4.7 * Do Newton’s laws mean anything, and if so, are they true?

On your first encounter with Newton’s first and second laws, you probably had a hard enough time just figuring out what they really meant and reconciling them with the whispers in your ear from the little Aristotelian devil sitting on your shoulder. This optional section is more likely to be of interest to you if you’re already beyond that point and are starting to worry about deeper questions. It addresses the logical foundations of Newton’s laws and sketches some of the empirical evidence for and against them. Section 5.7 gives a similar discussion for the third law, which we haven’t yet encountered.

**Newton’s first law**

Similar ideas are expressed by the principle of inertia (p. 71) and Newton’s first law (p. 131). Both of these assertions are false in a noninertial frame (p. 144). Let’s repack all of these ideas as follows:

**Newton’s first law (repackaged)**

When we find ourselves at any time and place in the universe, and we want to describe our immediate surroundings, we can always find some frame of reference that is inertial. An inertial frame is one in which an object acted on by zero total force responds by moving in a straight line at constant velocity.

A corollary of this definition of an inertial frame is that given any inertial frame F, any other frame F’ moving relative to it at constant velocity is also inertial. That is, we only need to find one inertial frame, and then we get infinitely many others for free.

**Ambiguities due to gravity**

But finding that first inertial frame can be as difficult as knowing when you’ve found your first true love. Suppose that Alice is doing experiments inside a certain laboratory (the “immediate surroundings”), and unknown to her, her lab happens to be an elevator that is in a state of free fall. (We assume that someone will gently decelerate the lab before it hits the bottom of the shaft, so she isn’t doomed.) Meanwhile, her twin sister Betty is doing similar experiments sealed inside a lab somewhere in the depths of outer space, where there is no gravity. Every experiment comes out exactly the same, regardless of whether it is performed by Alice or by Betty. Alice releases a pencil and sees it float in front of her; Newton says this is because she and the pencil are both falling with the same acceleration. Betty does the same experiment and gets the same result, but according to Newton the reason is now completely differ-
ent: Betty and the pencil are not accelerating at all, because there is no gravity. It appears, then, that the distinction between inertial and noninertial frames is not always possible to make.

A more realistic drawing of Braginskii and Panov’s experiment. The whole thing was encased in a tall vacuum tube, which was placed in a sealed basement whose temperature was controlled to within 0.02°C. The total mass of the platinum and aluminum test masses, plus the tungsten wire and the balance arms, was only 4.4 g. To detect tiny motions, a laser beam was bounced off of a mirror attached to the wire. There was so little friction that the balance would have taken on the order of several years to calm down completely after being put in place; to stop these vibrations, static electrical forces were applied through the two circular plates to provide very gentle twists on the ellipsoidal mass between them. After Braginskii and Panov.

One way to recover this distinction would be if we had access to some exotic matter — call it FloatyStuff™ — that had the ordinary amount of inertia, but was completely unaffected by gravity. Normally when we release a material object in a gravitational field, it experiences a force $mg$, and then by Newton’s second law its acceleration is $a = F/m = mg/m = g$. The $m$’s cancel, which is the reason that everything falls with the same acceleration (in the absence of other forces such as air resistance). If Alice and Betty both release a blob of FloatyStuff, they observe different results. Unfortunately, nobody has ever found anything like FloatyStuff. In fact, extremely delicate experiments have shown that the proportionality between weight and inertia holds to the incredible precision of one part in $10^{12}$. Figure n shows a crude test of this type, figure o a concept better suited to high-precision tests, and p a diagram of the actual apparatus used in one such experiment.\(^5\)

If we could tell Newton the story of Alice and Betty, he would probably propose a different solution: don’t seal the twins in boxes. Let them look around at all the nearby objects that could be making gravitational forces on their pencils. Alice will see such an object (the planet earth), so she’ll know that her pencil is subject to a nonzero force and that her frame is noninertial. Betty will not see any planet, so she’ll know that her frame is inertial.

The problem with Newton’s solution is that gravity can act from very far away. For example, Newton didn’t know that our solar system was embedded in the Milky Way Galaxy, so he imagined that the gravitational forces it felt from the uniform background of stars would almost perfectly cancel out by symmetry. But in reality, the galactic core is off in the direction of the constellation Sagittarius, and our solar system experiences a nonzero force in that direction, which keeps us from flying off straight and leaving

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\(^5\)V.B. Braginskii and V.I. Panov, Soviet Physics JETP 34, 463 (1972).
the galaxy. No problem, says Newton, that just means we should have taken our galaxy’s center of mass to define an inertial frame. But our galaxy turns out to be free-falling toward a distant-future collision with the Andromeda Galaxy. We can keep on zooming out, and the residual gravitational accelerations get smaller and smaller, but there is no guarantee that the process will ever terminate with a perfect inertial frame. We do find, however, that the accelerations seem to get pretty small on large scales. For example, our galaxy’s acceleration due to the gravitational attraction of the Andromeda Galaxy is only about $10^{-13} \text{ m/s}^2$.

Furthermore, these accelerations don’t necessarily hurt us, even if we don’t know about them and fail to take them into account. Alice, free-falling in her elevator, gets perfectly valid experimental results, identical to Betty’s in outer space. The only real problem would be if Alice did an experiment sensitive enough to be affected by the tiny difference in gravity between the floor and ceiling of the elevator. (The ocean tides are caused by small differences of this type in the moon’s gravity.) For a small enough laboratory, i.e., on a local scale, we expect such effects to be negligible for most purposes.

An example of an empirical test

These ambiguities in defining an inertial frame are not severe enough to prevent us from performing highly precise tests of the first law. One important type of test comes from observations in which the “laboratory” is our solar system. External bodies do produce gravitational forces that intrude into the solar system, but these forces are quite weak, and their differences from one side of the solar system to another are weaker still. Therefore the workings of the solar system can be considered as a local experiment.

The left panel of figure q shows a mirror on the moon. By reflecting laser pulses from the mirror, the distance from the earth to the moon has been measured to the phenomenal precision of a few centimeters, or about one part in $10^{10}$. This distance changes for a variety of known reasons. The biggest effect is that the moon’s orbit is not a circle but an ellipse (see ch. 10), with its long axis about 11% longer than its short one. A variety of other effects can also be accounted for, including such exotic phenomena as the slightly nonspherical shape of the earth, and the gravitational forces of bodies as small and distant as Pluto. Suppose for simplicity that all these effects had never existed, so that the moon was initially placed in a perfectly circular orbit around the earth, and the earth in a perfectly circular orbit around the sun.

If we then observed something like what is shown in the right panel of figure q, Newton’s first law would be disproved. If space itself is symmetrical in all directions, then there is no reason for the moon’s orbit to poof up near the top of the diagram and con-
Left: The Apollo 11 mission left behind a mirror, which in this photo shows the reflection of the black sky. Right: A highly exaggerated example of an observation that would disprove Newton’s first law. The radius of the moon’s orbit gets bigger and smaller over the course of a year.

The only possible explanation would be that there was some “special” or “preferred” frame of reference of the type envisioned by Aristotle, and that our solar system was moving relative to it. One could then imagine that the gravitational force of the earth on the moon could be affected by the moon’s motion relative to this frame. The lunar laser ranging data contain no measurable effect of the type shown in figure q, so that if the moon’s orbit is distorted in this way (or in a variety of other ways), the distortion must be less than a few centimeters. This constitutes a very strict upper limit on violation of Newton’s first law by gravitational forces. If the first law is violated, and the violation causes a fractional change in gravity that is proportional to the velocity relative to the hypothetical preferred frame, then the change is no more than about one part in $10^7$, even if the velocity is comparable to the speed of light. This is only one particular experiment involving gravity, but many different types of experiments have been done, and none have given any evidence for a preferred frame.

**Newton’s second law**

Newton’s second law, $a = F/m$, is false in general.

It fails at the microscopic level because particles are not just par-

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articles, they’re also waves. One consequence is that they do not have exactly well defined positions, so that the acceleration $a$ appearing in $a = F/m$ is not even well defined.

Example 13 on p. 141 shows an example of the failure of the second law as electrons approach the speed of light. We’ve seen in section 2.6 that relativity forbids objects from moving at speeds faster than the speed of light. We will see in section 14.7 that an object’s inertia $F/a$ is larger for an object moving closer to the speed of light (relative to the observer who measures $F$ and $a$). It is not a velocity-independent constant $m$ as in the usual interpretation of the second law. The second law is nevertheless highly accurate within its domain of validity, i.e., small relative speeds.

It is difficult in general to design an unambiguous test of the second law because if we like, we can take $F = ma$ to be a definition of force. For example, in the experiment with the electrons, we could simply say that the force must have been mysteriously decreasing, despite our best efforts to keep it constant. One way around this is to use the fact that the second law should be reproducible. For example, if the earth makes a certain gravitational force on the moon at a certain point in the moon’s orbit, then it is to be expected that one month later, when the moon has revolved once around the earth and is back at the same point, we should again obtain the same force and acceleration. If, for example, we saw that the moon had a slightly larger acceleration this time around, we could interpret this as evidence for a gradual trend of reducing mass, or increasing force. But either way, this trend over time would be a violation of Newton’s laws, because it would not have been caused by any change in the conditions to which the moon was subjected. Lunar laser ranging experiments of the type described above show that if any such trend in acceleration exists, it must be less than about one part in $10^{12}$ per year.

Newton’s second law refers to the total force acting on an object, so it predicts that forces are exactly additive. If we apply two forces of exactly 1 N to an object, the result is supposed to be exactly 2 N, not 1.999997 N or 2.00002 N. An alternative physical theory called MOND has been proposed, in an attempt to avoid the need for invoking exotic “dark matter” in order to correctly describe the rotation of galaxies. MOND is only approximately additive, so one of its predictions is that the gravitational forces exerted by the galaxy on the planets of the solar system would interact in complicated ways with the solar system’s internal gravitational forces, producing tiny discrepancies with the predictions of Newton’s laws. High-precision data from spacecraft have failed to detect this effect, and have limited any such anomalous accelerations of the planets to no more than about $10^{-14}$ m/s$^2$.

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7Iorio, arxiv.org/abs/0906.2937
Summary

Selected vocabulary
- weight . . . . . . . . the force of gravity on an object, equal to $mg$
- inertial frame . . a frame of reference that is not accelerating, one in which Newton’s first law is true
- noninertial frame an accelerating frame of reference, in which Newton’s first law is violated

Notation
- $F_W$ . . . . . . . . weight

Other terminology and notation
- net force . . . . another way of saying “total force”

Summary

Newton’s first law of motion states that if all the forces acting on an object cancel each other out, then the object continues in the same state of motion. This is essentially a more refined version of Galileo’s principle of inertia, which did not refer to a numerical scale of force.

Newton’s second law of motion allows the prediction of an object’s acceleration given its mass and the total force on it, $a_{cm} = F_{\text{total}}/m$. This is only the one-dimensional version of the law; the full-three dimensional treatment will come in chapter 8, Vectors. Without the vector techniques, we can still say that the situation remains unchanged by including an additional set of vectors that cancel among themselves, even if they are not in the direction of motion.

Newton’s laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.

Even in one-dimensional motion, it is seldom possible to solve real-world problems and predict the motion of an object in closed form. However, there are straightforward numerical techniques for solving such problems.
Problems

Key
✓ A computerized answer check is available online.
∫ A problem that requires calculus.
★ A difficult problem.

1 A car is accelerating forward along a straight road. If the force of the road on the car’s wheels, pushing it forward, is a constant 3.0 kN, and the car’s mass is 1000 kg, then how long will the car take to go from 20 m/s to 50 m/s?

2 (a) Let $T$ be the maximum tension that an elevator’s cable can withstand without breaking, i.e., the maximum force it can exert. If the motor is programmed to give the car an acceleration $a$ ($a > 0$ is upward), what is the maximum mass that the car can have, including passengers, if the cable is not to break?
(b) Interpret the equation you derived in the special cases of $a = 0$ and of a downward acceleration of magnitude $g$.

3 An object is observed to be moving at constant speed in a certain direction. Can you conclude that no forces are acting on it? Explain. [Based on a problem by Serway and Faughn.]

4 You are given a large sealed box, and are not allowed to open it. Which of the following experiments measure its mass, and which measure its weight? [Hint: Which experiments would give different results on the moon?]
(a) Put it on a frozen lake, throw a rock at it, and see how fast it scoots away after being hit.
(b) Drop it from a third-floor balcony, and measure how loud the sound is when it hits the ground.
(c) As shown in the figure, connect it with a spring to the wall, and watch it vibrate.

5 While escaping from the palace of the evil Martian emperor, Sally Spacehound jumps from a tower of height $h$ down to the ground. Ordinarily the fall would be fatal, but she fires her blaster rifle straight down, producing an upward force of magnitude $F_B$. This force is insufficient to levitate her, but it does cancel out some of the force of gravity. During the time $t$ that she is falling, Sally is unfortunately exposed to fire from the emperor’s minions, and can’t dodge their shots. Let $m$ be her mass, and $g$ the strength of gravity on Mars.
(a) Find the time $t$ in terms of the other variables.
(b) Check the units of your answer to part a.
(c) For sufficiently large values of $F_B$, your answer to part a becomes nonsense — explain what’s going on.
At low speeds, every car’s acceleration is limited by traction, not by the engine’s power. Suppose that at low speeds, a certain car is normally capable of an acceleration of 3 m/s². If it is towing a trailer with half as much mass as the car itself, what acceleration can it achieve? [Based on a problem from PSSC Physics.]

7 A helicopter of mass $m$ is taking off vertically. The only forces acting on it are the earth’s gravitational force and the force, $F_{\text{air}}$, of the air pushing up on the propeller blades.
(a) If the helicopter lifts off at $t = 0$, what is its vertical speed at time $t$?
(b) Check that the units of your answer to part a make sense.
(c) Discuss how your answer to part a depends on all three variables, and show that it makes sense. That is, for each variable, discuss what would happen to the result if you changed it while keeping the other two variables constant. Would a bigger value give a smaller result, or a bigger result? Once you’ve figured out this mathematical relationship, show that it makes sense physically.
(d) Plug numbers into your equation from part a, using $m = 2300$ kg, $F_{\text{air}} = 27000$ N, and $t = 4.0$ s.

8 A uranium atom deep in the earth spits out an alpha particle. An alpha particle is a fragment of an atom. This alpha particle has initial speed $v$, and travels a distance $d$ before stopping in the earth.
(a) Find the force, $F$, from the dirt that stopped the particle, in terms of $v, d$, and its mass, $m$. Don’t plug in any numbers yet. Assume that the force was constant.
(b) Show that your answer has the right units.
(c) Discuss how your answer to part a depends on all three variables, and show that it makes sense. That is, for each variable, discuss what would happen to the result if you changed it while keeping the other two variables constant. Would a bigger value give a smaller result, or a bigger result? Once you’ve figured out this mathematical relationship, show that it makes sense physically.
(d) Evaluate your result for $m = 6.7 \times 10^{-27}$ kg, $v = 2.0 \times 10^4$ km/s, and $d = 0.71$ mm.

9 A blimp is initially at rest, hovering, when at $t = 0$ the pilot turns on the engine driving the propeller. The engine cannot instantly get the propeller going, but the propeller speeds up steadily. The steadily increasing force between the air and the propeller is given by the equation $F = kt$, where $k$ is a constant. If the mass of the blimp is $m$, find its position as a function of time. (Assume that during the period of time you’re dealing with, the blimp is not yet moving fast enough to cause a significant backward force due to air resistance.)

10 Some garden shears are like a pair of scissors: one sharp blade
slices past another. In the “anvil” type, however, a sharp blade presses against a flat one rather than going past it. A gardening book says that for people who are not very physically strong, the anvil type can make it easier to cut tough branches, because it concentrates the force on one side. Evaluate this claim based on Newton’s laws. [Hint: Consider the forces acting on the branch, and the motion of the branch.]

11 In the 1964 Olympics in Tokyo, the best men’s high jump was 2.18 m. Four years later in Mexico City, the gold medal in the same event was for a jump of 2.24 m. Because of Mexico City’s altitude (2400 m), the acceleration of gravity there is lower than that in Tokyo by about 0.01 m/s². Suppose a high-jumper has a mass of 72 kg.

(a) Compare his mass and weight in the two locations.
(b) Assume that he is able to jump with the same initial vertical velocity in both locations, and that all other conditions are the same except for gravity. How much higher should he be able to jump in Mexico City? ✓

(Actually, the reason for the big change between ’64 and ’68 was the introduction of the “Fosbury flop.”)

12 The factorial of an integer \( n \), written \( n! \), is defined as the product of all the positive integers less than or equal to \( n \). For example, \( 3! = 1 \times 2 \times 3 = 6 \). Write a Python program to compute the factorial of a number. Test it with a small number whose factorial you can check by hand. Then use it to compute 30!. (Python computes integer results with unlimited precision, so you won’t get any problems with rounding or overflows.) Turn in a printout of your program and its output, including the test.

13 A ball falls from a height \( h \). Without air resistance, the time it takes to reach the floor is \( \sqrt{2h/g} \). Now suppose that air resistance is not negligible. For a smooth sphere of radius \( r \), moving at speed \( v \) through air of density \( \rho \), the force of air resistance is \( (\pi/4)\rho v^2 r^2 \).

Modify the program meteor on page 150 to handle this problem, and find the resulting change in the fall time in the case of a 21 g ball of radius 1.0 cm, falling from a height of 1.0 m. The density of air at sea level is about 1.2 kg/m³. You will need to use a very large value of \( n \) to achieve the required precision. Turn in a printout of both your program and its output. Answer: 0.34 ms.

14 The tires used in Formula 1 race cars can generate traction (i.e., force from the road) that is as much as 1.9 times greater than with the tires typically used in a passenger car. Suppose that we’re trying to see how fast a car can cover a fixed distance starting from rest, and traction is the limiting factor. By what factor is this time reduced when switching from ordinary tires to Formula 1 tires?
15  At the turn of the 20th century, Samuel Langley engaged in a bitter rivalry with the Wright brothers to develop human flight. Langley’s design used a catapult for launching. For safety, the catapult was built on the roof of a houseboat, so that any crash would be into the water. This design required reaching cruising speed within a fixed, short distance, so large accelerations were required, and the forces frequently damaged the craft, causing dangerous and embarrassing accidents. Langley achieved several uncrewed, unguided flights, but never succeeded with a human pilot. If the force of the catapult is fixed by the structural strength of the plane, and the distance for acceleration by the size of the houseboat, by what factor is the launch velocity reduced when the plane’s 340 kg is augmented by the 60 kg mass of a small man? \[ \sqrt{\frac{16}{1}} \]

16  A bullet of mass \( m \) is fired from a pistol, accelerating from rest to a speed \( v \) in the barrel’s length \( L \).
(a) What is the force on the bullet? (Assume this force is constant.) \( \sqrt{\text{[answer]}} \)
(b) Check that the units of your answer to part a make sense.
(c) Check that the dependence of your answer on each of the three variables makes sense.
[problem by B. Shotwell]

17  Blocks of mass \( M_1 \), \( M_2 \), and \( M_3 \) are stacked on a table as shown in the figure. Let the upward direction be positive.
(a) What is the force on block 2 from block 3? \( \sqrt{\text{[answer]}} \)
(b) What is the force on block 2 from block 1? \( \sqrt{\text{[answer]}} \)
[problem by B. Shotwell]
Exercise 4: Force and motion

Equipment:

- 2-meter pieces of butcher paper
- wood blocks with hooks
- string
- masses to put on top of the blocks to increase friction
- spring scales (preferably calibrated in Newtons)

Suppose a person pushes a crate, sliding it across the floor at a certain speed, and then repeats the same thing but at a higher speed. This is essentially the situation you will act out in this exercise. What do you think is different about her force on the crate in the two situations? Discuss this with your group and write down your hypothesis:

1. First you will measure the amount of friction between the wood block and the butcher paper when the wood and paper surfaces are slipping over each other. The idea is to attach a spring scale to the block and then slide the butcher paper under the block while using the scale to keep the block from moving with it. Depending on the amount of force your spring scale was designed to measure, you may need to put an extra mass on top of the block in order to increase the amount of friction. It is a good idea to use long piece of string to attach the block to the spring scale, since otherwise one tends to pull at an angle instead of directly horizontally.

First measure the amount of friction force when sliding the butcher paper as slowly as possible:

Now measure the amount of friction force at a significantly higher speed, say 1 meter per second. (If you try to go too fast, the motion is jerky, and it is impossible to get an accurate reading.)

Discuss your results. Why are we justified in assuming that the string’s force on the block (i.e., the scale reading) is the same amount as the paper’s frictional force on the block?

2. Now try the same thing but with the block moving and the paper standing still. Try two different speeds.

Do your results agree with your original hypothesis? If not, discuss what’s going on. How does the block “know” how fast to go?
Chapter 5
Analysis of Forces

5.1 Newton’s third law

Newton created the modern concept of force starting from his insight that all the effects that govern motion are interactions between two objects: unlike the Aristotelian theory, Newtonian physics has no phenomena in which an object changes its own motion.
Is one object always the “order-giver” and the other the “order-follower”? As an example, consider a batter hitting a baseball. The bat definitely exerts a large force on the ball, because the ball accelerates drastically. But if you have ever hit a baseball, you also know that the ball makes a force on the bat — often with painful results if your technique is as bad as mine!

How does the ball’s force on the bat compare with the bat’s force on the ball? The bat’s acceleration is not as spectacular as the ball’s, but maybe we shouldn’t expect it to be, since the bat’s mass is much greater. In fact, careful measurements of both objects’ masses and accelerations would show that $m_{\text{ball}}a_{\text{ball}}$ is very nearly equal to $-m_{\text{bat}}a_{\text{bat}}$, which suggests that the ball’s force on the bat is of the same magnitude as the bat’s force on the ball, but in the opposite direction.

Figures a and b show two somewhat more practical laboratory experiments for investigating this issue accurately and without too much interference from extraneous forces.

In experiment a, a large magnet and a small magnet are weighed separately, and then one magnet is hung from the pan of the top balance so that it is directly above the other magnet. There is an attraction between the two magnets, causing the reading on the top scale to increase and the reading on the bottom scale to decrease. The large magnet is more “powerful” in the sense that it can pick up a heavier paperclip from the same distance, so many people have a strong expectation that one scale’s reading will change by a far different amount than the other. Instead, we find that the two changes are equal in magnitude but opposite in direction: the force of the bottom magnet pulling down on the top one has the same strength as the force of the top one pulling up on the bottom one.

In experiment b, two people pull on two spring scales. Regardless of who tries to pull harder, the two forces as measured on the spring scales are equal. Interposing the two spring scales is necessary in order to measure the forces, but the outcome is not some artificial result of the scales’ interactions with each other. If one person slaps another hard on the hand, the slapper’s hand hurts just as much as the slappee’s, and it doesn’t matter if the recipient of the slap tries to be inactive. (Punching someone in the mouth causes just as much force on the fist as on the lips. It’s just that the lips are more delicate. The forces are equal, but not the levels of pain and injury.)

Newton, after observing a series of results such as these, decided that there must be a fundamental law of nature at work:
Newton’s third law

Forces occur in equal and opposite pairs: whenever object A exerts a force on object B, object B must also be exerting a force on object A. The two forces are equal in magnitude and opposite in direction.

Two modern, high-precision tests of the third law are described on p. 189.

In one-dimensional situations, we can use plus and minus signs to indicate the directions of forces, and Newton’s third law can be written succinctly as $F_A \text{ on } B = -F_B \text{ on } A$. Section 5.7 gives a more detailed discussion of the logical and empirical underpinnings of the third law.

self-check A

Figure d analyzes swimming using Newton’s third law. Do a similar analysis for a sprinter leaving the starting line. > Answer, p. 568

There is no cause and effect relationship between the two forces in Newton’s third law. There is no “original” force, and neither one is a response to the other. The pair of forces is a relationship, like marriage, not a back-and-forth process like a tennis match. Newton came up with the third law as a generalization about all the types of forces with which he was familiar, such as frictional and gravitational forces. When later physicists discovered a new type of force, such as the force that holds atomic nuclei together, they had to check whether it obeyed Newton’s third law. So far, no violation of the third law has ever been discovered, whereas the first and second laws were shown to have limitations by Einstein and the pioneers of atomic physics.

The English vocabulary for describing forces is unfortunately rooted in Aristotelianism, and often implies incorrectly that forces are one-way relationships. It is unfortunate that a half-truth such as “the table exerts an upward force on the book” is so easily expressed, while a more complete and correct description ends up sounding awkward or strange: “the table and the book interact via a force,” or “the table and book participate in a force.”

To students, it often sounds as though Newton’s third law implies nothing could ever change its motion, since the two equal and opposite forces would always cancel. The two forces, however, are always on two different objects, so it doesn’t make sense to add them in the first place — we only add forces that are acting on the same object. If two objects are interacting via a force and no other forces are involved, then both objects will accelerate — in opposite directions!
It doesn’t make sense for the man to talk about using the woman’s money to cancel out his bar tab, because there is no good reason to combine his debts and her assets. Similarly, it doesn’t make sense to refer to the equal and opposite forces of Newton’s third law as canceling. It only makes sense to add up forces that are acting on the same object, whereas two forces related to each other by Newton’s third law are always acting on two different objects.

A mnemonic for using Newton’s third law correctly

Mnemonics are tricks for memorizing things. For instance, the musical notes that lie between the lines on the treble clef spell the word FACE, which is easy to remember. Many people use the mnemonic “SOHCAHTOA” to remember the definitions of the sine, cosine, and tangent in trigonometry. I have my own modest offering, POFOSTITO, which I hope will make it into the mnemonics hall of fame. It’s a way to avoid some of the most common problems with applying Newton’s third law correctly:

- P air of
- O pposite
- F orces
- O f the
- S ame
- T ype
- I nvolving
- T wo
- O bjects

**Example 1**

A book lying on a table

A book is lying on a table. What force is the Newton’s-third-law partner of the earth’s gravitational force on the book?

Answer: Newton’s third law works like “B on A, A on B,” so the partner must be the book’s gravitational force pulling upward on the planet earth. Yes, there is such a force! No, it does not cause the earth to do anything noticeable.

Incorrect answer: The table’s upward force on the book is the Newton’s-third-law partner of the earth’s gravitational force on the book.

This answer violates two out of three of the commandments of POFOSTITO. The forces are not of the same type, because the table’s upward force on the book is not gravitational. Also, three
Newton's third law and action at a distance

Newton’s third law is completely symmetric in the sense that neither force constitutes a delayed response to the other. Newton’s third law does not even mention time, and the forces are supposed to agree at any given instant. This creates an interesting situation when it comes to noncontact forces. Suppose two people are holding magnets, and when one person waves or wiggles her magnet, the other person feels an effect on his. In this way they can send signals to each other from opposite sides of a wall, and if Newton’s third law is correct, it would seem that the signals are transmitted instantly, with no time lag. The signals are indeed transmitted quite quickly, but experiments with electrically controlled magnets show that the signals do not leap the gap instantly: they travel at the same speed as light, which is an extremely high speed but not an infinite one.

Is this a contradiction to Newton’s third law? Not really. According to current theories, there are no true noncontact forces. Action at a distance does not exist. Although it appears that the wiggling of one magnet affects the other with no need for anything to be in contact with anything, what really happens is that wiggling a magnet creates a ripple in the magnetic field pattern that exists even in empty space. The magnet shoves the ripples out with a kick and receives a kick in return, in strict obedience to Newton’s third law. The ripples spread out in all directions, and the ones that hit the other magnet then interact with it, again obeying Newton’s third law.
Discussion questions

A  When you fire a gun, the exploding gases push outward in all directions, causing the bullet to accelerate down the barrel. What third-law pairs are involved? [Hint: Remember that the gases themselves are an object.]

B  Tam Anh grabs Sarah by the hand and tries to pull her. She tries to remain standing without moving. A student analyzes the situation as follows. “If Tam Anh’s force on Sarah is greater than her force on him, he can get her to move. Otherwise, she’ll be able to stay where she is.” What’s wrong with this analysis?

C  You hit a tennis ball against a wall. Explain any and all incorrect ideas in the following description of the physics involved: “According to Newton’s third law, there has to be a force opposite to your force on the ball. The opposite force is the ball’s mass, which resists acceleration, and also air resistance.”

5.2 Classification and behavior of forces

One of the most basic and important tasks of physics is to classify the forces of nature. I have already referred informally to “types” of forces such as friction, magnetism, gravitational forces, and so on. Classification systems are creations of the human mind, so there is always some degree of arbitrariness in them. For one thing, the level of detail that is appropriate for a classification system depends on what you’re trying to find out. Some linguists, the “lumpers,” like to emphasize the similarities among languages, and a few extremists have even tried to find signs of similarities between words in languages as different as English and Chinese, lumping the world’s languages into only a few large groups. Other linguists, the “splitters,” might be more interested in studying the differences in pronunciation between English speakers in New York and Connecticut. The splitters call the lumpers sloppy, but the lumpers say that science isn’t worthwhile unless it can find broad, simple patterns within the seemingly complex universe.

Scientific classification systems are also usually compromises between practicality and naturalness. An example is the question of how to classify flowering plants. Most people think that biological classification is about discovering new species, naming them, and classifying them in the class-order-family-genus-species system according to guidelines set long ago. In reality, the whole system is in a constant state of flux and controversy. One very practical way of classifying flowering plants is according to whether their petals are separate or joined into a tube or cone — the criterion is so clear that it can be applied to a plant seen from across the street. But here practicality conflicts with naturalness. For instance, the begonia has
separate petals and the pumpkin has joined petals, but they are so similar in so many other ways that they are usually placed within the same order. Some taxonomists have come up with classification criteria that they claim correspond more naturally to the apparent relationships among plants, without having to make special exceptions, but these may be far less practical, requiring for instance the examination of pollen grains under an electron microscope.

In physics, there are two main systems of classification for forces. At this point in the course, you are going to learn one that is very practical and easy to use, and that splits the forces up into a relatively large number of types: seven very common ones that we’ll discuss explicitly in this chapter, plus perhaps ten less important ones such as surface tension, which we will not bother with right now.

Physicists, however, are obsessed with finding simple patterns, so recognizing as many as fifteen or twenty types of forces strikes them as distasteful and overly complex. Since about the year 1900, physics has been on an aggressive program to discover ways in which these many seemingly different types of forces arise from a smaller number of fundamental ones. For instance, when you press your hands together, the force that keeps them from passing through each other may seem to have nothing to do with electricity, but at the atomic level, it actually does arise from electrical repulsion between atoms. By about 1950, all the forces of nature had been explained as arising from four fundamental types of forces at the atomic and nuclear level, and the lumping-together process didn’t stop there. By the 1960’s the length of the list had been reduced to three, and some theorists even believe that they may be able to reduce it to two or one. Although the unification of the forces of nature is one of the most beautiful and important achievements of physics, it makes much more sense to start this course with the more practical and easy system of classification. The unified system of four forces will be one of the highlights of the end of your introductory physics sequence.

The practical classification scheme which concerns us now can be laid out in the form of the tree shown in figure i. The most specific types of forces are shown at the tips of the branches, and it is these types of forces that are referred to in the POFOSTITO mnemonic. For example, electrical and magnetic forces belong to the same general group, but Newton’s third law would never relate an electrical force to a magnetic force.

The broadest distinction is that between contact and noncontact forces, which has been discussed in ch. 4. Among the contact forces, we distinguish between those that involve solids only and those that have to do with fluids, a term used in physics to include both gases and liquids.
It should not be necessary to memorize this diagram by rote. It is better to reinforce your memory of this system by calling to mind your commonsense knowledge of certain ordinary phenomena. For instance, we know that the gravitational attraction between us and the planet earth will act even if our feet momentarily leave the ground, and that although magnets have mass and are affected by gravity, most objects that have mass are nonmagnetic.

**Hit an a wall**

▷ A bullet, flying horizontally, hits a steel wall. What type of force is there between the bullet and the wall?

▷ Starting at the bottom of the tree, we determine that the force is a contact force, because it only occurs once the bullet touches the wall. Both objects are solid. The wall forms a vertical plane. If the nose of the bullet was some shape like a sphere, you might imagine that it would only touch the wall at one point. Realistically, however, we know that a lead bullet will flatten out a lot on impact, so there is a surface of contact between the two, and its
orientation is vertical. The effect of the force on the bullet is to stop the horizontal motion of the bullet, and this horizontal acceleration must be produced by a horizontal force. The force is therefore perpendicular to the surface of contact, and it’s also repulsive (tending to keep the bullet from entering the wall), so it must be a normal force.

Diagram i is meant to be as simple as possible while including most of the forces we deal with in everyday life. If you were an insect, you would be much more interested in the force of surface tension, which allowed you to walk on water. I have not included the nuclear forces, which are responsible for holding the nuclei of atoms, because they are not evident in everyday life.

You should not be afraid to invent your own names for types of forces that do not fit into the diagram. For instance, the force that holds a piece of tape to the wall has been left off of the tree, and if you were analyzing a situation involving scotch tape, you would be absolutely right to refer to it by some commonsense name such as “sticky force.”

On the other hand, if you are having trouble classifying a certain force, you should also consider whether it is a force at all. For instance, if someone asks you to classify the force that the earth has because of its rotation, you would have great difficulty creating a place for it on the diagram. That’s because it’s a type of motion, not a type of force!

**Normal forces**

A normal force, \( F_N \), is a force that keeps one solid object from passing through another. “Normal” is simply a fancy word for “perpendicular,” meaning that the force is perpendicular to the surface of contact. Intuitively, it seems the normal force magically adjusts itself to provide whatever force is needed to keep the objects from occupying the same space. If your muscles press your hands together gently, there is a gentle normal force. Press harder, and the normal force gets stronger. How does the normal force know how strong to be? The answer is that the harder you jam your hands together, the more compressed your flesh becomes. Your flesh is acting like a spring: more force is required to compress it more. The same is true when you push on a wall. The wall flexes imperceptibly in proportion to your force on it. If you exerted enough force, would it be possible for two objects to pass through each other? No, typically the result is simply to strain the objects so much that one of them breaks.

**Gravitational forces**

As we’ll discuss in more detail later in the course, a gravitational force exists between any two things that have mass. In everyday life,
A model that correctly explains many properties of friction. The microscopic bumps and holes in two surfaces dig into each other.

Static and kinetic friction

If you have pushed a refrigerator across a kitchen floor, you have felt a certain series of sensations. At first, you gradually increased your force on the refrigerator, but it didn’t move. Finally, you supplied enough force to unstick the fridge, and there was a sudden jerk as the fridge started moving. Once the fridge was unstuck, you could reduce your force significantly and still keep it moving.

While you were gradually increasing your force, the floor’s frictional force on the fridge increased in response. The two forces on the fridge canceled, and the fridge didn’t accelerate. How did the floor know how to respond with just the right amount of force? Figure j shows one possible model of friction that explains this behavior. (A scientific model is a description that we expect to be incomplete, approximate, or unrealistic in some ways, but that nevertheless succeeds in explaining a variety of phenomena.) Figure j/1 shows a microscopic view of the tiny bumps and holes in the surfaces of the floor and the refrigerator. The weight of the fridge presses the two surfaces together, and some of the bumps in one surface will settle as deeply as possible into some of the holes in the other surface. In j/2, your leftward force on the fridge has caused it to ride up a little higher on the bump in the floor labeled with a small arrow. Still more force is needed to get the fridge over the bump and allow it to start moving. Of course, this is occurring simultaneously at millions of places on the two surfaces.

Once you had gotten the fridge moving at constant speed, you found that you needed to exert less force on it. Since zero total force is needed to make an object move with constant velocity, the floor’s rightward frictional force on the fridge has apparently decreased somewhat, making it easier for you to cancel it out. Our model also gives a plausible explanation for this fact: as the surfaces slide past each other, they don’t have time to settle down and mesh with one another, so there is less friction.

Even though this model is intuitively appealing and fairly successful, it should not be taken too seriously, and in some situations it is misleading. For instance, fancy racing bikes these days are made with smooth tires that have no tread — contrary to what we’d expect from our model, this does not cause any decrease in friction. Machinists know that two very smooth and clean metal
Many landfowl, even those that are competent fliers, prefer to escape from a predator by running upward rather than by flying. This partridge is running up a vertical tree trunk. Humans can’t walk up walls because there is no normal force and therefore no frictional force; when $F_N = 0$, the maximum force of static friction, $F_{s,max}$, is also zero. The partridge, however, has wings that it can flap in order to create a force between it and the air. Typically when a bird flaps its wings, the resulting force from the air is in the direction that would tend to lift the bird up. In this situation, however, the partridge changes its style of flapping so that the direction is reversed. The normal force between the feet and the tree allows a nonzero static frictional force. The mechanism is similar to that of a spoiler fin on a racing car. Some evolutionary biologists believe that when vertebrate flight first evolved, in dinosaurs, there was first a stage in which the wings were used only as an aid in running up steep inclines, and only later a transition to flight. (Redrawn from a figure by K.P. Dial.)

The maximum possible force of static friction depends on what kinds of surfaces they are, and also on how hard they are being pressed together. The approximate mathematical relationships can be expressed as follows:

$$F_{s,max} = \mu_s F_N,$$

where $\mu_s$ is a unitless number, called the coefficient of static friction, which depends on what kinds of surfaces they are. The maximum force that static friction can supply, $\mu_s F_N$, represents the boundary between static and kinetic friction. It depends on the normal force, which is numerically equal to whatever force is pressing the two surfaces together. In terms of our model, if the two surfaces are being pressed together more firmly, a greater sideways force will be required in order to make the irregularities in the surfaces ride up and over each other.

Note that just because we use an adjective such as “applied” to refer to a force, that doesn’t mean that there is some special type of force called the “applied force.” The applied force could be any type of force, or it could be the sum of more than one force trying to make an object move.

**self-check C**

The arrows in figure m show the forces of the tree trunk on the partridge. Describe the forces the bird makes on the tree.  

The force of kinetic friction on each of the two objects is in the direction that resists the slippage of the surfaces. Its magnitude is

---

**self-check B**

1. When a baseball player slides in to a base, is the friction static, or kinetic?
2. A mattress stays on the roof of a slowly accelerating car. Is the friction static, or kinetic?
3. Does static friction create heat? Kinetic friction?  

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The force of kinetic friction on each of the two objects is in the direction that resists the slippage of the surfaces. Its magnitude is

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Section 5.2 Classification and behavior of forces
usually well approximated as

\[ F_k = \mu_k F_N \]

where \( \mu_k \) is the coefficient of kinetic friction. Kinetic friction is usually more or less independent of velocity.

We choose a coordinate system in which the applied force, i.e., the force trying to move the objects, is positive. The friction force is then negative, since it is in the opposite direction. As you increase the applied force, the force of static friction increases to match it and cancel it out, until the maximum force of static friction is surpassed. The surfaces then begin slipping past each other, and the friction force becomes smaller in absolute value.

\[ \text{self-check D} \]

Can a frictionless surface exert a normal force? Can a frictional force exist without a normal force?

If you try to accelerate or decelerate your car too quickly, the forces between your wheels and the road become too great, and they begin slipping. This is not good, because kinetic friction is weaker than static friction, resulting in less control. Also, if this occurs while you are turning, the car’s handling changes abruptly because the kinetic friction force is in a different direction than the static friction force had been: contrary to the car’s direction of motion, rather than contrary to the forces applied to the tire.

Most people respond with disbelief when told of the experimental evidence that both static and kinetic friction are approximately independent of the amount of surface area in contact. Even after doing a hands-on exercise with spring scales to show that it is true, many students are unwilling to believe their own observations, and insist that bigger tires “give more traction.” In fact, the main reason why you would not want to put small tires on a big heavy car is that the tires would burst!

Although many people expect that friction would be proportional to surface area, such a proportionality would make predictions contrary to many everyday observations. A dog’s feet, for example, have very little surface area in contact with the ground compared to a human’s feet, and yet we know that a dog can often win a tug-of-war with a person.
The reason a smaller surface area does not lead to less friction is that the force between the two surfaces is more concentrated, causing their bumps and holes to dig into each other more deeply.

**Self-check E**
Find the direction of each of the forces in figure o.  

**Answer, p. 568**

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**Locomotives**

Looking at a picture of a locomotive, p, we notice two obvious things that are different from an automobile. Where a car typically has two drive wheels, a locomotive normally has many — ten in this example. (Some also have smaller, unpowered wheels in front of and behind the drive wheels, but this example doesn’t.) Also, cars these days are generally built to be as light as possible for their size, whereas locomotives are very massive, and no effort seems to be made to keep their weight low. (The steam locomotive in the photo is from about 1900, but this is true even for modern diesel and electric trains.)

The reason locomotives are built to be so heavy is for traction. The upward normal force of the rails on the wheels, $F_N$, cancels the downward force of gravity, $F_W$, so ignoring plus and minus signs, these two forces are equal in absolute value, $F_N = F_W$. Given this amount of normal force, the maximum force of static friction is $F_s = \mu_s F_N = \mu_s F_W$. This static frictional force, of the rails pushing forward on the wheels, is the only force that can accelerate the train, pull it uphill, or cancel out the force of air resistance while cruising at constant speed. The coefficient of static friction for steel on steel is about 1/4, so no locomotive can pull with a force greater than about 1/4 of its own weight. If the
Fluid friction depends on the fluid's pattern of flow, so it is more complicated than friction between solids, and there are no simple, universally applicable formulas to calculate it. From top to bottom: supersonic wind tunnel, vortex created by a crop duster, series of vortices created by a single object, turbulence.

...engine is capable of supplying more than that amount of force, the result will be simply to break static friction and spin the wheels.

The reason this is all so different from the situation with a car is that a car isn't pulling something else. If you put extra weight in a car, you improve the traction, but you also increase the inertia of the car, and make it just as hard to accelerate. In a train, the inertia is almost all in the cars being pulled, not in the locomotive.

The other fact we have to explain is the large number of driving wheels. First, we have to realize that increasing the number of driving wheels neither increases nor decreases the total amount of static friction, because static friction is independent of the amount of surface area in contact. (The reason four-wheel-drive is good in a car is that if one or more of the wheels is slipping on ice or in mud, the other wheels may still have traction. This isn't typically an issue for a train, since all the wheels experience the same conditions.) The advantage of having more driving wheels on a train is that it allows us to increase the weight of the locomotive without crushing the rails, or damaging bridges.

**Fluid friction**

Try to drive a nail into a waterfall and you will be confronted with the main difference between solid friction and fluid friction. Fluid friction is purely kinetic; there is no static fluid friction. The nail in the waterfall may tend to get dragged along by the water flowing past it, but it does not stick in the water. The same is true for gases such as air: recall that we are using the word “fluid” to include both gases and liquids.

Unlike kinetic friction between solids, fluid friction increases rapidly with velocity. It also depends on the shape of the object, which is why a fighter jet is more streamlined than a Model T. For objects of the same shape but different sizes, fluid friction typically scales up with the cross-sectional area of the object, which is one of the main reasons that an SUV gets worse mileage on the freeway than a compact car.
Discussion questions

A A student states that when he tries to push his refrigerator, the reason it won’t move is because Newton’s third law says there’s an equal and opposite frictional force pushing back. After all, the static friction force is equal and opposite to the applied force. How would you convince him he is wrong?

B Kinetic friction is usually more or less independent of velocity. However, inexperienced drivers tend to produce a jerk at the last moment of deceleration when they stop at a stop light. What does this tell you about the kinetic friction between the brake shoes and the brake drums?

C Some of the following are correct descriptions of types of forces that could be added on as new branches of the classification tree. Others are not really types of forces, and still others are not force phenomena at all. In each case, decide what’s going on, and if appropriate, figure out how you would incorporate them into the tree.

- sticky force: makes tape stick to things
- opposite force: the force that Newton’s third law says relates to every force you make
- flowing force: the force that water carries with it as it flows out of a hose
- surface tension: lets insects walk on water
- horizontal force: a force that is horizontal
- motor force: the force that a motor makes on the thing it is turning
- canceled force: a force that is being canceled out by some other force

5.3 Analysis of forces

Newton’s first and second laws deal with the total of all the forces exerted on a specific object, so it is very important to be able to figure out what forces there are. Once you have focused your attention on one object and listed the forces on it, it is also helpful to describe all the corresponding forces that must exist according to Newton’s third law. We refer to this as “analyzing the forces” in which the object participates.

s / What do the golf ball and the shark have in common? Both use the same trick to reduce fluid friction. The dimples on the golf ball modify the pattern of flow of the air around it, counterintuitively reducing friction. Recent studies have shown that sharks can accomplish the same thing by raising, or “bristling,” the scales on their skin at high speeds.

The wheelbases of the Hummer H3 and the Toyota Prius are surprisingly similar, differing by only 10%. The main difference in shape is that the Hummer is much taller and wider. It presents a much greater cross-sectional area to the wind, and this is the main reason that it uses about 2.5 times more gas on the freeway.
A barge example 6

A barge is being pulled to the right along a canal by teams of horses on the shores. Analyze all the forces in which the barge participates.

<table>
<thead>
<tr>
<th>force acting on barge</th>
<th>force related to it by Newton’s third law</th>
</tr>
</thead>
<tbody>
<tr>
<td>ropes’ normal forces on barge, →</td>
<td>barge’s normal force on ropes, ←</td>
</tr>
<tr>
<td>water’s fluid friction force on barge, ←</td>
<td>barge’s fluid friction force on water, →</td>
</tr>
<tr>
<td>planet earth’s gravitational force on barge, ↓</td>
<td>barge’s gravitational force on earth, ↑</td>
</tr>
<tr>
<td>water’s “floating” force on barge, ↑</td>
<td>barge’s “floating” force on water, ↓</td>
</tr>
</tbody>
</table>

Here I’ve used the word “floating” force as an example of a sensible invented term for a type of force not classified on the tree on p. 170. A more formal technical term would be “hydrostatic force.”

Note how the pairs of forces are all structured as “A’s force on B, B’s force on A”: ropes on barge and barge on ropes; water on barge and barge on water. Because all the forces in the left column are forces acting on the barge, all the forces in the right column are forces being exerted by the barge, which is why each entry in the column begins with “barge.”

Often you may be unsure whether you have forgotten one of the forces. Here are three strategies for checking your list:

1. See what physical result would come from the forces you’ve found so far. Suppose, for instance, that you’d forgotten the “floating” force on the barge in the example above. Looking at the forces you’d found, you would have found that there was a downward gravitational force on the barge which was not canceled by any upward force. The barge isn’t supposed to sink, so you know you need to find a fourth, upward force.

2. Another technique for finding missing forces is simply to go through the list of all the common types of forces and see if any of them apply.

3. Make a drawing of the object, and draw a dashed boundary line around it that separates it from its environment. Look for points on the boundary where other objects come in contact with your object. This strategy guarantees that you’ll find every contact force that acts on the object, although it won’t help you to find non-contact forces.

Fifi example 7

▷ Fifi is an industrial espionage dog who loves doing her job and looks great doing it. She leaps through a window and lands at initial horizontal speed $v_0$ on a conveyor belt which is itself moving at the greater speed $v_b$. Unfortunately the coefficient of kinetic friction $\mu_k$ between her foot-pads and the belt is fairly low, so she skids for a time $\Delta t$, during which the effect on her coiffure is un désastre. Find $\Delta t$. 

178 Chapter 5 Analysis of Forces
We analyze the forces:

<table>
<thead>
<tr>
<th>force acting on Fifi</th>
<th>force related to it by Newton’s third law</th>
</tr>
</thead>
<tbody>
<tr>
<td>planet earth’s gravitational force $F_W = mg$ on Fifi, ↓</td>
<td>Fifi’s gravitational force on earth, ↑</td>
</tr>
<tr>
<td>belt’s kinetic frictional force $F_k$ on Fifi, →</td>
<td>Fifi’s kinetic frictional force on belt, ←</td>
</tr>
<tr>
<td>belt’s normal force $F_N$ on Fifi, ↑</td>
<td>Fifi’s normal force on belt, ↓</td>
</tr>
</tbody>
</table>

Checking the analysis of the forces as described on p. 178:

(1) The physical result makes sense. The left-hand column consists of forces $\downarrow \rightarrow \uparrow$. We’re describing the time when she’s moving horizontally on the belt, so it makes sense that we have two vertical forces that could cancel. The rightward force is what will accelerate her until her speed matches that of the belt.

(2) We’ve included every relevant type of force from the tree on p. 170.

(3) We’ve included forces from the belt, which is the only object in contact with Fifi.

The purpose of the analysis is to let us set up equations containing enough information to solve the problem. Using the generalization of Newton’s second law given on p. 138, we use the horizontal force to determine the horizontal acceleration, and separately require the vertical forces to cancel out.

Let positive $x$ be to the right. Newton’s second law gives

$$(\rightarrow) \quad a = \frac{F_k}{m}$$

Although it’s the horizontal motion we care about, the only way to find $F_k$ is via the relation $F_k = \mu_k F_N$, and the only way to find $F_N$ is from the $\uparrow \downarrow$ forces. The two vertical forces must cancel, which means they have to be of equal strength:

$$(\uparrow \downarrow) \quad F_N - mg = 0.$$ 

Using the constant-acceleration equation $a = \Delta v / \Delta t$, we have

$$\Delta t = \frac{\Delta v}{a} = \frac{v_b - v_0}{\mu_k mg/m} = \frac{v_b - v_0}{\mu_k g}.$$ 

The units check out:

$$s = \frac{m/s}{m/s^2},$$
where $\mu_k$ is omitted as a factor because it’s unitless.

We should also check that the dependence on the variables makes sense. If Fifi puts on her rubber ninja booties, increasing $\mu_k$, then dividing by a larger number gives a smaller result for $\Delta t$; this makes sense physically, because the greater friction will cause her to come up to the belt’s speed more quickly. The dependence on $g$ is similar; more gravity would press her harder against the belt, improving her traction. Increasing $v_b$ increases $\Delta t$, which makes sense because it will take her longer to get up to a bigger speed. Since $v_0$ is subtracted, the dependence of $\Delta t$ on it is the other way around, and that makes sense too, because if she can land with a greater speed, she has less speeding up left to do.

In figure u, the three horses are arranged symmetrically at 120 degree intervals, and are all pulling on the central knot. Let’s say the knot is at rest and at least momentarily in equilibrium. The analysis of forces on the knot is as follows.
In our previous examples, the forces have all run along two perpendicular lines, and they often canceled in pairs. This example shows that neither of these always happens. Later in the book we'll see how to handle forces that are at arbitrary angles, using mathematical objects called vectors. But even without knowing about vectors, we already know what directions to draw the arrows in the table, since a rope can only pull parallel to itself at its ends. And furthermore, we can say something about the forces: by symmetry, we expect them all to be equal in strength. (If the knot was not in equilibrium, then this symmetry would be broken.)

This analysis also demonstrates that it's all right to leave out details if they aren't of interest and we don't intend to include them in our model. We called the forces normal forces, but we can't actually tell whether they are normal forces or frictional forces. They are probably some combination of those, but we don't include such details in this model, since aren't interested in describing the internal physics of the knot. This is an example of a more general fact about science, which is that science doesn't describe reality. It describes simplified models of reality, because reality is always too complex to model exactly.

<table>
<thead>
<tr>
<th>force acting on knot</th>
<th>force related to it by Newton's third law</th>
</tr>
</thead>
<tbody>
<tr>
<td>top rope's normal force on knot,      ↑</td>
<td>knot's normal force on top rope,        ↓</td>
</tr>
<tr>
<td>left rope's normal force on knot,      ↖</td>
<td>knot's normal force on left rope,      →</td>
</tr>
<tr>
<td>right rope's normal force on knot,      ↘</td>
<td>knot's normal force on right rope,      ↙</td>
</tr>
</tbody>
</table>
Discussion questions

A In the example of the barge going down the canal, I referred to a “floating” or “hydrostatic” force that keeps the boat from sinking. If you were adding a new branch on the force-classification tree to represent this force, where would it go?

B The earth’s gravitational force on you, i.e., your weight, is always equal to $mg$, where $m$ is your mass. So why can you get a shovel to go deeper into the ground by jumping onto it? Just because you’re jumping, that doesn’t mean your mass or weight is any greater, does it?

5.4 Transmission of forces by low-mass objects

You’re walking your dog. The dog wants to go faster than you do, and the leash is taut. Does Newton’s third law guarantee that your force on your end of the leash is equal and opposite to the dog’s force on its end? If they’re not exactly equal, is there any reason why they should be approximately equal?

If there was no leash between you, and you were in direct contact with the dog, then Newton’s third law would apply, but Newton’s third law cannot relate your force on the leash to the dog’s force on the leash, because that would involve three separate objects. Newton’s third law only says that your force on the leash is equal and opposite to the leash’s force on you,

$$F_{yL} = -F_{Ly},$$

and that the dog’s force on the leash is equal and opposite to its force on the dog

$$F_{dL} = -F_{Ld}.$$

Still, we have a strong intuitive expectation that whatever force we make on our end of the leash is transmitted to the dog, and vice-versa. We can analyze the situation by concentrating on the forces that act on the leash, $F_{dL}$ and $F_{yL}$. According to Newton’s second law, these relate to the leash’s mass and acceleration:

$$F_{dL} + F_{yL} = m_L a_L.$$

The leash is far less massive than any of the other objects involved, and if $m_L$ is very small, then apparently the total force on the leash is also very small, $F_{dL} + F_{yL} \approx 0$, and therefore

$$F_{dL} \approx -F_{yL}.$$

Thus even though Newton’s third law does not apply directly to these two forces, we can approximate the low-mass leash as if it was not intervening between you and the dog. It’s at least approximately as if you and the dog were acting directly on each other, in which case Newton’s third law would have applied.
In general, low-mass objects can be treated approximately as if they simply transmitted forces from one object to another. This can be true for strings, ropes, and cords, and also for rigid objects such as rods and sticks.

\[ F = T \]

If we imagine dividing a taut rope up into small segments, then any segment has forces pulling outward on it at each end. If the rope is of negligible mass, then all the forces equal \(+T\) or \(-T\), where \(T\), the tension, is a single number.

If you look at a piece of string under a magnifying glass as you pull on the ends more and more strongly, you will see the fibers straightening and becoming taut. Different parts of the string are apparently exerting forces on each other. For instance, if we think of the two halves of the string as two objects, then each half is exerting a force on the other half. If we imagine the string as consisting of many small parts, then each segment is transmitting a force to the next segment, and if the string has very little mass, then all the forces are equal in magnitude. We refer to the magnitude of the forces as the tension in the string, \(T\).

The term “tension” refers only to internal forces within the string. If the string makes forces on objects at its ends, then those forces are typically normal or frictional forces (example 9).
**Example 9.** The forces between the rope and other objects are normal and frictional forces.

> Analyze the forces in figures x/1 and x/2.

> In all cases, a rope can only make “pulling” forces, i.e., forces that are parallel to its own length and that are toward itself, not away from itself. You can’t push with a rope!

In x/1, the rope passes through a type of hook, called a carabiner, used in rock climbing and mountaineering. Since the rope can only pull along its own length, the direction of its force on the carabiner must be down and to the right. This is perpendicular to the surface of contact, so the force is a normal force.

<table>
<thead>
<tr>
<th>force acting on carabiner</th>
<th>force related to it by Newton’s third law</th>
</tr>
</thead>
<tbody>
<tr>
<td>rope’s normal force on carabiner</td>
<td>carabiner’s normal force on rope</td>
</tr>
</tbody>
</table>

(There are presumably other forces acting on the carabiner from other hardware above it.)

In figure x/2, the rope can only exert a net force at its end that is parallel to itself and in the pulling direction, so its force on the hand is down and to the left. This is parallel to the surface of contact, so it must be a frictional force. If the rope isn’t slipping through the hand, we have static friction. Friction can’t exist without normal forces. These forces are perpendicular to the surface of contact. For simplicity, we show only two pairs of these normal forces, as if the hand were a pair of pliers.

<table>
<thead>
<tr>
<th>force acting on person</th>
<th>force related to it by Newton’s third law</th>
</tr>
</thead>
<tbody>
<tr>
<td>rope’s static frictional force on person</td>
<td>person’s static frictional force on rope</td>
</tr>
<tr>
<td>rope’s normal force on person</td>
<td>person’s normal force on rope</td>
</tr>
</tbody>
</table>

(There are presumably other forces acting on the person as well, such as gravity.)

If a rope goes over a pulley or around some other object, then the tension throughout the rope is approximately equal so long as the pulley has negligible mass and there is not too much friction. A rod or stick can be treated in much the same way as a string, but it is possible to have either compression or tension.

**Discussion question**

A When you step on the gas pedal, is your foot’s force being transmitted in the sense of the word used in this section?
5.5 Objects under strain

A string lengthens slightly when you stretch it. Similarly, we have already discussed how an apparently rigid object such as a wall is actually flexing when it participates in a normal force. In other cases, the effect is more obvious. A spring or a rubber band visibly elongates when stretched.

Common to all these examples is a change in shape of some kind: lengthening, bending, compressing, etc. The change in shape can be measured by picking some part of the object and measuring its position, \( x \). For concreteness, let’s imagine a spring with one end attached to a wall. When no force is exerted, the unfixed end of the spring is at some position \( x_0 \). If a force acts at the unfixed end, its position will change to some new value of \( x \). The more force, the greater the departure of \( x \) from \( x_0 \).

Back in Newton’s time, experiments like this were considered cutting-edge research, and his contemporary Hooke is remembered today for doing them and for coming up with a simple mathematical generalization called Hooke’s law:

\[
F \approx k(x - x_0). \quad \text{[force required to stretch a spring; valid for small forces only]}
\]

Here \( k \) is a constant, called the spring constant, that depends on how stiff the object is. If too much force is applied, the spring exhibits more complicated behavior, so the equation is only a good approximation if the force is sufficiently small. Usually when the force is so large that Hooke’s law is a bad approximation, the force ends up permanently bending or breaking the spring.

Although Hooke’s law may seem like a piece of trivia about springs, it is actually far more important than that, because all
solid objects exert Hooke’s-law behavior over some range of sufficiently small forces. For example, if you push down on the hood of a car, it dips by an amount that is directly proportional to the force. (But the car’s behavior would not be as mathematically simple if you dropped a boulder on the hood!)

- **Solved problem: Combining springs** page 198, problem 26
- **Solved problem: Young’s modulus** page 198, problem 28

**Discussion question**

A car is connected to its axles through big, stiff springs called shock absorbers, or “shocks.” Although we’ve discussed Hooke’s law above only in the case of stretching a spring, a car’s shocks are continually going through both stretching and compression. In this situation, how would you interpret the positive and negative signs in Hooke’s law?

### 5.6 Simple Machines: the pulley

Even the most complex machines, such as cars or pianos, are built out of certain basic units called *simple machines*. The following are some of the main functions of simple machines:

- transmitting a force: The chain on a bicycle transmits a force from the crank set to the rear wheel.
- changing the direction of a force: If you push down on a seesaw, the other end goes up.
- changing the speed and precision of motion: When you make the “come here” motion, your biceps only moves a couple of centimeters where it attaches to your forearm, but your arm moves much farther and more rapidly.
- changing the amount of force: A lever or pulley can be used to increase or decrease the amount of force.

You are now prepared to understand one-dimensional simple machines, of which the pulley is the main example.

---

**A pulley example 10**

Farmer Bill says this pulley arrangement doubles the force of his tractor. Is he just a dumb hayseed, or does he know what he’s doing?
To use Newton’s first law, we need to pick an object and consider the sum of the forces on it. Since our goal is to relate the tension in the part of the cable attached to the stump to the tension in the part attached to the tractor, we should pick an object to which both those cables are attached, i.e., the pulley itself. The tension in a string or cable remains approximately constant as it passes around an idealized pulley.\(^1\) There are therefore two leftward forces acting on the pulley, each equal to the force exerted by the tractor. Since the acceleration of the pulley is essentially zero, the forces on it must be canceling out, so the rightward force of the pulley-stump cable on the pulley must be double the force exerted by the tractor. Yes, Farmer Bill knows what he’s talking about.

More complicated pulley systems can be constructed to give greater amplification of forces or to redirect forces in different directions. For an idealized system,\(^2\) the fundamental principles are:

1. The total force acting on any pulley is zero.\(^3\)
2. The tension in any given piece of rope is constant throughout its length.
3. The length of every piece of rope remains the same.

\(^1\)A compound pulley

Find the mechanical advantage \(T_5/F\) of the pulley system. The bar is massless.

By rule 2, \(T_1 = T_2\), and by rule 1, \(F = T_1 + T_2\), so \(T_1 = T_2 = F/2\). Similarly, \(T_3 = T_4 = F/4\). Since the bar is massless, the same reasoning that led to rule 1 applies to the bar as well, and \(T_5 = T_1 + T_3\). The mechanical advantage is \(T_5/F = 3/4\), i.e., this pulley system reduces the input force.

\(^2\)How far does the tractor go compared to the stump? example 12

To move the stump in figure z by 1 cm, how far must the tractor move?

Applying rule 3 to the the right-hand piece of rope, we find that the pulley moves 1 cm. The upper leg of the U-shaped rope therefore shortens by 1 cm, so the lower leg must lengthen by 1 cm. Since the pulley moves 1 cm to the left, and the lower leg extending from it also lengths by 1 cm, the tractor must move 2 cm.

\(^3\)This was asserted in section 5.4 without proof. Essentially it holds because of symmetry. E.g., if the U-shaped piece of rope in figure z had unequal tension in its two legs, then this would have to be caused by some asymmetry between clockwise and counterclockwise rotation. But such an asymmetry can only be caused by friction or inertia, which we assume don’t exist.

\(^1\)In such a system: (1) The ropes and pulleys have negligible mass. (2) Friction in the pulleys’ bearings is negligible. (3) The ropes don’t stretch.

\(^3\)\(F = ma\), and \(m = 0\) since the pulley’s mass is assumed to be negligible.
Examples 10 and 12 showed that the pulley system in figure 7 amplifies the force by a factor of 2, but it reduces the motion by 1/2. This is an example of a more general inverse proportionality for all such systems. Superficially, it follows from rules 1-3 above. If, for example, we try to construct a pulley system that doubles the force while keeping the motion the same, we will find that the rules seem to mysteriously conspire against us, and every attempt ends in failure. We could in fact prove as a mathematical theorem that the inverse proportionality always holds if we assume these rules.

But these rules are only an idealized mathematical model of a specific type of simple machine. What about other machines built out of other parts such as levers, screws, or gears? Through trial and error we will find that the inverse proportionality holds for them as well, so there must be some more fundamental principles involved. These principles, which we won’t discuss formally until ch. 11 and 13, are conservation of energy and the equation for mechanical work. Informally, imagine that we had a machine that violated this rule. We could then insert it into a setup like the one in figure ab. When we release the single weight at the top, it drops to the ground while lifting the pan, which holds double the weight, all the way to the top. This is the ultimate free lunch. Once the pair of weights is up at the top, we can use them to hoist four more, then 8, 16, and so on. This is known as a perpetual motion machine.

If this seems to be too good to be true, it is. Just as small machines can be put together to make bigger ones, any machine can also be broken down into smaller and smaller ones. This process can be continued until we get down to the level of atoms. The law of conservation of energy essentially says that atoms don’t act like perpetual motion machines, and therefore any machine built out of atoms also fails to be a perpetual motion machine.
Does Newton’s third law mean anything, and if so, is it true?

This section discusses Newton’s third law in the same spirit as section 4.7 on the first and second laws.

Ernst Mach gave a cogent critique of the third law’s logical assumptions in his book *The Science of Mechanics*. The book is available online for free at archive.org, and is very readable. To understand Mach’s criticism, consider the experiment illustrated in figure a on p. 164, in which a large magnet and a small magnet are found to exert equal forces on one another. I use this as a student lab, and I find that most students are surprised by the result. Nevertheless, the lab can be considered a swindle, for the following reason. If we wanted to, we could cut the large magnet apart into smaller pieces, each of which was the same size as the small magnet. In fact, the large magnets I use for this lab were constructed simply by taking six small ones, stacking them together, and wrapping them in plastic. To represent this symbolically, let the small magnet be [A] and the large one [BCDEFG]. Since A and B are identical, and they are oriented in the same way, it follows simply by symmetry that A’s force on B and B’s on A obey the third law. The same holds for A on C and C on A, and so on. Since Newton claims that forces combine by addition, it follows that the result of the experiment must be in accord with the third law, despite the superficial asymmetry.

Now suppose that material objects 1 and 2 have the same chemical composition. By a similar argument it seems likely that $F_{12}$ and $F_{21}$ obey Newton’s third law.

This argument shows how pointless it can be to attempt to test a scientific theory unless you have in your possession a sensible alternative theory that predicts something different. One could spend decades doing experiments of the kind described above without realizing that the tests were all trivially guaranteed to give null results, even if nature was really described by a theory that violated Newton’s third law.

Here is an example of a fairly sane theory that could violate Newton’s third law. Einstein’s famous $E = mc^2$ states that a certain amount of energy $E$ is equivalent to a certain amount of mass $m$, with $c$ being the speed of light. (We won’t formally encounter energy until ch. 11, or the reasons for $E = mc^2$ until section 12.5, but for now just think of energy as the kind of thing you intuitively associate with food calories or a tank full of gasoline, and take $E = mc^2$ for granted.) Einstein claimed that this would hold for three different kinds of mass: the mass measured by an object’s inertia, the “active” gravitational mass $m_a$ that determines the gravitational forces it makes on other objects, and the “passive” gravitational mass $m_p$.
that measures how strongly it feels gravity. Einstein’s reason for predicting similar behavior for \( m_a \) and \( m_p \) was that anything else would have violated Newton’s third law for gravitational forces.

Suppose instead that an object’s energy content contributes only to \( m_p \), not to \( m_a \). Atomic nuclei get something like 1% of their mass from the energy of the electric fields inside their nuclei, but this percentage varies with the number of protons, so if we have objects \( m \) and \( M \) with different chemical compositions, it follows that in this theory \( m_p/m_a \) will not be the same as \( M_p/M_a \), and in this non-Einsteinian version of relativity, Newton’s third law is violated.

This was tested in a Princeton PhD-thesis experiment by Kreuzer\(^4\) in 1966. Kreuzer carried out an experiment, figure ad, using masses made of two different substances. The first substance was teflon. The second substance was a mixture of the liquids trichloroethylene and dibromoethane, with the proportions chosen so as to give a passive-mass density as close as possible to that of teflon, as determined by the neutral buoyancy of the teflon masses suspended inside the liquid. If the active-mass densities of these substances are not strictly proportional to their passive-mass densities, then moving the chunk of teflon back and forth in figure ad/2 would change the gravitational force acting on the nearby small sphere. No such change was observed, and the results verified \( m_p/m_a = M_p/M_a \) to within one part in \( 10^6 \), in agreement with Einstein and Newton. If electrical energy had not contributed at all to active mass, then a violation of the third law would have been detected at the level of about one part in \( 10^2 \).

The Kreuzer result was improved in 1986 by Bartlett and van Buren\(^5\) using lunar laser ranging data similar to those described in section 4.7. Since the moon has an asymmetrical distribution of iron and aluminum, a theory with \( m_p/m_a \neq M_p/M_a \) would cause it to have an anomalous acceleration along a certain line. The lack of any such observed acceleration limits violations of Newton’s third law to about one part in \( 10^{10} \).

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\(^4\)Kreuzer, Phys. Rev. 169 (1968) 1007
Summary

Selected vocabulary

repulsive . . . . describes a force that tends to push the two participating objects apart
attractive . . . . describes a force that tends to pull the two participating objects together
oblique . . . . . describes a force that acts at some other angle, one that is not a direct repulsion or attraction
normal force . . . the force that keeps two objects from occupying the same space
static friction . . a friction force between surfaces that are not slipping past each other
kinetic friction . . a friction force between surfaces that are slipping past each other
fluid . . . . . . . . . . a gas or a liquid
fluid friction . . . a friction force in which at least one of the object is is a fluid
spring constant . . the constant of proportionality between force and elongation of a spring or other object under strain

Notation

\[ F_N \] . . . . . . . . . a normal force
\[ F_s \] . . . . . . . . . a static frictional force
\[ F_k \] . . . . . . . . . a kinetic frictional force
\[ \mu_s \] . . . . . . . . . the coefficient of static friction; the constant of proportionality between the maximum static frictional force and the normal force; depends on what types of surfaces are involved
\[ \mu_k \] . . . . . . . . . the coefficient of kinetic friction; the constant of proportionality between the kinetic frictional force and the normal force; depends on what types of surfaces are involved
\[ k \] . . . . . . . . . . the spring constant; the constant of proportionality between the force exerted on an object and the amount by which the object is lengthened or compressed

Summary

Newton’s third law states that forces occur in equal and opposite pairs. If object A exerts a force on object B, then object B must simultaneously be exerting an equal and opposite force on object A. Each instance of Newton’s third law involves exactly two objects, and exactly two forces, which are of the same type.

There are two systems for classifying forces. We are presently using the more practical but less fundamental one. In this system, forces are classified by whether they are repulsive, attractive, or oblique; whether they are contact or noncontact forces; and whether
the two objects involved are solids or fluids.

Static friction adjusts itself to match the force that is trying to make the surfaces slide past each other, until the maximum value is reached,

\[ F_{s,max} = \mu_s F_N. \]

Once this force is exceeded, the surfaces slip past one another, and kinetic friction applies,

\[ F_k = \mu_k F_N. \]

Both types of frictional force are nearly independent of surface area, and kinetic friction is usually approximately independent of the speed at which the surfaces are slipping. The direction of the force is in the direction that would tend to stop or prevent slipping.

A good first step in applying Newton’s laws of motion to any physical situation is to pick an object of interest, and then to list all the forces acting on that object. We classify each force by its type, and find its Newton’s-third-law partner, which is exerted by the object on some other object.

When two objects are connected by a third low-mass object, their forces are transmitted to each other nearly unchanged.

Objects under strain always obey Hooke’s law to a good approximation, as long as the force is small. Hooke’s law states that the stretching or compression of the object is proportional to the force exerted on it,

\[ F \approx k(x - x_o). \]
Problems

Key
√ A computerized answer check is available online.
∫ A problem that requires calculus.
★ A difficult problem.

1 In each case, identify the force that causes the acceleration, and give its Newton’s-third-law partner. Describe the effect of the partner force. (a) A swimmer speeds up. (b) A golfer hits the ball off of the tee. (c) An archer fires an arrow. (d) A locomotive slows down. ▷ Solution, p. 556

2 Example 2 on page 167 involves a person pushing a box up a hill. The incorrect answer describes three forces. For each of these three forces, give the force that it is related to by Newton’s third law, and state the type of force. ▷ Solution, p. 556

3 (a) Compare the mass of a one-liter water bottle on earth, on the moon, and in interstellar space. ▷ Solution, p. 556 (b) Do the same for its weight.

In problems 4-8, analyze the forces using a table in the format shown in section 5.3. Analyze the forces in which the italicized object participates.

4 Some people put a spare car key in a little magnetic box that they stick under the chassis of their car. Let’s say that the box is stuck directly underneath a horizontal surface, and the car is parked. (See instructions above.)

5 Analyze two examples of objects at rest relative to the earth that are being kept from falling by forces other than the normal force. Do not use objects in outer space, and do not duplicate problem 4 or 8. (See instructions above.)

6 A person is rowing a boat, with her feet braced. She is doing the part of the stroke that propels the boat, with the ends of the oars in the water (not the part where the oars are out of the water). (See instructions above.)

7 A farmer is in a stall with a cow when the cow decides to press him against the wall, pinning him with his feet off the ground. Analyze the forces in which the farmer participates. (See instructions above.)

8 A propeller plane is cruising east at constant speed and altitude. (See instructions above.)
9 A little old lady and a pro football player collide head-on. Compare their forces on each other, and compare their accelerations. Explain.

10 The earth is attracted to an object with a force equal and opposite to the force of the earth on the object. If this is true, why is it that when you drop an object, the earth does not have an acceleration equal and opposite to that of the object?

11 When you stand still, there are two forces acting on you, the force of gravity (your weight) and the normal force of the floor pushing up on your feet. Are these forces equal and opposite? Does Newton’s third law relate them to each other? Explain.

12 Today’s tallest buildings are really not that much taller than the tallest buildings of the 1940’s. One big problem with making an even taller skyscraper is that every elevator needs its own shaft running the whole height of the building. So many elevators are needed to serve the building’s thousands of occupants that the elevator shafts start taking up too much of the space within the building. An alternative is to have elevators that can move both horizontally and vertically: with such a design, many elevator cars can share a few shafts, and they don’t get in each other’s way too much because they can detour around each other. In this design, it becomes impossible to hang the cars from cables, so they would instead have to ride on rails which they grab onto with wheels. Friction would keep them from slipping. The figure shows such a frictional elevator in its vertical travel mode. (The wheels on the bottom are for when it needs to switch to horizontal motion.)

(a) If the coefficient of static friction between rubber and steel is $\mu_s$, and the maximum mass of the car plus its passengers is $M$, how much force must there be pressing each wheel against the rail in order to keep the car from slipping? (Assume the car is not accelerating.)

(b) Show that your result has physically reasonable behavior with respect to $\mu_s$. In other words, if there was less friction, would the wheels need to be pressed more firmly or less firmly? Does your equation behave that way?

13 An ice skater builds up some speed, and then coasts across the ice passively in a straight line. (a) Analyze the forces, using a table in the format shown in section 5.3.

(b) If his initial speed is $v$, and the coefficient of kinetic friction is $\mu_k$, find the maximum theoretical distance he can glide before coming to a stop. Ignore air resistance.

(c) Show that your answer to part b has the right units.

(d) Show that your answer to part b depends on the variables in a way that makes sense physically.

(e) Evaluate your answer numerically for $\mu_k = 0.0046$, and a world-record speed of 14.58 m/s. (The coefficient of friction was measured by De Koning et al., using special skates worn by real speed skaters.)
(f) Comment on whether your answer in part e seems realistic. If it doesn’t, suggest possible reasons why.

14 A cop investigating the scene of an accident measures the length $L$ of a car’s skid marks in order to find out its speed $v$ at the beginning of the skid. Express $v$ in terms of $L$ and any other relevant variables.

15 Someone tells you she knows of a certain type of Central American earthworm whose skin, when rubbed on polished diamond, has $\mu_k > \mu_s$. Why is this not just empirically unlikely but logically suspect?

16 When I cook rice, some of the dry grains always stick to the measuring cup. To get them out, I turn the measuring cup upside-down and hit the “roof” with my hand so that the grains come off of the “ceiling.” (a) Explain why static friction is irrelevant here. (b) Explain why gravity is negligible. (c) Explain why hitting the cup works, and why its success depends on hitting the cup hard enough.

17 Pick up a heavy object such as a backpack or a chair, and stand on a bathroom scale. Shake the object up and down. What do you observe? Interpret your observations in terms of Newton’s third law.

18 The following reasoning leads to an apparent paradox; explain what’s wrong with the logic. A baseball player hits a ball. The ball and the bat spend a fraction of a second in contact. During that time they’re moving together, so their accelerations must be equal. Newton’s third law says that their forces on each other are also equal. But $a = F/m$, so how can this be, since their masses are unequal? (Note that the paradox isn’t resolved by considering the force of the batter’s hands on the bat. Not only is this force very small compared to the ball-bat force, but the batter could have just thrown the bat at the ball.)

19 A tugboat of mass $m$ pulls a ship of mass $M$, accelerating it. The speeds are low enough that you can ignore fluid friction acting on their hulls, although there will of course need to be fluid friction acting on the tug’s propellers.
(a) Analyze the forces in which the tugboat participates, using a table in the format shown in section 5.3. Don’t worry about vertical forces.
(b) Do the same for the ship.
(c) If the force acting on the tug’s propeller is $F$, what is the tension, $T$, in the cable connecting the two ships? [Hint: Write down two equations, one for Newton’s second law applied to each object. Solve these for the two unknowns $T$ and $a$.] √
(d) Interpret your answer in the special cases of $M = 0$ and $M = \infty$. Problems 195
Problem 20. Unequal masses $M$ and $m$ are suspended from a pulley as shown in the figure.

(a) Analyze the forces in which mass $m$ participates, using a table in the format shown in section 5.3. [The forces in which the other mass participates will of course be similar, but not numerically the same.]

(b) Find the magnitude of the accelerations of the two masses. [Hints: (1) Pick a coordinate system, and use positive and negative signs consistently to indicate the directions of the forces and accelerations. (2) The two accelerations of the two masses have to be equal in magnitude but of opposite signs, since one side eats up rope at the same rate at which the other side pays it out. (3) You need to apply Newton’s second law twice, once to each mass, and then solve the two equations for the unknowns: the acceleration, $a$, and the tension in the rope, $T$.]

(c) Many people expect that in the special case of $M = m$, the two masses will naturally settle down to an equilibrium position side by side. Based on your answer from part b, is this correct?

(d) Find the tension in the rope, $T$.

(e) Interpret your equation from part d in the special case where one of the masses is zero. Here “interpret” means to figure out what happens mathematically, figure out what should happen physically, and connect the two.

Problem 21. The figure shows a stack of two blocks, sitting on top of a table that is bolted to the floor. All three objects are made from identical wood, with their surfaces finished identically using the same sandpaper. We tap the middle block, giving it an initial velocity $v$ to the right. The tap is executed so rapidly that almost no initial velocity is imparted to the top block.

(a) Find the time that will elapse until the slipping between the top and middle blocks stops. Express your answer in terms of $v$, $m$, $M$, $g$, and the relevant coefficient of friction.

(b) Show that your answer makes sense in terms of units.

(c) Check that your result has the correct behavior when you make $m$ bigger or smaller. Explain. This means that you should discuss the mathematical behavior of the result, and then explain how this corresponds to what would really happen physically.

(d) Similarly, discuss what happens when you make $M$ bigger or smaller.

(e) Similarly, discuss what happens when you make $g$ bigger or smaller.
22 Mountain climbers with masses $m$ and $M$ are roped together while crossing a horizontal glacier when a vertical crevasse opens up under the climber with mass $M$. The climber with mass $m$ drops down on the snow and tries to stop by digging into the snow with the pick of an ice ax. Alas, this story does not have a happy ending, because this doesn’t provide enough friction to stop. Both $m$ and $M$ continue accelerating, with $M$ dropping down into the crevasse and $m$ being dragged across the snow, slowed only by the kinetic friction with coefficient $\mu_k$ acting between the ax and the snow. There is no significant friction between the rope and the lip of the crevasse.

(a) Find the acceleration $a$.
(b) Check the units of your result.
(c) Check the dependence of your equation on the variables. That means that for each variable, you should determine what its effect on $a$ should be physically, and then what your answer from part a says its effect would be mathematically.

23 Ginny has a plan. She is going to ride her sled while her dog Foo pulls her, and she holds on to his leash. However, Ginny hasn’t taken physics, so there may be a problem: she may slide right off the sled when Foo starts pulling.

(a) Analyze all the forces in which Ginny participates, making a table as in section 5.3.
(b) Analyze all the forces in which the sled participates.
(c) The sled has mass $m$, and Ginny has mass $M$. The coefficient of static friction between the sled and the snow is $\mu_1$, and $\mu_2$ is the corresponding quantity for static friction between the sled and her snow pants. Ginny must have a certain minimum mass so that she will not slip off the sled. Find this in terms of the other three variables.

(d) Interpreting your equation from part c, under what conditions will there be no physically realistic solution for $M$? Discuss what this means physically.

24 In the system shown in the figure, the pulleys on the left and right are fixed, but the pulley in the center can move to the left or right. The two masses are identical. Find the upward acceleration of the mass on the left, in terms of $g$ only. Assume all the ropes and pulleys are massless and frictionless. Hints: (1) Use rules 1-3 on p. 187. (2) The approach is similar to the one in problem 20, but the ratio of the accelerations isn’t 1:1.

25 Example 10 on page 186 describes a force-doubling setup involving a pulley. Make up a more complicated arrangement, using two pulleys, that would multiply the force by four. The basic idea is to take the output of one force doubler and feed it into the input of a second one.
26 The figure shows two different ways of combining a pair of identical springs, each with spring constant $k$. We refer to the top setup as parallel, and the bottom one as a series arrangement.

(a) For the parallel arrangement, analyze the forces acting on the connector piece on the left, and then use this analysis to determine the equivalent spring constant of the whole setup. Explain whether the combined spring constant should be interpreted as being stiffer or less stiff.

(b) For the series arrangement, analyze the forces acting on each spring and figure out the same things.  

27 Generalize the results of problem 26 to the case where the two spring constants are unequal.

28 (a) Using the solution of problem 26, which is given in the back of the book, predict how the spring constant of a fiber will depend on its length and cross-sectional area.

(b) The constant of proportionality is called the Young’s modulus, $E$, and typical values of the Young’s modulus are about $10^{10}$ to $10^{11}$. What units would the Young’s modulus have in the SI (meter-kilogram-second) system?

29 This problem depends on the results of problems 26 and 28, whose solutions are in the back of the book. When atoms form chemical bonds, it makes sense to talk about the spring constant of the bond as a measure of how “stiff” it is. Of course, there aren’t really little springs — this is just a mechanical model. The purpose of this problem is to estimate the spring constant, $k$, for a single bond in a typical piece of solid matter. Suppose we have a fiber, like a hair or a piece of fishing line, and imagine for simplicity that it is made of atoms of a single element stacked in a cubical manner, as shown in the figure, with a center-to-center spacing $b$. A typical value for $b$ would be about $10^{-10}$ m.

(a) Find an equation for $k$ in terms of $b$, and in terms of the Young’s modulus, $E$, defined in problem 16 and its solution.

(b) Estimate $k$ using the numerical data given in problem 28.

(c) Suppose you could grab one of the atoms in a diatomic molecule like $H_2$ or $O_2$, and let the other atom hang vertically below it. Does the bond stretch by any appreciable fraction due to gravity?

30 A cross-country skier is gliding on a level trail, with negligible friction. Then, when he is at position $x = 0$, the tip of his skis enters a patch of dirt. As he rides onto the dirt, more and more of his weight is being supported by the dirt. The skis have length $\ell$, so if he reached $x = \ell$ without stopping, his weight would be completely on the dirt. This problem deals with the motion for $x < \ell$.

(a) Find the acceleration in terms of $x$, as well as any other relevant constants.

(b) This is a second-order differential equation. You should be able to find the solution simply by thinking about some commonly oc-
curing functions that you know about, and finding two that have the right properties. If these functions are \( x = f(t) \) and \( x = g(t) \), then the most general solution to the equations of motion will be of the form \( x = af + bg \), where \( a \) and \( b \) are constants to be determined from the initial conditions.

(c) Suppose that the initial velocity \( v_0 \) at \( x = 0 \) is such that he stops at \( x < \ell \). Find the time until he stops, and show that, counterintuitively, this time is independent of \( v_0 \). Explain physically why this is true.

\[ \star \]

31 The two masses are identical. Find the upward acceleration of the mass on the right, in terms of \( g \) only. Assume all the ropes and pulleys, as well as the cross-bar, are massless, and the pulleys are frictionless. The right-hand mass has been positioned away from the bar’s center, so that the bar will not twist. Hints: (1) Use rules 1-3 on p. 187. (2) The approach is similar to the one in problem 20, but the ratio of the accelerations isn’t 1:1.

\[ \checkmark \]

32 Find the upward acceleration of mass \( m_1 \) in the figure.

\[ \checkmark \star \]

33 The figure shows a mountaineer doing a vertical rappel. Her anchor is a big boulder. The American Mountain Guides Association suggests as a rule of thumb that in this situation, the boulder should be at least as big as a refrigerator, and should be sitting on a surface that is horizontal rather than sloping. The goal of this problem is to estimate what coefficient of static friction \( \mu_s \) between the boulder and the ledge is required if this setup is to hold the person’s body weight. For comparison, reference books meant for civil engineers building walls out of granite blocks state that granite on granite typically has a \( \mu_s \approx 0.6 \). We expect the result of our calculation to be much less than this, both because a large margin of safety is desired and because the coefficient could be much lower if, for example, the surface was sandy rather than clean. We will assume that there is no friction where the rope goes over the lip of the cliff, although in reality this friction significantly reduces the load on the boulder.

(a) Let \( m \) be the mass of the climber, \( V \) the volume of the boulder, \( \rho \) its density, and \( g \) the strength of the gravitational field. Find the minimum value of \( \mu_s \).

\[ \checkmark \]

(b) Show that the units of your answer make sense.

(c) Check that its dependence on the variables makes sense.

(d) Evaluate your result numerically. The volume of my refrigerator is about 0.7 m\(^3\), the density of granite is about 2.7 g/cm\(^3\), and standards bodies use a body mass of 80 kg for testing climbing equipment.

\[ \checkmark \]
Problem 34: A toy manufacturer is playtesting teflon booties that slip on over your shoes. In the parking lot, giggling engineers find that when they start with an initial speed of 1.2 m/s, they glide for 2.0 m before coming to a stop. What is the coefficient of friction between the asphalt and the booties? [problem by B. Shotwell]

Problem 35: Blocks $M_1$ and $M_2$ are stacked as shown, with $M_2$ on top. $M_2$ is connected by a string to the wall, and $M_1$ is pulled to the right with a force $F$ big enough to get $M_1$ to move. The coefficient of kinetic friction has the same value $\mu_k$ among all surfaces (i.e., the block-block and ground-block interfaces).

(a) Analyze the forces in which each block participates, as in section 5.3. [problem by B. Shotwell]
(b) Determine the tension in the string.
(c) Find the acceleration of the block of mass $M_1$. [problem by B. Shotwell]

Problem 36: A person can pull with a maximum force $F$. What is the maximum mass that the person can lift with the pulley setup shown in the figure? [problem by B. Shotwell]