of relativity (section 2.6). Newton’s laws are only a good approximation for velocities that are small compared to $c$, the maximum speed of cause and effect. But light travels at $c$, so Newton’s laws are not a good approximation to the behavior of light.

For insight into the behavior of things that go at exactly $c$, let’s consider a case where something goes very close to $c$. A typical 22-caliber rifle shoots a bullet with a mass of about 3 g at a speed of about 400 m/s. Now consider the firing of such a rifle as seen through an ultra-powerful telescope by an alien in a distant galaxy. We happen to be firing in the direction away from the alien, who gets a view from over our shoulder. Since the universe is expanding, our two galaxies are receding from each other. In the alien’s frame, our own galaxy is the one that is moving — let’s say at $c - (200 \text{ m/s})$. If the two velocities simply added, the bullet would be moving at $c + (200 \text{ m/s})$. But velocities don’t simply add and subtract relativistically (p. 89), and applying the correct equation for relativistic combination of velocities, we find that in the alien’s frame, the bullet flies at only $c - (199.9995 \text{ m/s})$. That is, according to the alien, the energy in the gunpowder only succeeded in accelerating the bullet by 0.0005 m/s! If we insisted on believing in $KE = (1/2)mv^2$, this would clearly violate conservation of energy in the alien’s frame of reference. $KE$ must not only get bigger faster than $(1/2)mv^2$ as $v$ approaches $c$, it must blow up to infinity. This gives a mechanical explanation for why no material object can ever reach or exceed $c$, which is reassuring because speeds greater than $c$ lead to violation of causality.

**Ultrarelativistic motion**

The bullet as seen in the alien’s frame of reference is an example of an ultrarelativistic particle, meaning one moving very close to $c$. We can fairly easily infer quite a bit about how kinetic energy must behave at ultrarelativistic speeds. We know that it must get larger and larger, and the question is how large it is when the speed differs from $c$ by some small amount.

A good way of thinking about an ultrarelativistic particle is that it’s a particle with a very small mass. For example, the subatomic particle called the neutrino has a very small mass, thousands of times smaller than that of the electron. Neutrinos are emitted in radioactive decay, and because the neutrino’s mass is so small, the amount of energy available in these decays is always enough to accelerate it to very close to the speed of light. Nobody has ever succeeded in observing a neutrino that was not ultrarelativistic. When a particle’s mass is very small, the mass becomes difficult to measure. For almost 70 years after the neutrino was discovered, its mass was
thought to be zero. Similarly, we currently believe that a ray of light has no mass, but it is always possible that its mass will be found to be nonzero at some point in the future. A ray of light can be modeled as an ultrarelativistic particle.

Let’s compare ultrarelativistic particles with train cars. A single car with kinetic energy $E$ has different properties than a train of two cars each with kinetic energy $E/2$. The single car has half the mass and a speed that is greater by a factor of $\sqrt{2}$. But the same is not true for ultrarelativistic particles. Since an idealized ultrarelativistic particle has a mass too small to be detectable in any experiment, we can’t detect the difference between $m$ and $2m$. Furthermore, ultrarelativistic particles move at close to $c$, so there is no observable difference in speed. Thus we expect that a single ultrarelativistic particle with energy $E$ compared with two such particles, each with energy $E/2$, should have all the same properties as measured by a mechanical detector.

An idealized zero-mass particle also has no frame in which it can be at rest. It always travels at $c$, and no matter how fast we chase after it, we can never catch up. We can, however, observe it in different frames of reference, and we will find that its energy is different. For example, distant galaxies are receding from us at substantial fractions of $c$, and when we observe them through a telescope, they appear very dim not just because they are very far away but also because their light has less energy in our frame than in a frame at rest relative to the source. This effect must be such that changing frames of reference according to a specific Lorentz transformation always changes the energy of the particle by a fixed factor, regardless of the particle’s original energy; for if not, then the effect of a Lorentz transformation on a single particle of energy $E$ would be different from its effect on two particles of energy $E/2$.

How does this energy-shift factor depend on the velocity $v$ of the Lorentz transformation? Actually, it is more convenient to express this in terms of a different variable rather than $v$. In nonrelativistic physics, we change frames of reference simply by adding a constant onto all our velocities, but this is only a low-velocity approximation. For this reason, it will be more convenient to work with a variable $s$, defined as the factor by which the long diagonal of a parallelogram like the ones in section 2.6 stretches under a Lorentz transformation. For example, we found in problem 21 on p. 99 that a velocity of $0.6c$ corresponds to a stretch factor $s = 2$. The convenient thing about stretch factors is that when we change to a new frame of reference, they simply multiply. For example, in problem 21 you found the result of combining a velocity of $0.6c$ with another velocity of $0.6c$ by drawing a parallelogram with its long axis stretched by a factor of $2 \times 2 = 4$. The relation between $s$ and $v$ is given by $s = \sqrt{(1 + v)/(1 - v)}$ (in units with $c = 1$; see problems 18 on p. 98 and 22 on p. 100).
example 11

What happens when the velocity is small compared to \( c \)? In units where \( c = 1 \), this means that \( v \) is small compared to 1. The stretch factor \( s = \sqrt{(1 + v)/(1 - v)} \) can then be approximated by taking \( 1/(1 - v) \approx 1 + v \) and \( \sqrt{1 + \epsilon} \approx 1 + \epsilon/2 \), so that \( s \approx 1 + v \).

Let’s write \( f(s) \) for the energy-shift factor that results from a given Lorentz transformation. Since a Lorentz transformation \( s_1 \) followed by a second transformation \( s_2 \) is equivalent to a single transformation by \( s_1 s_2 \), we must have \( f(s_1 s_2) = f(s_1) f(s_2) \). This tightly constrains the form of the function \( f \); it must be something like \( f(s) = s^n \), where \( n \) is a constant. The interpretation of \( n \) is that under a Lorentz transformation corresponding to 1% of \( c \), energies of ultrarelativistic particles change by about \( n\% \) (making the approximation that \( v = .01 \) gives \( s \approx 1.01 \)). We postpone until p. 421 the proof that \( n = 1 \), which is also in agreement with experiments with rays of light.

Our final result is that the energy of an ultrarelativistic particle is simply proportional to its Lorentz “stretch factor” \( s \). Even in the case where the particle is truly massless, so that \( s \) doesn’t have any finite value, we can still find how the energy differs according to different observers by finding the \( s \) of the Lorentz transformation between the two observers’ frames of reference.

example 12

For quantum-mechanical reasons, a hydrogen atom can only exist in states with certain specific energies. By conservation of energy, the atom can therefore only absorb or emit light that has an energy equal to the difference between two such atomic energies. The outer atmosphere of a star is mostly made of monoatomic hydrogen, and one of the energies that a hydrogen atom can absorb or emit is \( 3.0276 \times 10^{-19} \) J. When we observe light from stars in the Andromeda Galaxy, it has an energy of \( 3.0306 \times 10^{-19} \) J. If this is assumed to be due entirely to the motion of the Milky Way and Andromeda Galaxy relative to one another, along the line connecting them, find the direction and magnitude of this velocity.

The energy is shifted upward, which means that the Andromeda Galaxy is moving toward us. (Galaxies at cosmological distances are always observed to be receding from one another, but this doesn’t necessarily hold for galaxies as close as these.) Relating the energy shift to the velocity, we have

\[
\frac{E'}{E} = s = \sqrt{(1 + v)/(1 - v)}.
\]

Since the shift is only about one part per thousand, the velocity is small compared to \( c \) — or small compared to 1 in units where \( c = 1 \). Therefore we can approximate as in example 11, \( s \approx 1 + v \),
and we find

\[ v \approx s - 1 = \frac{E'}{E} - 1 = 1.0 \times 10^{-3}. \]

This is in units where \( c = 1 \). Converting to SI units, where \( c \neq 1 \), we have \( v = (1.0 \times 10^{-3})c = 300 \text{ km/s} \). Although the Andromeda Galaxy’s tangential motion is not accurately known, it is considered likely that it will collide with the Milky Way in a few billion years.

**A symmetry property of the energy shift example 13**

Suppose that A and B are at rest relative to one another, but C is moving along the line between A and B. A sends a pulse of laser light to C, who then measures its energy and transmits another pulse to B having the same energy. The pulse accumulates two energy shifts, and the result is their product \( s(v)s(-v) \). But C didn’t actually need to absorb the original pulse and retransmit it; the results would have been the same if C had just stayed out of the way. Therefore this product must equal 1, so we must have \( s(-v)s(v) = 1 \), which can be verified directly from the equation.

**The Ives-Stilwell experiment example 14**

The result of example 13 was the basis of one of the earliest laboratory tests of special relativity, by Ives and Stilwell in 1938. They observed the light emitted by a beam of excited \( \text{H}_2^+ \) and \( \text{H}_3^+ \) ions with speeds of a few tenths of a percent of \( c \). Measuring the light from both ahead of and behind the beams, they found that the product \( s(v)s(-v) \) was equal to 1, as predicted by relativity. If relativity had been false, then one would have expected the product to differ from 1 by an amount that would have been detectable in their experiment. In 2003, Saathoff et al. carried out an extremely precise version of the Ives-Stilwell technique with \( \text{Li}^+ \) ions moving at 6.4% of \( c \). The energies observed, in units of \( 10^{-28} \text{ J} \), were:

\[
\begin{align*}
E_0 & = 3620927488 \pm 3 \\
(\text{unshifted energy}) \\
E_0s(v) & = 3859620256 \pm 0.6 \\
(\text{shifted energy, forward}) \\
E_0s(-v) & = 3396996334 \pm 3 \\
(\text{shifted energy, backward}) \\
\sqrt{E_0s(v) \cdot E_0s(-v)} & = 3620927487 \pm 2
\end{align*}
\]

The results show incredibly precise agreement between \( E_0 \) and \( \sqrt{E_0s(v) \cdot E_0s(-v)} \), as expected relativistically because \( s(v)s(-v) \) is supposed to equal 1. The agreement extends to 9 significant figures, whereas if relativity had been false there should have been a relative disagreement of about \( v^2 = .004 \), i.e., a discrepancy in the third significant figure. The spectacular agreement with theory has made this experiment a lightning rod for anti-relativity kooks.
Summary

Selected vocabulary

energy . . . . . . A numerical scale used to measure the heat, motion, or other properties that would require fuel or physical effort to put into an object; a scalar quantity with units of joules (J).

power . . . . . . The rate of transferring energy; a scalar quantity with units of watts (W).

kinetic energy . . The energy an object possesses because of its motion.

heat . . . . . . A form of energy that relates to temperature. Heat is different from temperature because an object with twice as much mass requires twice as much heat to increase its temperature by the same amount. Heat is measured in joules, temperature in degrees. (In standard terminology, there is another, finer distinction between heat and thermal energy, which is discussed below. In this book, I informally refer to both as heat.)

temperature . . . What a thermometer measures. Objects left in contact with each other tend to reach the same temperature. Cf. heat. As discussed in more detail in chapter 2, temperature is essentially a measure of the average kinetic energy per molecule.

Notation

E . . . . . . . . energy
J . . . . . . . . joules, the SI unit of energy
KE . . . . . . . . kinetic energy
P . . . . . . . . power
W . . . . . . . . watts, the SI unit of power; equivalent to J/s

Other terminology and notation

Q or ∆Q . . . . the amount of heat transferred into or out of an object

K or T . . . . alternative symbols for kinetic energy, used in the scientific literature and in most advanced textbooks

thermal energy . Careful writers make a distinction between heat and thermal energy, but the distinction is often ignored in casual speech, even among physicists. Properly, thermal energy is used to mean the total amount of energy possessed by an object, while heat indicates the amount of thermal energy transferred in or out. The term heat is used in this book to include both meanings.
Summary

Heating an object, making it move faster, or increasing its distance from another object that is attracting it are all examples of things that would require fuel or physical effort. All these things can be quantified using a single scale of measurement, and we describe them all as forms of energy. The SI unit of energy is the Joule. The reason why energy is a useful and important quantity is that it is always conserved. That is, it cannot be created or destroyed but only transferred between objects or changed from one form to another. Conservation of energy is the most important and broadly applicable of all the laws of physics, more fundamental and general even than Newton’s laws of motion.

Heating an object requires a certain amount of energy per degree of temperature and per unit mass, which depends on the substance of which the object consists. Heat and temperature are completely different things. Heat is a form of energy, and its SI unit is the joule (J). Temperature is not a measure of energy. Heating twice as much of something requires twice as much heat, but double the amount of a substance does not have double the temperature.

The energy that an object possesses because of its motion is called kinetic energy. Kinetic energy is related to the mass of the object and the magnitude of its velocity vector by the equation

\[ KE = \frac{1}{2}mv^2. \]

Power is the rate at which energy is transformed from one form to another or transferred from one object to another,

\[ P = \frac{dE}{dt} \]

The SI unit of power is the watt (W).

The equation \( KE = (1/2)mv^2 \) is a nonrelativistic approximation, valid at speeds that are small compared to \( c \). In the opposite limit, of a particle with a speed very close to \( c \), the energy is proportional to the “stretch factor” of the Lorentz transformation, \( s = \sqrt{(1 + v)/(1 - v)} \) (in units with \( c = 1 \)), for \( v \to +c \) and \( 1/s \) for \( v \to -c \). This gives a mechanical explanation for why no material object can ever reach or exceed \( c \), which is reassuring because speeds greater than \( c \) lead to violation of causality.
Problems

Key
√ A computerized answer check is available online.
∫ A problem that requires calculus.
★ A difficult problem.

1 Can kinetic energy ever be less than zero? Explain. [Based on a problem by Serway and Faughn.]

2 Estimate the kinetic energy of an Olympic sprinter.

3 You are driving your car, and you hit a brick wall head on, at full speed. The car has a mass of 1500 kg. The kinetic energy released is a measure of how much destruction will be done to the car and to your body. Calculate the energy released if you are traveling at (a) 40 mi/hr, and again (b) if you’re going 80 mi/hr. What is counterintuitive about this, and what implication does this have for driving at high speeds?

4 The following table gives the amount of energy required in order to heat, melt, or boil a gram of water.

<table>
<thead>
<tr>
<th>Process</th>
<th>Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat 1 g of ice by 1°C</td>
<td>2.05</td>
</tr>
<tr>
<td>Melt 1 g of ice</td>
<td>333</td>
</tr>
<tr>
<td>Heat 1 g of water by 1°C</td>
<td>4.19</td>
</tr>
<tr>
<td>Boil 1 g of water</td>
<td>2500</td>
</tr>
<tr>
<td>Heat 1 g of steam by 1°C</td>
<td>2.01</td>
</tr>
</tbody>
</table>

(a) How much energy is required in order to convert 1.00 g of ice at -20 °C into steam at 137 °C? ★
(b) What is the minimum amount of hot water that could melt 1.00 g of ice? ★

5 A closed system can be a bad thing — for an astronaut sealed inside a space suit, getting rid of body heat can be difficult. Suppose a 60-kg astronaut is performing vigorous physical activity, expending 200 W of power. If none of the heat can escape from her space suit, how long will it take before her body temperature rises by 6°C(11°F), an amount sufficient to kill her? Assume that the amount of heat required to raise her body temperature by 1°C is the same as it would be for an equal mass of water. Express your answer in units of minutes.
6 A bullet flies through the air, passes through a paperback book, and then continues to fly through the air beyond the book. When is there a force? When is there energy?  
▷ Solution, p. 562

7 Experiments show that the power consumed by a boat’s engine is approximately proportional to the third power of its speed. (We assume that it is moving at constant speed.) (a) When a boat is cruising at constant speed, what type of energy transformation do you think is being performed? (b) If you upgrade to a motor with double the power, by what factor is your boat’s cruising speed increased? [Based on a problem by Arnold Arons.]

▷ Solution, p. 563

8 Object A has a kinetic energy of 13.4 J. Object B has a mass that is greater by a factor of 3.77, but is moving more slowly by a factor of 2.34. What is object B’s kinetic energy? [Based on a problem by Arnold Arons.]

▷ Solution, p. 563

9 Example 10 on page 325 showed that the power produced by a wind turbine is proportional to the cube of the wind speed \(v\). Von Kármán found empirically that when a fluid flows turbulently over a surface, the speed of the fluid is often well approximated by \(v \propto z^{1/7}\), where \(z\) is the distance from the surface. Wind turbine towers are often constructed at heights of 50 m, but surveys of wind speeds are usually conducted at heights of about 3 m. By what factor should the predicted wind power density be scaled up relative to the survey data?

10 The moon doesn’t really just orbit the Earth. By Newton’s third law, the moon’s gravitational force on the earth is the same as the earth’s force on the moon, and the earth must respond to the moon’s force by accelerating. If we consider the earth and moon in isolation and ignore outside forces, then Newton’s first law says their common center of mass doesn’t accelerate, i.e., the earth wobbles around the center of mass of the earth-moon system once per month, and the moon also orbits around this point. The moon’s mass is 81 times smaller than the earth’s. Compare the kinetic energies of the earth and moon. (We know that the center of mass is a kind of balance point, so it must be closer to the earth than to the moon. In fact, the distance from the earth to the center of mass is 1/81 of the distance from the moon to the center of mass, which makes sense intuitively, and can be proved rigorously using the equation on page 408.)

11 My 1.25 kW microwave oven takes 126 seconds to bring 250 g of water from room temperature to a boil. What percentage of the power is being wasted? Where might the rest of the energy be going?

▷ Solution, p. 563
12 The multiflash photograph shows a collision between two pool balls. The ball that was initially at rest shows up as a dark image in its initial position, because its image was exposed several times before it was struck and began moving. By making measurements on the figure, determine numerically whether or not energy appears to have been conserved in the collision. What systematic effects would limit the accuracy of your test? [From an example in PSSC Physics.]

13 This problem is a numerical example of the imaginary experiment discussed on p. 322 regarding the relationship between energy and relative motion. Let’s say that the pool balls both have masses of 1.00 kg. Suppose that in the frame of reference of the pool table, the cue ball moves at a speed of 1.00 m/s toward the eight ball, which is initially at rest. The collision is head-on, and as you can verify for yourself the next time you’re playing pool, the result of such a collision is that the incoming ball stops dead and the ball that was struck takes off with the same speed originally possessed by the incoming ball. (This is actually a bit of an idealization. To keep things simple, we’re ignoring the spin of the balls, and we assume that no energy is liberated by the collision as heat or sound.) (a) Calculate the total initial kinetic energy and the total final kinetic energy, and verify that they are equal. (b) Now carry out the whole calculation again in the frame of reference that is moving in the same direction that the cue ball was initially moving, but at a speed of 0.50 m/s. In this frame of reference, both balls have nonzero initial and final velocities, which are different from what they were in the table’s frame. [See also problem 15 on p. 428.]
One theory about the destruction of the space shuttle Columbia in 2003 is that one of its wings had been damaged on liftoff by a chunk of foam insulation that fell off of one of its external fuel tanks. The New York Times reported on June 5, 2003, that NASA engineers had recreated the impact to see if it would damage a mock-up of the shuttle’s wing. “Before last week’s test, many engineers at NASA said they thought lightweight foam could not harm the seemingly tough composite panels, and privately predicted that the foam would bounce off harmlessly, like a Nerf ball.” In fact, the 0.80 kg piece of foam, moving at 240 m/s, did serious damage. A member of the board investigating the disaster said this demonstrated that “people’s intuitive sense of physics is sometimes way off.” (a) Compute the kinetic energy of the foam, and (b) compare with the energy of an 80 kg boulder moving at 2.4 m/s (the speed it would have if you dropped it from about knee-level). (c) The boulder is a hundred times more massive, but its speed is a hundred times smaller, so what’s counterintuitive about your results?

The figure above is from a classic 1920 physics textbook by Millikan and Gale. It represents a method for raising the water from the pond up to the water tower, at a higher level, without using a pump. Water is allowed into the drive pipe, and once it is flowing fast enough, it forces the valve at the bottom closed. Explain how this works in terms of conservation of mass and energy. (Cf. example 1 on page 313.)
All stars, including our sun, show variations in their light output to some degree. Some stars vary their brightness by a factor of two or even more, but our sun has remained relatively steady during the hundred years or so that accurate data have been collected. Nevertheless, it is possible that climate variations such as ice ages are related to long-term irregularities in the sun’s light output. If the sun was to increase its light output even slightly, it could melt enough Antarctic ice to flood all the world’s coastal cities. The total sunlight that falls on Antarctica amounts to about $1 \times 10^{16}$ watts. Presently, this heat input to the poles is balanced by the loss of heat via winds, ocean currents, and emission of infrared light, so that there is no net melting or freezing of ice at the poles from year to year. Suppose that the sun changes its light output by some small percentage, but there is no change in the rate of heat loss by the polar caps. Estimate the percentage by which the sun’s light output would have to increase in order to melt enough ice to raise the level of the oceans by 10 meters over a period of 10 years. (This would be enough to flood New York, London, and many other cities.) Melting 1 kg of ice requires $3 \times 10^4$ J.

Estimate the kinetic energy of a buzzing fly’s wing. (You may wish to review section 1.3 on order-of-magnitude estimates.)

A blade of grass moves upward as it grows. Estimate its kinetic energy. (You may wish to review section 1.3 on order-of-magnitude estimates.)
Do these forms of energy have anything in common?

Chapter 12

Simplifying the Energy Zoo

Variety is the spice of life, not of science. The figure shows a few examples from the bewildering array of forms of energy that surrounds us. The physicist’s psyche rebels against the prospect of a long laundry list of types of energy, each of which would require its own equations, concepts, notation, and terminology. The point at which we’ve arrived in the study of energy is analogous to the period in the 1960’s when a half a dozen new subatomic particles were being discovered every year in particle accelerators. It was an embarrassment. Physicists began to speak of the “particle zoo,” and it seemed that the subatomic world was distressingly complex. The particle zoo was simplified by the realization that most of the
new particles being whipped up were simply clusters of a previously unsuspected set of more fundamental particles (which were whimsically dubbed quarks, a made-up word from a line of poetry by James Joyce, “Three quarks for Master Mark.”) The energy zoo can also be simplified, and it is the purpose of this chapter to demonstrate the hidden similarities between forms of energy as seemingly different as heat and motion.

A vivid demonstration that heat is a form of motion. A small amount of boiling water is poured into the empty can, which rapidly fills up with hot steam. The can is then sealed tightly, and soon crumples. This can be explained as follows. The high temperature of the steam is interpreted as a high average speed of random motions of its molecules. Before the lid was put on the can, the rapidly moving steam molecules pushed their way out of the can, forcing the slower air molecules out of the way. As the steam inside the can thinned out, a stable situation was soon achieved, in which the force from the less dense steam molecules moving at high speed balanced against the force from the more dense but slower air molecules outside. The cap was put on, and after a while the steam inside the can reached the same temperature as the air outside. The force from the cool, thin steam no longer matched the force from the cool, dense air outside, and the imbalance of forces crushed the can.

12.1 Heat is kinetic energy

What is heat really? Is it an invisible fluid that your bare feet soak up from a hot sidewalk? Can one ever remove all the heat from an object? Is there a maximum to the temperature scale?

The theory of heat as a fluid seemed to explain why colder objects absorbed heat from hotter ones, but once it became clear that heat was a form of energy, it began to seem unlikely that a material substance could transform itself into and out of all those other forms of energy like motion or light. For instance, a compost pile gets hot, and we describe this as a case where, through the action of bacteria, chemical energy stored in the plant cuttings is transformed into heat energy. The heating occurs even if there is no nearby warmer object that could have been leaking “heat fluid” into the pile.

An alternative interpretation of heat was suggested by the theory that matter is made of atoms. Since gases are thousands of times less dense than solids or liquids, the atoms (or clusters of atoms called molecules) in a gas must be far apart. In that case, what is keeping all the air molecules from settling into a thin film on the floor of the room in which you are reading this book? The simplest explanation is that they are moving very rapidly, continually ricocheting off of
the floor, walls, and ceiling. Though bizarre, the cloud-of-bullets image of a gas did give a natural explanation for the surprising ability of something as tenuous as a gas to exert huge forces. Your car’s tires can hold it up because you have pumped extra molecules into them. The inside of the tire gets hit by molecules more often than the outside, forcing it to stretch and stiffen.

The outward forces of the air in your car’s tires increase even further when you drive on the freeway for a while, heating up the rubber and the air inside. This type of observation leads naturally to the conclusion that hotter matter differs from colder in that its atoms’ random motion is more rapid. In a liquid, the motion could be visualized as people in a milling crowd shoving past each other more quickly. In a solid, where the atoms are packed together, the motion is a random vibration of each atom as it knocks against its neighbors.

We thus achieve a great simplification in the theory of heat. Heat is simply a form of kinetic energy, the total kinetic energy of random motion of all the atoms in an object. With this new understanding, it becomes possible to answer at one stroke the questions posed at the beginning of the section. Yes, it is at least theoretically possible to remove all the heat from an object. The coldest possible temperature, known as absolute zero, is that at which all the atoms have zero velocity, so that their kinetic energies, \((1/2)mv^2\), are all zero. No, there is no maximum amount of heat that a certain quantity of matter can have, and no maximum to the temperature scale, since arbitrarily large values of \(v\) can create arbitrarily large amounts of kinetic energy per atom.

The kinetic theory of heat also provides a simple explanation of the true nature of temperature. Temperature is a measure of the amount of energy per molecule, whereas heat is the total amount of energy possessed by all the molecules in an object.

There is an entire branch of physics, called thermodynamics, that deals with heat and temperature and forms the basis for technologies such as refrigeration.

Thermodynamics is not covered in this book, and I have provided here only a brief overview of the thermodynamic concepts that relate directly to energy, glossing over at least one point that would be dealt with more carefully in a thermodynamics course: it is really only true for a gas that all the heat is in the form of kinetic energy. In solids and liquids, the atoms are close enough to each other to exert intense electrical forces on each other, and there is therefore another type of energy involved, the energy associated with the atoms’ distances from each other. Strictly speaking, heat energy is defined not as energy associated with random motion of molecules but as any form of energy that can be conducted between objects in contact, without any force.
12.2 Potential energy: energy of distance or closeness

We have already seen many examples of energy related to the distance between interacting objects. When two objects participate in an attractive noncontact force, energy is required to bring them farther apart. In both of the perpetual motion machines that started off the previous chapter, one of the types of energy involved was the energy associated with the distance between the balls and the earth, which attract each other gravitationally. In the perpetual motion machine with the magnet on the pedestal, there was also energy associated with the distance between the magnet and the iron ball, which were attracting each other.

The opposite happens with repulsive forces: two socks with the same type of static electric charge will repel each other, and cannot be pushed closer together without supplying energy.

In general, the term potential energy, with algebra symbol $PE$, is used for the energy associated with the distance between two objects that attract or repel each other via a force that depends on the distance between them. Forces that are not determined by distance do not have potential energy associated with them. For instance, the normal force acts only between objects that have zero distance between them, and depends on other factors besides the fact that the distance is zero. There is no potential energy associated with the normal force.

The following are some commonplace examples of potential energy:

**gravitational potential energy:** The skateboarder in the photo has risen from the bottom of the pool, converting kinetic energy into gravitational potential energy. After being at rest for an instant, he will go back down, converting PE back into KE.

**magnetic potential energy:** When a magnetic compass needle is allowed to rotate, the poles of the compass change their distances from the earth’s north and south magnetic poles, converting magnetic potential energy into kinetic energy. (Eventually the kinetic energy is all changed into heat by friction, and the needle settles down in the position that minimizes its potential energy.)

**electrical potential energy:** Socks coming out of the dryer cling together because of attractive electrical forces. Energy is required in order to separate them.

**potential energy of bending or stretching:** The force between the two ends of a spring depends on the distance between
them, i.e., on the length of the spring. If a car is pressed down on its shock absorbers and then released, the potential energy stored in the spring is transformed into kinetic and gravitational potential energy as the car bounces back up.

I have deliberately avoided introducing the term potential energy up until this point, because it tends to produce unfortunate connotations in the minds of students who have not yet been inoculated with a careful description of the construction of a numerical energy scale. Specifically, there is a tendency to generalize the term inappropriately to apply to any situation where there is the “potential” for something to happen: “I took a break from digging, but I had potential energy because I knew I’d be ready to work hard again in a few minutes.”

An equation for gravitational potential energy

All the vital points about potential energy can be made by focusing on the example of gravitational potential energy. For simplicity, we treat only vertical motion, and motion close to the surface of the earth, where the gravitational force is nearly constant. (The generalization to the three dimensions and varying forces is more easily accomplished using the concept of work, which is the subject of the next chapter.)

To find an equation for gravitational PE, we examine the case of free fall, in which energy is transformed between kinetic energy and gravitational PE. Whatever energy is lost in one form is gained in an equal amount in the other form, so using the notation \( \Delta KE \) to stand for \( KE_f - KE_i \) and a similar notation for PE, we have

\[
\Delta KE = -\Delta PE_{grav}.
\]

It will be convenient to refer to the object as falling, so that PE is being changed into KE, but the math applies equally well to an object slowing down on its way up. We know an equation for kinetic energy,

\[
KE = \frac{1}{2}mv^2,
\]

so if we can relate \( v \) to height, \( y \), we will be able to relate \( \Delta PE \) to \( y \), which would tell us what we want to know about potential energy. The \( y \) component of the velocity can be connected to the height via the constant acceleration equation

\[
v_f^2 = v_i^2 + 2a\Delta y,
\]

and Newton’s second law provides the acceleration,

\[
a = F/m,
\]
in terms of the gravitational force.

The algebra is simple because both equation [2] and equation [3] have velocity to the second power. Equation [2] can be solved for \( v^2 \) to give \( v^2 = 2KE/m \), and substituting this into equation [3], we find

\[
2\frac{KE_f}{m} = 2\frac{KE_i}{m} + 2a\Delta y.
\]

Making use of equations [1] and [4] gives the simple result

\[
\Delta PE_{\text{grav}} = -F\Delta y. \quad \text{[change in gravitational PE resulting from a change in height \( \Delta y \); \( F \) is the gravitational force on the object, i.e., its weight; valid only near the surface of the earth, where \( F \) is constant]}
\]

**Example 1: Dropping a rock**

- If you drop a 1-kg rock from a height of 1 m, how many joules of KE does it have on impact with the ground? (Assume that any energy transformed into heat by air friction is negligible.)

- If we choose the \( y \) axis to point up, then \( F_y \) is negative, and equals \(- (1 \text{ kg})(g) = -9.8 \text{ N.} \) A decrease in \( y \) is represented by a negative value of \( \Delta y \), \( \Delta y = -1 \text{ m}, \) so the change in potential energy is \(-(-9.8 \text{ N})(-1 \text{ m}) \approx -10 \text{ J.} \) (The proof that newtons multiplied by meters give units of joules is left as a homework problem.) Conservation of energy says that the loss of this amount of PE must be accompanied by a corresponding increase in KE of 10 J.

It may be dismaying to note how many minus signs had to be handled correctly even in this relatively simple example: a total of four. Rather than depending on yourself to avoid any mistakes with signs, it is better to check whether the final result make sense physically. If it doesn’t, just reverse the sign.

Although the equation for gravitational potential energy was derived by imagining a situation where it was transformed into kinetic energy, the equation can be used in any context, because all the types of energy are freely convertible into each other.
Gravitational PE converted directly into heat example 2

A 50-kg firefighter slides down a 5-m pole at constant velocity. How much heat is produced?

Since she slides down at constant velocity, there is no change in KE. Heat and gravitational PE are the only forms of energy that change. Ignoring plus and minus signs, the gravitational force on her body equals \( mg \), and the amount of energy transformed is

\[(mg)(5 \text{ m}) = 2500 \text{ J}.
\]

On physical grounds, we know that there must have been an increase (positive change) in the heat energy in her hands and in the flagpole.

Here are some questions and answers about the interpretation of the equation \( \Delta PE_{grav} = -F\Delta y \) for gravitational potential energy.

**Question:** In a nutshell, why is there a minus sign in the equation?
**Answer:** It is because we increase the PE by moving the object in the opposite direction compared to the gravitational force.

**Question:** Why do we only get an equation for the change in potential energy? Don’t I really want an equation for the potential energy itself?
**Answer:** No, you really don’t. This relates to a basic fact about potential energy, which is that it is not a well defined quantity in the absolute sense. Only changes in potential energy are unambiguously defined. If you and I both observe a rock falling, and agree that it deposits 10 J of energy in the dirt when it hits, then we will be forced to agree that the 10 J of KE must have come from a loss of 10 joules of PE. But I might claim that it started with 37 J of PE and ended with 27, while you might swear just as truthfully that it had 109 J initially and 99 at the end. It is possible to pick some specific height as a reference level and say that the PE is zero there, but it’s easier and safer just to work with changes in PE and avoid absolute PE altogether.

**Question:** You referred to potential energy as the energy that two objects have because of their distance from each other. If a rock falls, the object is the rock. Where’s the other object?
**Answer:** Newton’s third law guarantees that there will always be two objects. The other object is the planet earth.

**Question:** If the other object is the earth, are we talking about the distance from the rock to the center of the earth or the distance from the rock to the surface of the earth?
**Answer:** It doesn’t matter. All that matters is the change in distance, \( \Delta y \), not \( y \). Measuring from the earth’s center or its surface are just two equally valid choices of a reference point for defining absolute PE.

**Question:** Which object contains the PE, the rock or the earth?
**Answer:** We may refer casually to the PE of the rock, but technically the PE is a relationship between the earth and the rock, and we should refer to the earth and the rock together as possessing the PE.

**Question:** How would this be any different for a force other than gravity?

**Answer:** It wouldn’t. The result was derived under the assumption of constant force, but the result would be valid for any other situation where two objects interacted through a constant force. Gravity is unusual, however, in that the gravitational force on an object is so nearly constant under ordinary conditions. The magnetic force between a magnet and a refrigerator, on the other hand, changes drastically with distance. The math is a little more complex for a varying force, but the concepts are the same.

**Question:** Suppose a pencil is balanced on its tip and then falls over. The pencil is simultaneously changing its height and rotating, so the height change is different for different parts of the object. The bottom of the pencil doesn’t lose any height at all. What do you do in this situation?

**Answer:** The general philosophy of energy is that an object’s energy is found by adding up the energy of every little part of it. You could thus add up the changes in potential energy of all the little parts of the pencil to find the total change in potential energy. Luckily there’s an easier way! The derivation of the equation for gravitational potential energy used Newton’s second law, which deals with the acceleration of the object’s center of mass (i.e., its balance point). If you just define $\Delta y$ as the height change of the center of mass, everything works out. A huge Ferris wheel can be rotated without putting in or taking out any PE, because its center of mass is staying at the same height.

**Self-check A**

A ball thrown straight up will have the same speed on impact with the ground as a ball thrown straight down at the same speed. How can this be explained using potential energy?  

**Discussion question**

A You throw a steel ball up in the air. How can you prove based on conservation of energy that it has the same speed when it falls back into your hand? What if you throw a feather up — is energy not conserved in this case?
12.3 All energy is potential or kinetic

In the same way that we found that a change in temperature is really only a change in kinetic energy at the atomic level, we now find that every other form of energy turns out to be a form of potential energy. Boiling, for instance, means knocking some of the atoms (or molecules) out of the liquid and into the space above, where they constitute a gas. There is a net attractive force between essentially any two atoms that are next to each other, which is why matter always prefers to be packed tightly in the solid or liquid state unless we supply enough potential energy to pull it apart into a gas. This explains why water stops getting hotter when it reaches the boiling point: the power being pumped into the water by your stove begins going into potential energy rather than kinetic energy.

As shown in figure e, every stored form of energy that we encounter in everyday life turns out to be a form of potential energy at the atomic level. The forces between atoms are electrical and magnetic in nature, so these are actually electrical and magnetic potential energies.

Even if we wish to include nuclear reactions in the picture, there still turn out to be only four fundamental types of energy:

- kinetic energy (including heat)
- gravitational PE
- electrical and magnetic PE (including light)
- nuclear PE

How does light fit into this picture? Optional section 11.6 discussed the idea of modeling a ray of light as a stream of massless particles. But the way in which we described the energy of such particles was completely different from the use of $KE = (1/2)mv^2$ for objects made of atoms. Since the purpose of this chapter has been to bring every form of energy under the same roof, this inconsistency feels unsatisfying. Section 12.5 eliminates this inconsistency.

Discussion question

A Referring back to the pictures at the beginning of the chapter, how do all these forms of energy fit into the shortened list of categories given above?

12.4 Applications

Heat transfer

Conduction

When you hold a hot potato in your hand, energy is transferred from the hot object to the cooler one. Our microscopic picture of this process (figure b, p. 341) tells us that the heat transfer can
only occur at the surface of contact, where one layer of atoms in the potato skin make contact with one such layer in the hand. This type of heat transfer is called conduction, and its rate is proportional to both the surface area and the temperature difference.

Convection

In a gas or a liquid, a faster method of heat transfer can occur, because hotter or colder parts of the fluid can flow, physically transporting their heat energy from one place to another. This mechanism of heat transfer, convection, is at work in Los Angeles when hot Santa Ana winds blow in from the Mojave Desert. On a cold day, the reason you feel warmer when there is no wind is that your skin warms a thin layer of air near it by conduction. If a gust of wind comes along, convection robs you of this layer. A thermos bottle has inner and outer walls separated by a layer of vacuum, which prevents heat transport by conduction or convection, except for a tiny amount of conduction through the thin connection between the walls, near the neck, which has a small cross-sectional area.

Radiation

The glow of the sun or a candle flame is an example of heat transfer by radiation. In this context, “radiation” just means anything that radiates outward from a source, including, in these examples, ordinary visible light. The power is proportional to the surface area of the radiating object. It also depends very dramatically on the radiator’s absolute temperature, \( P \propto T^4 \).

We can easily understand the reason for radiation based on the picture of heat as random kinetic energy at the atomic scale. Atoms are made out of subatomic particles, such as electrons and nuclei, that carry electric charge. When a charged particle vibrates, it creates wave disturbances in the electric and magnetic fields, and the waves have a frequency (number of vibrations per second) that matches the frequency of the particle’s motion. If this frequency is in the right range, they constitute visible light. When an object is closer to room temperature, it glows in the invisible infrared part of the spectrum.

Earth’s energy equilibrium

Our planet receives a nearly constant amount of energy from the sun (about \( 1.8 \times 10^{17} \) W). If it hadn’t had any mechanism for getting rid of that energy, the result would have been some kind of catastrophic explosion soon after its formation. Even a 10% imbalance between energy input and output, if maintained steadily from the time of the Roman Empire until the present, would have been enough to raise the oceans to a boil. So evidently the earth does dump this energy somehow. How does it do it? Our planet is surrounded by the vacuum of outer space, like the ultimate thermos bottle. Therefore it can’t expel heat by conduction or convection,
but it does radiate in the infrared, and this is the only available mechanism for cooling.

**Global warming**

It was realized starting around 1930 that this created a dangerous vulnerability in our biosphere. Our atmosphere is only about 0.04% carbon dioxide, but carbon dioxide is an extraordinarily efficient absorber of infrared light. It is, however, transparent to visible light. Therefore any increase in the concentration of carbon dioxide would decrease the efficiency of cooling by radiation, while allowing in just as much heat input from visible light. When we burn fossil fuels such as gasoline or coal, we release into the atmosphere carbon that had previously been locked away underground. This results in a shift to a new energy balance. The average temperature $T$ of the land and oceans increases until the $T^4$ dependence of radiation compensates for the additional absorption of infrared light.

By about 1980, a clear scientific consensus had emerged that this effect was real, that it was caused by human activity, and that it had resulted in an abrupt increase in the earth’s average temperature. We know, for example, from radioisotope studies that the effect has not been caused by the release of carbon dioxide in volcanic eruptions. The temperature increase has been verified by multiple independent methods, including studies of tree rings and coral reefs. Detailed computer models have correctly predicted a number of effects that were later verified empirically, including a rise in sea levels, and day-night and pole-equator variations. There is no longer any controversy among climate scientists about the existence or cause of the effect.

One solution to the problem is to replace fossil fuels with renewable sources of energy such as solar power and wind. However, these cannot be brought online fast enough to prevent severe warming in the next few decades, so nuclear power is also a critical piece of the puzzle.
12.5 \* E=mc^2

In section 11.6 we found the relativistic expression for kinetic energy in the limiting case of an ultrarelativistic particle, i.e., one with a speed very close to \( c \): its energy is proportional to the “stretch factor” of the Lorentz transformation, \( s = \sqrt{\frac{1+v}{1-v}} \) (in units with \( c = 1 \)), for \( v \to +c \) and \( 1/s \) for \( v \to -c \). What about intermediate cases, like \( v = c/2 \)?

When we are forced to tinker with a time-honored theory, our first instinct should always be to tinker as conservatively as possible. Although we’ve been forced to admit that kinetic energy doesn’t vary as \( v^2/2 \) at relativistic speeds, the next most conservative thing we could do would be to assume that the only change necessary is to replace the factor of \( v^2/2 \) in the nonrelativistic expression for kinetic energy with some other function, which would have to act like \( s \) or \( 1/s \) for \( v \to \pm c \). I suspect that this is what Einstein thought when he completed his original paper on relativity in 1905, because it wasn’t until later that year that he published a second paper showing that this still wasn’t enough of a change to produce a working theory. We now know that there is something more that needs to be changed about prerelativistic physics, and this is the assumption that mass is only a property of material particles such as atoms (figure j). Call this the “atoms-only hypothesis.”

Now that we know the correct relativistic way of finding the energy of a ray of light, it turns out that we can use that to find what we were originally seeking, which was the energy of a material object. The following discussion closely follows Einstein’s.

Suppose that a material object \( O \) of mass \( m_o \), initially at rest in a certain frame \( A \), emits two rays of light, each with energy \( E/2 \). By conservation of energy, the object must have lost an amount of energy equal to \( E \). By symmetry, \( O \) remains at rest.

We now switch to a different frame of reference \( B \) moving at some arbitrary speed corresponding to a stretch factor \( S \). The change of frames means that we’re chasing one ray, so that its energy is scaled down to \( (E/2)S^{-1} \), while running away from the other, whose energy...