Marie and Pierre Curie were the first to purify radium in significant quantities. Radium's intense radioactivity made possible the experiments that led to the modern planetary model of the atom, in which electrons orbit a nucleus made of protons and neutrons.

Chapter 26
The Atom and $E=mc^2$

26.1 Atoms
I was brought up to look at the atom as a nice, hard fellow, red or grey in color according to taste.  

_Rutherford_

The chemical elements
How would one find out what types of atoms there were? Today, it doesn’t seem like it should have been very difficult to work out an experimental program to classify the types of atoms. For each type of atom, there should be a corresponding element, i.e., a pure substance made out of nothing but that type of atom. Atoms are supposed to be unsplittable, so a substance like milk could not possibly be elemental, since churning it vigorously causes it to split up into two separate substances: butter and whey. Similarly, rust could not be an element, because it can be made by combining two substances: iron and oxygen. Despite its apparent reasonableness, no such program was carried out until the eighteenth century.

By 1900, however, chemists had done a reasonably good job of find-
Out what the elements were. They also had determined the ratios of the different atoms’ masses fairly accurately. A typical technique would be to measure how many grams of sodium (Na) would combine with one gram of chlorine (Cl) to make salt (NaCl). (This assumes you’ve already decided based on other evidence that salt consisted of equal numbers of Na and Cl atoms.) The masses of individual atoms, as opposed to the mass ratios, were known only to within a few orders of magnitude based on indirect evidence, and plenty of physicists and chemists denied that individual atoms were anything more than convenient symbols.

The following table gives the atomic masses of all the elements, on a standard scale in which the mass of hydrogen is very close to 1.0. The absolute calibration of the whole scale was only very roughly known for a long time, but was eventually tied down, with the mass of a hydrogen atom being determined to be about $1.7 \times 10^{-27}$ kg.

<table>
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<th>Ag</th>
<th>107.9</th>
<th>Eu</th>
<th>152.0</th>
<th>Mo</th>
<th>95.9</th>
<th>Sc</th>
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<td>N</td>
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<td>S</td>
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<td>Zr</td>
<td>91.2</td>
</tr>
<tr>
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<td>Mn</td>
<td>54.9</td>
<td>Sb</td>
<td>121.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Making sense of the elements

As the information accumulated, the challenge was to find a way of systematizing it; the modern scientist’s aesthetic sense rebels against complication. This hodgepodge of elements was an embarrassment. One contemporary observer, William Crookes, described the elements as extending “before us as stretched the wide Atlantic before the gaze of Columbus, mocking, taunting and murmuring strange riddles, which no man has yet been able to solve.” It wasn’t long before people started recognizing that many atoms’ masses were nearly integer multiples of the mass of hydrogen, the lightest element. A few excitable types began speculating that hydrogen was the basic building block, and that the heavier elements were made of clusters of hydrogen. It wasn’t long, however, before their parade was rained on by more accurate measurements, which showed that not all of the elements had atomic masses that were near integer
multiples of hydrogen, and even the ones that were close to being integer multiples were off by one percent or so.

A modern periodic table. Elements in the same column have similar chemical properties. The modern atomic numbers, discussed on p. 769, were not known in Mendeleev’s time, since the table could be flipped in various ways.

Chemistry professor Dmitri Mendeleev, preparing his lectures in 1869, wanted to find some way to organize his knowledge for his students to make it more understandable. He wrote the names of all the elements on cards and began arranging them in different ways on his desk, trying to find an arrangement that would make sense of the muddle. The row-and-column scheme he came up with is essentially our modern periodic table. The columns of the modern version represent groups of elements with similar chemical properties, and each row is more massive than the one above it. Going across each row, this almost always resulted in placing the atoms in sequence by weight as well. What made the system significant was its predictive value. There were three places where Mendeleev had to leave gaps in his checkerboard to keep chemically similar elements in the same column. He predicted that elements would exist to fill these gaps, and extrapolated or interpolated from other elements in the same column to predict their numerical properties, such as masses, boiling points, and densities. Mendeleev’s professional stock skyrocketed when his three elements (later named gallium, scandium and germanium) were discovered and found to have very nearly the properties he had predicted.

One thing that Mendeleev’s table made clear was that mass was not the basic property that distinguished atoms of different elements. To make his table work, he had to deviate from ordering the elements strictly by mass. For instance, iodine atoms are lighter than tellurium, but Mendeleev had to put iodine after tellurium so that it would lie in a column with chemically similar elements.

**Direct proof that atoms existed**

The success of the kinetic theory of heat was taken as strong evidence that, in addition to the motion of any object as a whole, there is an invisible type of motion all around us: the random motion of atoms within each object. But many conservatives were not con-
vinced that atoms really existed. Nobody had ever seen one, after all. It wasn’t until generations after the kinetic theory of heat was developed that it was demonstrated conclusively that atoms really existed and that they participated in continuous motion that never died out.

The smoking gun to prove atoms were more than mathematical abstractions came when some old, obscure observations were reexamined by an unknown Swiss patent clerk named Albert Einstein. A botanist named Brown, using a microscope that was state of the art in 1827, observed tiny grains of pollen in a drop of water on a microscope slide, and found that they jumped around randomly for no apparent reason. Wondering at first if the pollen he’d assumed to be dead was actually alive, he tried looking at particles of soot, and found that the soot particles also moved around. The same results would occur with any small grain or particle suspended in a liquid. The phenomenon came to be referred to as Brownian motion, and its existence was filed away as a quaint and thoroughly unimportant fact, really just a nuisance for the microscopist.

It wasn’t until 1906 that Einstein found the correct interpretation for Brown’s observation: the water molecules were in continuous random motion, and were colliding with the particle all the time, kicking it in random directions. After all the millennia of speculation about atoms, at last there was solid proof. Einstein’s calculations dispelled all doubt, since he was able to make accurate predictions of things like the average distance traveled by the particle in a certain amount of time. (Einstein received the Nobel Prize not for his theory of relativity but for his papers on Brownian motion and the photoelectric effect.)

**Discussion questions**

A How could knowledge of the size of an individual aluminum atom be used to infer an estimate of its mass, or vice versa?

B How could one test Einstein’s interpretation of Brownian motion by observing it at different temperatures?

### 26.2 Quantization of charge

Proving that atoms actually existed was a big accomplishment, but demonstrating their existence was different from understanding their properties. Note that the Brown-Einstein observations had nothing at all to do with electricity, and yet we know that matter is inherently electrical, and we have been successful in interpreting certain electrical phenomena in terms of mobile positively and negatively charged particles. Are these particles atoms? Parts of atoms? Particles that are entirely separate from atoms? It is perhaps premature to attempt to answer these questions without any conclusive evidence in favor of the charged-particle model of electricity.
Strong support for the charged-particle model came from a 1911 experiment by physicist Robert Millikan at the University of Chicago. Consider a jet of droplets of perfume or some other liquid made by blowing it through a tiny pinhole. The droplets emerging from the pinhole must be smaller than the pinhole, and in fact most of them are even more microscopic than that, since the turbulent flow of air tends to break them up. Millikan reasoned that the droplets would acquire a little bit of electric charge as they rubbed against the channel through which they emerged, and if the charged-particle model of electricity was right, the charge might be split up among so many minuscule liquid drops that a single drop might have a total charge amounting to an excess of only a few charged particles — perhaps an excess of one positive particle on a certain drop, or an excess of two negative ones on another.

Millikan’s ingenious apparatus, e, consisted of two metal plates, which could be electrically charged as needed. He sprayed a cloud of oil droplets into the space between the plates, and selected one drop through a microscope for study. First, with no charge on the plates, he would determine the drop’s mass by letting it fall through the air and measuring its terminal velocity, i.e., the velocity at which the force of air friction canceled out the force of gravity. The force of air drag on a slowly moving sphere had already been found by experiment to be \(bv^2\), where \(b\) was a constant. Setting the total force equal to zero when the drop is at terminal velocity gives

\[bv^2 - mg = 0,\]

and setting the known density of oil equal to the drop’s mass divided by its volume gives a second equation,

\[\rho = \frac{m}{\frac{4}{3} \pi r^3}.\]

Everything in these equations can be measured directly except for \(m\) and \(r\), so these are two equations in two unknowns, which can be solved in order to determine how big the drop is.

Next Millikan charged the metal plates, adjusting the amount of charge so as to exactly counteract gravity and levitate the drop. If, for instance, the drop being examined happened to have a total charge that was negative, then positive charge put on the top plate would attract it, pulling it up, and negative charge on the bottom plate would repel it, pushing it up. (Theoretically only one plate would be necessary, but in practice a two-plate arrangement like this gave electrical forces that were more uniform in strength throughout the space where the oil drops were.) When the drop was being levitated, the gravitational and electric forces canceled, so \(qE = mg\). Since the mass of the drop, the gravitational field \(g\), and the electric field \(E\) were all known, the charge \(q\) could be determined.
Table f shows a few of the results from Millikan’s 1911 paper. (Millikan took data on both negatively and positively charged drops, but in his paper he gave only a sample of his data on negatively charged drops, so these numbers are all negative.) Even a quick look at the data leads to the suspicion that the charges are not simply a series of random numbers. For instance, the second charge is almost exactly equal to half the first one. Millikan explained the observed charges as all being integer multiples of a single number, \(1.64 \times 10^{-19} \text{ C}\). In the second column, dividing by this constant gives numbers that are essentially integers, allowing for the random errors present in the experiment. Millikan states in his paper that these results were a

\[
\begin{array}{lcc}
q (\text{C}) & 1.64 \\
-1.970 \times 10^{-18} & -12.02 \\
-0.987 \times 10^{-18} & -6.02 \\
-2.773 \times 10^{-18} & -16.93
\end{array}
\]

\(f\) A few samples of Millikan’s data.

...direct and tangible demonstration...of the correctness of the view advanced many years ago and supported by evidence from many sources that all electrical charges, however produced, are exact multiples of one definite, elementary electrical charge, or in other words, that an electrical charge instead of being spread uniformly over the charged surface has a definite granular structure, consisting, in fact, of...specks, or atoms of electricity, all precisely alike, peppered over the surface of the charged body.

In other words, he had provided direct evidence for the charged-particle model of electricity and against models in which electricity was described as some sort of fluid. The basic charge is notated \(e\), and the modern value is \(e = 1.60 \times 10^{-19} \text{ C}\). The word “quantized” is used in physics to describe a quantity that can only have certain numerical values, and cannot have any of the values between those. In this language, we would say that Millikan discovered that charge is quantized. The charge \(e\) is referred to as the quantum of charge.

**self-check A**

Is money quantized? What is the quantum of money?  
Answer, p. 1042

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**A historical note on Millikan’s fraud**

Very few undergraduate physics textbooks mention the well-documented fact that although Millikan’s conclusions were correct, he was guilty of scientific fraud. His technique was difficult and painstaking to perform, and his original notebooks, which have been preserved, show that the data were far less perfect than he claimed in his published scientific papers. In his publications, he stated categorically that every single oil drop observed had a charge that was a multiple of \(e\), with no Exceptions or omissions. But his notebooks are replete with notations such as “beautiful data, keep,” and “bad run, throw out.” Millikan, then, appears to have earned his Nobel Prize by advocating a correct position with dishonest descriptions of his data.
26.3 The electron

Cathode rays

Nineteenth-century physicists spent a lot of time trying to come up with wild, random ways to play with electricity. The best experiments of this kind were the ones that made big sparks or pretty colors of light.

One such parlor trick was the cathode ray. To produce it, you first had to hire a good glassblower and find a good vacuum pump. The glassblower would create a hollow tube and embed two pieces of metal in it, called the electrodes, which were connected to the outside via metal wires passing through the glass. Before letting him seal up the whole tube, you would hook it up to a vacuum pump, and spend several hours huffing and puffing away at the pump’s hand crank to get a good vacuum inside. Then, while you were still pumping on the tube, the glassblower would melt the glass and seal the whole thing shut. Finally, you would put a large amount of positive charge on one wire and a large amount of negative charge on the other. Metals have the property of letting charge move through them easily, so the charge deposited on one of the wires would quickly spread out because of the repulsion of each part of it for every other part. This spreading-out process would result in nearly all the charge ending up in the electrodes, where there is more room to spread out than there is in the wire. For obscure historical reasons a negative electrode is called a cathode and a positive one is an anode.

Figure g shows the light-emitting stream that was observed. If, as shown in this figure, a hole was made in the anode, the beam would extend on through the hole until it hit the glass. Drilling a hole in the cathode, however would not result in any beam coming out on the left side, and this indicated that the stuff, whatever it was, was coming from the cathode. The rays were therefore christened “cathode rays.” (The terminology is still used today in the term “cathode ray tube” or “CRT” for the picture tube of a TV or computer monitor.)

Were cathode rays a form of light, or of matter?

Were cathode rays a form of light, or matter? At first no one really cared what they were, but as their scientific importance became more apparent, the light-versus-matter issue turned into a controversy along nationalistic lines, with the Germans advocating light and the English holding out for matter. The supporters of the material interpretation imagined the rays as consisting of a stream of atoms ripped from the substance of the cathode.

One of our defining characteristics of matter is that material objects cannot pass through each other. Experiments showed that cathode...
rays could penetrate at least some small thickness of matter, such as a metal foil a tenth of a millimeter thick, implying that they were a form of light.

Other experiments, however, pointed to the contrary conclusion. Light is a wave phenomenon, and one distinguishing property of waves is demonstrated by speaking into one end of a paper towel roll. The sound waves do not emerge from the other end of the tube as a focused beam. Instead, they begin spreading out in all directions as soon as they emerge. This shows that waves do not necessarily travel in straight lines. If a piece of metal foil in the shape of a star or a cross was placed in the way of the cathode ray, then a “shadow” of the same shape would appear on the glass, showing that the rays traveled in straight lines. This straight-line motion suggested that they were a stream of small particles of matter.

These observations were inconclusive, so what was really needed was a determination of whether the rays had mass and weight. The trouble was that cathode rays could not simply be collected in a cup and put on a scale. When the cathode ray tube is in operation, one does not observe any loss of material from the cathode, or any crust being deposited on the anode.

Nobody could think of a good way to weigh cathode rays, so the next most obvious way of settling the light/matter debate was to check whether the cathode rays possessed electrical charge. Light was known to be uncharged. If the cathode rays carried charge, they were definitely matter and not light, and they were presumably being made to jump the gap by the simultaneous repulsion of the negative charge in the cathode and attraction of the positive charge in the anode. The rays would overshoot the anode because of their momentum. (Although electrically charged particles do not normally leap across a gap of vacuum, very large amounts of charge were being used, so the forces were unusually intense.)

**Thomson’s experiments**

Physicist J.J. Thomson at Cambridge carried out a series of definitive experiments on cathode rays around the year 1897. By turning them slightly off course with electrical forces, he showed that they were indeed electrically charged, which was strong evidence that they were material. Not only that, but he proved that they had mass, and measured the ratio of their mass to their charge, $m/q$. Since their mass was not zero, he concluded that they were a form of matter, and presumably made up of a stream of microscopic, negatively charged particles. When Millikan published his results fourteen years later, it was reasonable to assume that the charge of one such particle equaled minus one fundamental charge, $q = -e$, and from the combination of Thomson’s and Millikan’s results one could therefore determine the mass of a single cathode ray particle.
The basic technique for determining \( m/q \) was simply to measure the angle through which the charged plates bent the beam. The electric force acting on a cathode ray particle while it was between the plates was

\[
F_{\text{elec}} = qE.
\]

By Newton’s second law, \( a = F/m \), we can find \( m/q \):

\[
\frac{m}{q} = \frac{E}{a}
\]

There was just one catch. Thomson needed to know the cathode ray particles’ velocity in order to figure out their acceleration. At that point, however, nobody had even an educated guess as to the speed of the cathode rays produced in a given vacuum tube. The beam appeared to leap across the vacuum tube practically instantaneously, so it was no simple matter of timing it with a stopwatch!

Thomson’s clever solution was to observe the effect of both electric and magnetic forces on the beam. The magnetic force exerted by a particular magnet would depend on both the cathode ray’s charge and its speed:

\[
F_{\text{mag}} = qvB
\]

Thomson played with the electric and magnetic forces until either one would produce an equal effect on the beam, allowing him to solve for the speed,

\[
v = \frac{E}{B}.
\]

Knowing the speed (which was on the order of 10% of the speed of light for his setup), he was able to find the acceleration and thus the mass-to-charge ratio \( m/q \). Thomson’s techniques were relatively crude (or perhaps more charitably we could say that they stretched the state of the art of the time), so with various methods he came up with \( m/q \) values that ranged over about a factor of two, even for cathode rays extracted from a cathode made of a single material. The best modern value is \( m/q = 5.69 \times 10^{-12} \text{ kg/C} \), which is consistent with the low end of Thomson’s range.
The cathode ray as a subatomic particle: the electron

What was significant about Thomson's experiment was not the actual numerical value of \( m/q \), however, so much as the fact that, combined with Millikan's value of the fundamental charge, it gave a mass for the cathode ray particles that was thousands of times smaller than the mass of even the lightest atoms. Even without Millikan's results, which were 14 years in the future, Thomson recognized that the cathode rays' \( m/q \) was thousands of times smaller than the \( m/q \) ratios that had been measured for electrically charged atoms in chemical solutions. He correctly interpreted this as evidence that the cathode rays were smaller building blocks — he called them electrons — out of which atoms themselves were formed. This was an extremely radical claim, coming at a time when atoms had not yet been proven to exist! Even those who used the word “atom” often considered them no more than mathematical abstractions, not literal objects. The idea of searching for structure inside of “un-splittable” atoms was seen by some as lunacy, but within ten years Thomson’s ideas had been amply verified by many more detailed experiments.

Discussion questions

A Thomson started to become convinced during his experiments that the “cathode rays” observed coming from the cathodes of vacuum tubes were building blocks of atoms — what we now call electrons. He then carried out observations with cathodes made of a variety of metals, and found that \( m/q \) was roughly the same in every case, considering his limited accuracy. Given his suspicion, why did it make sense to try different metals? How would the consistent values of \( m/q \) test his hypothesis?

B My students have frequently asked whether the \( m/q \) that Thomson measured was the value for a single electron, or for the whole beam. Can you answer this question?

C Thomson found that the \( m/q \) of an electron was thousands of times smaller than that of charged atoms in chemical solutions. Would this imply that the electrons had more charge? Less mass? Would there be no way to tell? Explain. Remember that Millikan's results were still many years in the future, so \( q \) was unknown.

D Can you guess any practical reason why Thomson couldn't just let one electron fly across the gap before disconnecting the battery and turning off the beam, and then measure the amount of charge deposited on the anode, thus allowing him to measure the charge of a single electron directly?

E Why is it not possible to determine \( m \) and \( q \) themselves, rather than just their ratio, by observing electrons’ motion in electric and magnetic fields?

The raisin cookie model

Based on his experiments, Thomson proposed a picture of the atom which became known as the raisin cookie model. In the neutral atom, \( j \), there are four electrons with a total charge of \(-4e\), sitting
The raisin cookie model of the atom with four units of charge, which we now know to be beryllium. It was known that chemical reactions could not change one element into another, so in Thomson’s scenario, each element’s cookie sphere had a permanently fixed radius, mass, and positive charge, different from those of other elements. The electrons, however, were not a permanent feature of the atom, and could be tacked on or pulled out to make charged ions. Although we now know, for instance, that a neutral atom with four electrons is the element beryllium, scientists at the time did not know how many electrons the various neutral atoms possessed.

This model is clearly different from the one you’ve learned in grade school or through popular culture, where the positive charge is concentrated in a tiny nucleus at the atom’s center. An equally important change in ideas about the atom has been the realization that atoms and their constituent subatomic particles behave entirely differently from objects on the human scale. For instance, we’ll see later that an electron can be in more than one place at one time. The raisin cookie model was part of a long tradition of attempts to make mechanical models of phenomena, and Thomson and his contemporaries never questioned the appropriateness of building a mental model of an atom as a machine with little parts inside. Today, mechanical models of atoms are still used (for instance the tinker-toy-style molecular modeling kits like the ones used by Watson and Crick to figure out the double helix structure of DNA), but scientists realize that the physical objects are only aids to help our brains’ symbolic and visual processes think about atoms.

Although there was no clear-cut experimental evidence for many of the details of the raisin cookie model, physicists went ahead and started working out its implications. For instance, suppose you had a four-electron atom. All four electrons would be repelling each other, but they would also all be attracted toward the center of the “cookie” sphere. The result should be some kind of stable, symmetric arrangement in which all the forces canceled out. People sufficiently clever with math soon showed that the electrons in a four-electron atom should settle down at the vertices of a pyramid with one less side than the Egyptian kind, i.e., a regular tetrahedron. This deduction turns out to be wrong because it was based on incorrect features of the model, but the model also had many successes, a few of which we will now discuss.

1Flow of electrical charge in wires example 1
One of my former students was the son of an electrician, and had become an electrician himself. He related to me how his father had remained refused to believe all his life that electrons really flowed through wires. If they had, he reasoned, the metal would have gradually become more and more damaged, eventually crumbling to dust.
His opinion is not at all unreasonable based on the fact that electrons are material particles, and that matter cannot normally pass through matter without making a hole through it. Nineteenth-century physicists would have shared his objection to a charged-particle model of the flow of electrical charge. In the raisin-cookie model, however, the electrons are very low in mass, and therefore presumably very small in size as well. It is not surprising that they can slip between the atoms without damaging them.

Flow of electrical charge across cell membranes example 2
Your nervous system is based on signals carried by charge moving from nerve cell to nerve cell. Your body is essentially all liquid, and atoms in a liquid are mobile. This means that, unlike the case of charge flowing in a solid wire, entire charged atoms can flow in your nervous system.

Emission of electrons in a cathode ray tube example 3
Why do electrons detach themselves from the cathode of a vacuum tube? Certainly they are encouraged to do so by the repulsion of the negative charge placed on the cathode and the attraction from the net positive charge of the anode, but these are not strong enough to rip electrons out of atoms by main force — if they were, then the entire apparatus would have been instantly vaporized as every atom was simultaneously ripped apart!

The raisin cookie model leads to a simple explanation. We know that heat is the energy of random motion of atoms. The atoms in any object are therefore violently jostling each other all the time, and a few of these collisions are violent enough to knock electrons out of atoms. If this occurs near the surface of a solid object, the electron may come loose. Ordinarily, however, this loss of electrons is a self-limiting process; the loss of electrons leaves the object with a net positive charge, which attracts the lost sheep home to the fold. (For objects immersed in air rather than vacuum, there will also be a balanced exchange of electrons between the air and the object.)

This interpretation explains the warm and friendly yellow glow of the vacuum tubes in an antique radio. To encourage the emission of electrons from the vacuum tubes’ cathodes, the cathodes are intentionally warmed up with little heater coils.

Discussion questions
A Today many people would define an ion as an atom (or molecule) with missing electrons or extra electrons added on. How would people have defined the word “ion” before the discovery of the electron?
B Since electrically neutral atoms were known to exist, there had to be positively charged subatomic stuff to cancel out the negatively charged electrons in an atom. Based on the state of knowledge immediately after the Millikan and Thomson experiments, was it possible that the positively charged stuff had an unquantized amount of charge? Could it be quan-
26.4 The nucleus

Radioactivity

Becquerel’s discovery of radioactivity

How did physicists figure out that the raisin cookie model was incorrect, and that the atom’s positive charge was concentrated in a tiny, central nucleus? The story begins with the discovery of radioactivity by the French chemist Becquerel. Up until radioactivity was discovered, all the processes of nature were thought to be based on chemical reactions, which were rearrangements of combinations of atoms. Atoms exert forces on each other when they are close together, so sticking or unsticking them would either release or store electrical energy. That energy could be converted to and from other forms, as when a plant uses the energy in sunlight to make sugars and carbohydrates, or when a child eats sugar, releasing the energy in the form of kinetic energy.

Becquerel discovered a process that seemed to release energy from an unknown new source that was not chemical. Becquerel, whose father and grandfather had also been physicists, spent the first twenty years of his professional life as a successful civil engineer, teaching physics on a part-time basis. He was awarded the chair of physics at the Musée d’Histoire Naturelle in Paris after the death of his father, who had previously occupied it. Having now a significant amount of time to devote to physics, he began studying the interaction of light and matter. He became interested in the phenomenon of phosphorescence, in which a substance absorbs energy from light, then releases the energy via a glow that only gradually goes away. One of the substances he investigated was a uranium compound, the salt UKSO\textsubscript{5}. One day in 1896, cloudy weather interfered with his plan to expose this substance to sunlight in order to observe its fluorescence. He stuck it in a drawer, coincidentally on top of a blank photographic plate — the old-fashioned glass-backed counterpart of the modern plastic roll of film. The plate had been carefully wrapped, but several days later when Becquerel checked it in the darkroom before using it, he found that it was ruined, as if it had been completely exposed to light.

History provides many examples of scientific discoveries that happened this way: an alert and inquisitive mind decides to investigate a phenomenon that most people would not have worried about explaining. Becquerel first determined by further experiments that the effect was produced by the uranium salt, despite a thick wrapping of paper around the plate that blocked out all light. He tried a variety of compounds, and found that it was the uranium that did it: the effect was produced by any uranium compound, but not
by any compound that didn’t include uranium atoms. The effect could be at least partially blocked by a sufficient thickness of metal, and he was able to produce silhouettes of coins by interposing them between the uranium and the plate. This indicated that the effect traveled in a straight line, so that it must have been some kind of ray rather than, e.g., the seepage of chemicals through the paper. He used the word “radiations,” since the effect radiated out from the uranium salt.

At this point Becquerel still believed that the uranium atoms were absorbing energy from light and then gradually releasing the energy in the form of the mysterious rays, and this was how he presented it in his first published lecture describing his experiments. Interesting, but not earth-shattering. But he then tried to determine how long it took for the uranium to use up all the energy that had supposedly been stored in it by light, and he found that it never seemed to become inactive, no matter how long he waited. Not only that, but a sample that had been exposed to intense sunlight for a whole afternoon was no more or less effective than a sample that had always been kept inside. Was this a violation of conservation of energy? If the energy didn’t come from exposure to light, where did it come from?

Three kinds of “radiations”

Unable to determine the source of the energy directly, turn-of-the-century physicists instead studied the behavior of the “radiations” once they had been emitted. Becquerel had already shown that the radioactivity could penetrate through cloth and paper, so the first obvious thing to do was to investigate in more detail what thickness of material the radioactivity could get through. They soon learned that a certain fraction of the radioactivity’s intensity would be eliminated by even a few inches of air, but the remainder was not eliminated by passing through more air. Apparently, then, the radioactivity was a mixture of more than one type, of which one was blocked by air. They then found that of the part that could penetrate air, a further fraction could be eliminated by a piece of paper or a very thin metal foil. What was left after that, however, was a third, extremely penetrating type, some of whose intensity would still remain even after passing through a brick wall. They decided that this showed there were three types of radioactivity, and without having the faintest idea of what they really were, they made up names for them. The least penetrating type was arbitrarily labeled $\alpha$ (alpha), the first letter of the Greek alphabet, and so on through $\beta$ (beta) and finally $\gamma$ (gamma) for the most penetrating type.

Radium: a more intense source of radioactivity

The measuring devices used to detect radioactivity were crude: photographic plates or even human eyeballs (radioactivity makes flashes
of light in the jelly-like fluid inside the eye, which can be seen by the eyeball’s owner if it is otherwise very dark). Because the ways of detecting radioactivity were so crude and insensitive, further progress was hindered by the fact that the amount of radioactivity emitted by uranium was not really very great. The vital contribution of physicist/chemist Marie Curie and her husband Pierre was to discover the element radium, and to purify and isolate significant quantities of it. Radium emits about a million times more radioactivity per unit mass than uranium, making it possible to do the experiments that were needed to learn the true nature of radioactivity. The dangers of radioactivity to human health were then unknown, and Marie died of leukemia thirty years later. (Pierre was run over and killed by a horsecart.)

Tracking down the nature of alphas, betas, and gammas

As radium was becoming available, an apprentice scientist named Ernest Rutherford arrived in England from his native New Zealand and began studying radioactivity at the Cavendish Laboratory. The young colonial’s first success was to measure the mass-to-charge ratio of beta rays. The technique was essentially the same as the one Thomson had used to measure the mass-to-charge ratio of cathode rays by measuring their deflections in electric and magnetic fields. The only difference was that instead of the cathode of a vacuum tube, a nugget of radium was used to supply the beta rays. Not only was the technique the same, but so was the result. Beta rays had the same $m/q$ ratio as cathode rays, which suggested they were one and the same. Nowadays, it would make sense simply to use the term “electron,” and avoid the archaic “cathode ray” and “beta particle,” but the old labels are still widely used, and it is unfortunately necessary for physics students to memorize all three names for the same thing.

At first, it seemed that neither alphas or gammas could be deflected in electric or magnetic fields, making it appear that neither was electrically charged. But soon Rutherford obtained a much more powerful magnet, and was able to use it to deflect the alphas but not the gammas. The alphas had a much larger value of $m/q$ than the betas (about 4000 times greater), which was why they had been so hard to deflect. Gammas are uncharged, and were later found to be a form of light.

The $m/q$ ratio of alpha particles turned out to be the same as those of two different types of ions, $\text{He}^{++}$ (a helium atom with two missing electrons) and $\text{H}_2^+$ (two hydrogen atoms bonded into a molecule, with one electron missing), so it seemed likely that they were one or the other of those. The diagram shows a simplified version of Rutherford’s ingenious experiment proving that they were $\text{He}^{++}$ ions. The gaseous element radon, an alpha emitter, was introduced into one half of a double glass chamber. The glass wall dividing
the chamber was made extremely thin, so that some of the rapidly moving alpha particles were able to penetrate it. The other chamber, which was initially evacuated, gradually began to accumulate a population of alpha particles (which would quickly pick up electrons from their surroundings and become electrically neutral). Rutherford then determined that it was helium gas that had appeared in the second chamber. Thus alpha particles were proved to be He$^{++}$ ions. The nucleus was yet to be discovered, but in modern terms, we would describe a He$^{++}$ ion as the nucleus of a He atom.

To summarize, here are the three types of radiation emitted by radioactive elements, and their descriptions in modern terms:

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>α particle</td>
<td>stopped by a few inches of air</td>
<td>He nucleus</td>
</tr>
<tr>
<td>β particle</td>
<td>stopped by a piece of paper</td>
<td>electron</td>
</tr>
<tr>
<td>γ ray</td>
<td>penetrates thick shielding</td>
<td>a type of light</td>
</tr>
</tbody>
</table>

**Discussion question**

A. Most sources of radioactivity emit alphas, betas, and gammas, not just one of the three. In the radon experiment, how did Rutherford know that he was studying the alphas?

**The planetary model**

The stage was now set for the unexpected discovery that the positively charged part of the atom was a tiny, dense lump at the atom’s center rather than the “cookie dough” of the raisin cookie model. By 1909, Rutherford was an established professor, and had students working under him. For a raw undergraduate named Marsden, he picked a research project he thought would be tedious but straightforward.

Rutherford had already determined in his previous experiments the speed of the alpha particles emitted by radium, a fantastic $1.5 \times 10^7$ m/s. The experimenters in Rutherford’s group visualized them as very small, very fast cannonballs penetrating the “cookie dough” part of the big gold atoms. A piece of paper has a thickness of a hundred thousand atoms or so, which would be sufficient to stop them completely, but crashing through a thousand would only slow them a little and turn them slightly off of their original paths.

Marsden’s supposedly ho-hum assignment was to use the apparatus shown in figure p to measure how often alpha particles were deflected at various angles. A tiny lump of radium in a box emitted alpha
particles, and a thin beam was created by blocking all the alphas except those that happened to pass out through a tube. Typically deflected in the gold by only a small amount, they would reach a screen very much like the screen of a TV’s picture tube, which would make a flash of light when it was hit. Here is the first example we have encountered of an experiment in which a beam of particles is detected one at a time. This was possible because each alpha particle carried so much kinetic energy; they were moving at about the same speed as the electrons in the Thomson experiment, but had ten thousand times more mass.

Marsden sat in a dark room, watching the apparatus hour after hour and recording the number of flashes with the screen moved to various angles. The rate of the flashes was highest when he set the screen at an angle close to the line of the alphas’ original path, but if he watched an area farther off to the side, he would also occasionally see an alpha that had been deflected through a larger angle. After seeing a few of these, he got the crazy idea of moving the screen to see if even larger angles ever occurred, perhaps even angles larger than 90 degrees.

![Diagram of alpha particles being scattered by a gold nucleus.](image)

The crazy idea worked: a few alpha particles were deflected through angles of up to 180 degrees, and the routine experiment had become an epoch-making one. Rutherford said, “We have been able to get some of the alpha particles coming backwards. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.” Explanations were hard to come by in the raisin cookie model. What intense electrical forces could have caused some of the alpha particles, moving at such astronomical
speeds, to change direction so drastically? Since each gold atom was electrically neutral, it would not exert much force on an alpha particle outside it. True, if the alpha particle was very near to or inside of a particular atom, then the forces would not necessarily cancel out perfectly; if the alpha particle happened to come very close to a particular electron, the $1/r^2$ form of the Coulomb force law would make for a very strong force. But Marsden and Rutherford knew that an alpha particle was 8000 times more massive than an electron, and it is simply not possible for a more massive object to rebound backwards from a collision with a less massive object while conserving momentum and energy. It might be possible in principle for a particular alpha to follow a path that took it very close to one electron, and then very close to another electron, and so on, with the net result of a large deflection, but careful calculations showed that such multiple “close encounters” with electrons would be millions of times too rare to explain what was actually observed.

At this point, Rutherford and Marsden dusted off an unpopular and neglected model of the atom, in which all the electrons orbited around a small, positively charged core or “nucleus,” just like the planets orbiting around the sun. All the positive charge and nearly all the mass of the atom would be concentrated in the nucleus, rather than spread throughout the atom as in the raisin cookie model. The positively charged alpha particles would be repelled by the gold atom’s nucleus, but most of the alphas would not come close enough to any nucleus to have their paths drastically altered. The few that did come close to a nucleus, however, could rebound backwards from a single such encounter, since the nucleus of a heavy gold atom would be fifty times more massive than an alpha particle. It turned out that it was not even too difficult to derive a formula giving the relative frequency of deflections through various angles, and this calculation agreed with the data well enough (to within 15%), considering the difficulty in getting good experimental statistics on the rare, very large angles.

What had started out as a tedious exercise to get a student started in science had ended as a revolution in our understanding of nature. Indeed, the whole thing may sound a little too much like a moralistic fable of the scientific method with overtones of the Horatio Alger genre. The skeptical reader may wonder why the planetary model was ignored so thoroughly until Marsden and Rutherford’s discovery. Is science really more of a sociological enterprise, in which certain ideas become accepted by the establishment, and other, equally plausible explanations are arbitrarily discarded? Some social scientists are currently ruffling a lot of scientists’ feathers with critiques very much like this, but in this particular case, there were very sound reasons for rejecting the planetary model. As you’ll learn in more detail later in this course, any charged particle that undergoes an acceleration dissipate energy in the form of light. In the
planetary model, the electrons were orbiting the nucleus in circles or ellipses, which meant they were undergoing acceleration, just like the acceleration you feel in a car going around a curve. They should have dissipated energy as light, and eventually they should have lost all their energy. Atoms don’t spontaneously collapse like that, which was why the raisin cookie model, with its stationary electrons, was originally preferred. There were other problems as well. In the planetary model, the one-electron atom would have to be flat, which would be inconsistent with the success of molecular modeling with spherical balls representing hydrogen and atoms. These molecular models also seemed to work best if specific sizes were used for different atoms, but there is no obvious reason in the planetary model why the radius of an electron’s orbit should be a fixed number. In view of the conclusive Marsden-Rutherford results, however, these became fresh puzzles in atomic physics, not reasons for disbelieving the planetary model.

Some phenomena explained with the planetary model

The planetary model may not be the ultimate, perfect model of the atom, but don’t underestimate its power. It already allows us to visualize correctly a great many phenomena.

As an example, let’s consider the distinctions among nonmetals, metals that are magnetic, and metals that are nonmagnetic. As shown in figure s, a metal differs from a nonmetal because its outermost electrons are free to wander rather than owing their allegiance to a particular atom. A metal that can be magnetized is one that is willing to line up the rotations of some of its electrons so that their axes are parallel. Recall that magnetic forces are forces made by moving charges; we have not yet discussed the mathematics and geometry of magnetic forces, but it is easy to see how random orientations of the atoms in the nonmagnetic substance would lead to cancellation of the forces.

Even if the planetary model does not immediately answer such questions as why one element would be a metal and another a nonmetal, these ideas would be difficult or impossible to conceptualize in the raisin cookie model.

Discussion question

A In reality, charges of the same type repel one another and charges of different types are attracted. Suppose the rules were the other way around, giving repulsion between opposite charges and attraction between similar ones. What would the universe be like?

Atomic number

As alluded to in a discussion question in the previous section, scientists of this period had only a very approximate idea of how many units of charge resided in the nuclei of the various chemical elements. Although we now associate the number of units of nuclear
charge with the element’s position on the periodic table, and call it the atomic number, they had no idea that such a relationship existed. Mendeleev’s table just seemed like an organizational tool, not something with any necessary physical significance. And everything Mendeleev had done seemed equally valid if you turned the table upside-down or reversed its left and right sides, so even if you wanted to number the elements sequentially with integers, there was an ambiguity as to how to do it. Mendeleev’s original table was in fact upside-down compared to the modern one.

[a modern periodic table, labeled with atomic numbers. Mendeleev’s original table was upside-down compared to this one.]

In the period immediately following the discovery of the nucleus, physicists only had rough estimates of the charges of the various nuclei. In the case of the very lightest nuclei, they simply found the maximum number of electrons they could strip off by various methods: chemical reactions, electric sparks, ultraviolet light, and so on. For example they could easily strip off one or two electrons from helium, making He\(^+\) or He\(^++\), but nobody could make He\(^+++\), presumably because the nuclear charge of helium was only +2e. Unfortunately only a few of the lightest elements could be stripped completely, because the more electrons were stripped off, the greater the positive net charge remaining, and the more strongly the rest of the negatively charged electrons would be held on. The heavy elements’ atomic numbers could only be roughly extrapolated from the light elements, where the atomic number was about half the atom’s mass expressed in units of the mass of a hydrogen atom. Gold, for example, had a mass about 197 times that of hydrogen, so its atomic number was estimated to be about half that, or somewhere around 100. We now know it to be 79.

How did we finally find out? The riddle of the nuclear charges was at last successfully attacked using two different techniques, which gave consistent results. One set of experiments, involving x-rays, was performed by the young Henry Mosely, whose scientific brilliance was soon to be sacrificed in a battle between European imperialists over who would own the Dardanelles, during that pointless conflict
then known as the War to End All Wars, and now referred to as World War I.

Since Mosely’s analysis requires several concepts with which you are not yet familiar, we will instead describe the technique used by James Chadwick at around the same time. An added bonus of describing Chadwick’s experiments is that they presaged the important modern technique of studying collisions of subatomic particles. In grad school, I worked with a professor whose thesis adviser’s thesis adviser was Chadwick, and he related some interesting stories about the man. Chadwick was apparently a little nutty and a complete fanatic about science, to the extent that when he was held in a German prison camp during World War II, he managed to cajole his captors into allowing him to scrounge up parts from broken radios so that he could attempt to do physics experiments.

Chadwick’s experiment worked like this. Suppose you perform two Rutherford-type alpha scattering measurements, first one with a gold foil as a target as in Rutherford’s original experiment, and then one with a copper foil. It is possible to get large angles of deflection in both cases, but as shown in figure v, the alpha particle must be heading almost straight for the copper nucleus to get the same angle of deflection that would have occurred with an alpha that was much farther off the mark; the gold nucleus’ charge is so much greater than the copper’s that it exerts a strong force on the alpha particle even from far off. The situation is very much like that of a blindfolded person playing darts. Just as it is impossible to aim an alpha particle at an individual nucleus in the target, the blindfolded person cannot really aim the darts. Achieving a very close encounter with the copper atom would be akin to hitting an inner circle on the dartboard. It’s much more likely that one would have the luck to hit the outer circle, which covers a greater number of square inches. By analogy, if you measure the frequency with which alphas are scattered by copper at some particular angle, say
between 19 and 20 degrees, and then perform the same measurement at the same angle with gold, you get a much higher percentage for gold than for copper.

An alpha particle must be headed for the ring on the front of the imaginary cylindrical pipe in order to produce scattering at an angle between 19 and 20 degrees. The area of this ring is called the "cross-section" for scattering at 19-20° because it is the cross-sectional area of a cut through the pipe.

In fact, the numerical ratio of the two nuclei’s charges can be derived from this same experimentally determined ratio. Using the standard notation $Z$ for the atomic number (charge of the nucleus divided by $e$), the following equation can be proved (example 4):

$$\frac{Z_{\text{gold}}^2}{Z_{\text{copper}}^2} = \frac{\text{number of alphas scattered by gold at 19-20°}}{\text{number of alphas scattered by copper at 19-20°}}$$

By making such measurements for targets constructed from all the elements, one can infer the ratios of all the atomic numbers, and since the atomic numbers of the light elements were already known, atomic numbers could be assigned to the entire periodic table. According to Mosely, the atomic numbers of copper, silver and platinum were 29, 47, and 78, which corresponded well with their positions on the periodic table. Chadwick’s figures for the same elements were 29.3, 46.3, and 77.4, with error bars of about 1.5 times the fundamental charge, so the two experiments were in good agreement.

The point here is absolutely not that you should be ready to plug numbers into the above equation for a homework or exam question! My overall goal in this chapter is to explain how we know what we know about atoms. An added bonus of describing Chadwick’s experiment is that the approach is very similar to that used in modern particle physics experiments, and the ideas used in the analysis are closely related to the now-ubiquitous concept of a “cross-section.” In the dartboard analogy, the cross-section would be the area of the circular ring you have to hit. The reasoning behind the invention of the term “cross-section” can be visualized as shown in figure v. In this language, Rutherford’s invention of the planetary model came from his unexpected discovery that there was a nonzero cross-section.
for alpha scattering from gold at large angles, and Chadwick confirmed Mosely’s determinations of the atomic numbers by measuring cross-sections for alpha scattering.

Proof of the relationship between $Z$ and scattering example 4

The equation above can be derived by the following not very rigorous proof. To deflect the alpha particle by a certain angle requires that it acquire a certain momentum component in the direction perpendicular to its original momentum. Although the nucleus’s force on the alpha particle is not constant, we can pretend that it is approximately constant during the time when the alpha is within a distance equal to, say, 150% of its distance of closest approach, and that the force is zero before and after that part of the motion. (If we chose 120% or 200%, it shouldn’t make any difference in the final result, because the final result is a ratio, and the effects on the numerator and denominator should cancel each other.) In the approximation of constant force, the change in the alpha’s perpendicular momentum component is then equal to $F\Delta t$. The Coulomb force law says the force is proportional to $Z/r^2$. Although $r$ does change somewhat during the time interval of interest, it’s good enough to treat it as a constant number, since we’re only computing the ratio between the two experiments’ results. Since we are approximating the force as acting over the time during which the distance is not too much greater than the distance of closest approach, the time interval $\Delta t$ must be proportional to $r$, and the sideways momentum imparted to the alpha, $F\Delta t$, is proportional to $(Z/r^2)r$, or $Z/r$. If we’re comparing alphas scattered at the same angle from gold and from copper, then $\Delta p$ is the same in both cases, and the proportionality $\Delta p \propto Z/r$ tells us that the ones scattered from copper at that angle had to be headed in along a line closer to the central axis by a factor equaling $Z_{\text{gold}}/Z_{\text{copper}}$. If you imagine a “dartboard ring” that the alphas have to hit, then the ring for the gold experiment has the same proportions as the one for copper, but it is enlarged by a factor equal to $Z_{\text{gold}}/Z_{\text{copper}}$. That is, not only is the radius of the ring greater by that factor, but unlike the rings on a normal dartboard, the thickness of the outer ring is also greater in proportion to its radius. When you take a geometric shape and scale it up in size like a photographic enlargement, its area is increased in proportion to the square of the enlargement factor, so the area of the dartboard ring in the gold experiment is greater by a factor equal to $(Z_{\text{gold}}/Z_{\text{copper}})^2$. Since the alphas are aimed entirely randomly, the chances of an alpha hitting the ring are in proportion to the area of the ring, which proves the equation given above.

As an example of the modern use of scattering experiments and cross-section measurements, you may have heard of the recent experimental evidence for the existence of a particle called the top quark. Of the twelve subatomic particles currently believed to be the
smallest constituents of matter, six form a family called the quarks, distinguished from the other six by the intense attractive forces that make the quarks stick to each other. (The other six consist of the electron plus five other, more exotic particles.) The only two types of quarks found in naturally occurring matter are the “up quark” and “down quark,” which are what protons and neutrons are made of, but four other types were theoretically predicted to exist, for a total of six. (The whimsical term “quark” comes from a line by James Joyce reading “Three quarks for master Mark.”) Until recently, only five types of quarks had been proven to exist via experiments, and the sixth, the top quark, was only theorized. There was no hope of ever detecting a top quark directly, since it is radioactive, and only exists for a zillionth of a second before evaporating. Instead, the researchers searching for it at the Fermi National Accelerator Laboratory near Chicago measured cross-sections for scattering of nuclei off of other nuclei. The experiment was much like those of Rutherford and Chadwick, except that the incoming nuclei had to be boosted to much higher speeds in a particle accelerator. The resulting encounter with a target nucleus was so violent that both nuclei were completely demolished, but, as Einstein proved, energy can be converted into matter, and the energy of the collision creates a spray of exotic, radioactive particles, like the deadly shower of wood fragments produced by a cannon ball in an old naval battle. Among those particles were some top quarks. The cross-sections being measured were the cross-sections for the production of certain combinations of these secondary particles. However different the details, the principle was the same as that employed at the turn of the century: you smash things together and look at the fragments that fly off to see what was inside them. The approach has been compared to shooting a clock with a rifle and then studying the pieces that fly off to figure out how the clock worked.

Discussion questions

A The diagram, showing alpha particles being deflected by a gold nucleus, was drawn with the assumption that alpha particles came in on lines at many different distances from the nucleus. Why wouldn’t they all come in along the same line, since they all came out through the same tube?

B Why does it make sense that, as shown in the figure, the trajectories that result in $19^\circ$ and $20^\circ$ scattering cross each other?

C Rutherford knew the velocity of the alpha particles emitted by radium, and guessed that the positively charged part of a gold atom had a charge of about $+100e$ (we now know it is $+79e$). Considering the fact that some alpha particles were deflected by $180^\circ$, how could he then use conservation of energy to derive an upper limit on the size of a gold nucleus? (For simplicity, assume the size of the alpha particle is negligible compared to that of the gold nucleus, and ignore the fact that the gold nucleus recoils a little from the collision, picking up a little kinetic energy.)
The structure of nuclei

The proton

The fact that the nuclear charges were all integer multiples of \( e \) suggested to many physicists that rather than being a pointlike object, the nucleus might contain smaller particles having individual charges of \(+e\). Evidence in favor of this idea was not long in arriving. Rutherford reasoned that if he bombarded the atoms of a very light element with alpha particles, the small charge of the target nuclei would give a very weak repulsion. Perhaps those few alpha particles that happened to arrive on head-on collision courses would get so close that they would physically crash into some of the target nuclei. An alpha particle is itself a nucleus, so this would be a collision between two nuclei, and a violent one due to the high speeds involved. Rutherford hit pay dirt in an experiment with alpha particles striking a target containing nitrogen atoms. Charged particles were detected flying out of the target like parts flying off of cars in a high-speed crash. Measurements of the deflection of these particles in electric and magnetic fields showed that they had the same charge-to-mass ratio as singly-ionized hydrogen atoms. Rutherford concluded that these were the conjectured singly-charged particles that held the charge of the nucleus, and they were later named protons. The hydrogen nucleus consists of a single proton, and in general, an element’s atomic number gives the number of protons contained in each of its nuclei. The mass of the proton is about 1800 times greater than the mass of the electron.

The neutron

It would have been nice and simple if all the nuclei could have been built only from protons, but that couldn’t be the case. If you spend a little time looking at a periodic table, you will soon notice that although some of the atomic masses are very nearly integer multiples of hydrogen’s mass, many others are not. Even where the masses are close whole numbers, the masses of an element other than hydrogen is always greater than its atomic number, not equal to it. Helium, for instance, has two protons, but its mass is four times greater than that of hydrogen.

Chadwick cleared up the confusion by proving the existence of a new subatomic particle. Unlike the electron and proton, which are electrically charged, this particle is electrically neutral, and he named it the neutron. Chadwick’s experiment has been described in detail in section 14.2, but briefly the method was to expose a sample of the light element beryllium to a stream of alpha particles from a lump of radium. Beryllium has only four protons, so an alpha that happens to be aimed directly at a beryllium nucleus can actually hit it rather than being stopped short of a collision by electrical repulsion. Neutrons were observed as a new form of radiation emerging from the collisions, and Chadwick correctly inferred that they were previ-
Examples of the construction of atoms: hydrogen (top) and helium (bottom). On this scale, the electrons’ orbits would be the size of a college campus.

ously unsuspected components of the nucleus that had been knocked out. As described earlier, Chadwick also determined the mass of the neutron; it is very nearly the same as that of the proton.

To summarize, atoms are made of three types of particles:

<table>
<thead>
<tr>
<th></th>
<th>charge</th>
<th>mass in units of the proton’s mass</th>
<th>location in atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>$+e$</td>
<td>1</td>
<td>in nucleus</td>
</tr>
<tr>
<td>neutron</td>
<td>0</td>
<td>1.001</td>
<td>in nucleus</td>
</tr>
<tr>
<td>electron</td>
<td>$-e$</td>
<td>$1/1836$</td>
<td>orbiting nucleus</td>
</tr>
</tbody>
</table>

The existence of neutrons explained the mysterious masses of the elements. Helium, for instance, has a mass very close to four times greater than that of hydrogen. This is because it contains two neutrons in addition to its two protons. The mass of an atom is essentially determined by the total number of neutrons and protons. The total number of neutrons plus protons is therefore referred to as the atom’s mass number.

Isotopes

We now have a clear interpretation of the fact that helium is close to four times more massive than hydrogen, and similarly for all the atomic masses that are close to an integer multiple of the mass of hydrogen. But what about copper, for instance, which had an atomic mass 63.5 times that of hydrogen? It didn’t seem reasonable to think that it possessed an extra half of a neutron! The solution was found by measuring the mass-to-charge ratios of singly-ionized atoms (atoms with one electron removed). The technique is essentially that same as the one used by Thomson for cathode rays, except that whole atoms do not spontaneously leap out of the surface of an object as electrons sometimes do. Figure x shows an example of how the ions can be created and injected between the charged plates for acceleration.

Injecting a stream of copper ions into the device, we find a surprise — the beam splits into two parts! Chemists had elevated to dogma the assumption that all the atoms of a given element were identical, but we find that 69% of copper atoms have one mass, and 31% have another. Not only that, but both masses are very nearly integer multiples of the mass of hydrogen (63 and 65, respectively). Copper gets its chemical identity from the number of protons in its nucleus, 29, since chemical reactions work by electric forces. But apparently some copper atoms have $63 - 29 = 34$ neutrons while others have $65 - 29 = 36$. The atomic mass of copper, 63.5, reflects the proportions of the mixture of the mass-63 and mass-65 varieties. The different mass varieties of a given element are called isotopes of that element.
Isotopes can be named by giving the mass number as a subscript to the left of the chemical symbol, e.g., $^{65}\text{Cu}$. Examples:

<table>
<thead>
<tr>
<th></th>
<th>protons</th>
<th>neutrons</th>
<th>mass number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>1</td>
<td>0</td>
<td>0+1 = 1</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>2</td>
<td>2</td>
<td>2+2 = 4</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>6</td>
<td>6</td>
<td>6+6 = 12</td>
</tr>
<tr>
<td>$^{14}\text{C}$</td>
<td>6</td>
<td>8</td>
<td>6+8 = 14</td>
</tr>
<tr>
<td>$^{262}\text{Ha}$</td>
<td>105</td>
<td>157</td>
<td>105+157 = 262</td>
</tr>
</tbody>
</table>

**self-check B**

Why are the positive and negative charges of the accelerating plates reversed in the isotope-separating apparatus compared to the Thomson apparatus?  

▷ Answer, p. 1042

Chemical reactions are all about the exchange and sharing of electrons: the nuclei have to sit out this dance because the forces of electrical repulsion prevent them from ever getting close enough to make contact with each other. Although the protons do have a vitally important effect on chemical processes because of their electrical forces, the neutrons can have no effect on the atom’s chemical reactions. It is not possible, for instance, to separate $^{63}\text{Cu}$ from $^{65}\text{Cu}$ by chemical reactions. This is why chemists had never realized that different isotopes existed. (To be perfectly accurate, different isotopes do behave slightly differently because the more massive atoms move more sluggishly and therefore react with a tiny bit less intensity. This tiny difference is used, for instance, to separate out the isotopes of uranium needed to build a nuclear bomb. The smallness of this effect makes the separation process a slow and difficult one, which is what we have to thank for the fact that nuclear weapons have not been built by every terrorist cabal on the planet.)

**Sizes and shapes of nuclei**

Matter is nearly all nuclei if you count by weight, but in terms of volume nuclei don’t amount to much. The radius of an individual neutron or proton is very close to 1 fm (1 fm = 10$^{-15}$ m), so even a big lead nucleus with a mass number of 208 still has a diameter of only about 13 fm, which is ten thousand times smaller than the diameter of a typical atom. Contrary to the usual imagery of the nucleus as a small sphere, it turns out that many nuclei are somewhat elongated, like an American football, and a few have exotic asymmetric shapes like pears or kiwi fruits.

**Discussion questions**

A Suppose the entire universe was in a (very large) cereal box, and the nutritional labeling was supposed to tell a godlike consumer what percentage of the contents was nuclei. Roughly what would the percentage be like if the labeling was according to mass? What if it was by volume?
The strong nuclear force cuts off very sharply at a range of about 1 fm.

A nuclear power plant at Cattenom, France. Unlike the coal and oil plants that supply most of the U.S.’s electrical power, a nuclear power plant like this one releases no pollution or greenhouse gases into the Earth’s atmosphere, and therefore doesn’t contribute to global warming. The white stuff puffing out of this plant is non-radioactive water vapor. Although nuclear power plants generate long-lived nuclear waste, this waste arguably poses much less of a threat to the biosphere than greenhouse gases would.

**The strong nuclear force, alpha decay and fission**

Once physicists realized that nuclei consisted of positively charged protons and uncharged neutrons, they had a problem on their hands. The electrical forces among the protons are all repulsive, so the nucleus should simply fly apart! The reason all the nuclei in your body are not spontaneously exploding at this moment is that there is another force acting. This force, called the **strong nuclear force**, is always attractive, and acts between neutrons and neutrons, neutrons and protons, and protons and protons with roughly equal strength. The strong nuclear force does not have any effect on electrons, which is why it does not influence chemical reactions.

Unlike electric forces, whose strengths are given by the simple Coulomb force law, there is no simple formula for how the strong nuclear force depends on distance. Roughly speaking, it is effective over ranges of ~ 1 fm, but falls off extremely quickly at larger distances (much faster than $1/r^2$). Since the radius of a neutron or proton is about 1 fm, that means that when a bunch of neutrons and protons are packed together to form a nucleus, the strong nuclear force is effective only between neighbors.

Figure aa illustrates how the strong nuclear force acts to keep ordinary nuclei together, but is not able to keep very heavy nuclei from breaking apart. In aa/1, a proton in the middle of a carbon nucleus feels an attractive strong nuclear force (arrows) from each of its nearest neighbors. The forces are all in different directions, and tend to cancel out. The same is true for the repulsive electrical forces (not shown). In figure aa/2, a proton at the edge of the nucleus has neighbors only on one side, and therefore all the strong
The forces cancel. 2. The forces don’t cancel. 3. In a heavy nucleus, the large number of electrical repulsions can add up to a force that is comparable to the strong nuclear attraction. 4. Alpha emission. 5. Fission.

nuclear forces acting on it are tending to pull it back in. Although all the electrical forces from the other five protons (dark arrows) are all pushing it out of the nucleus, they are not sufficient to overcome the strong nuclear forces.

In a very heavy nucleus, a proton that finds itself near the edge has only a few neighbors close enough to attract it significantly via the strong nuclear force, but every other proton in the nucleus exerts a repulsive electrical force on it. If the nucleus is large enough, the total electrical repulsion may be sufficient to overcome the attraction of the strong force, and the nucleus may spit out a proton. Proton emission is fairly rare, however; a more common type of radioactive decay in heavy nuclei is alpha decay, shown in aa/4. The imbalance of the forces is similar, but the chunk that is ejected is an alpha particle (two protons and two neutrons) rather than a single proton.

It is also possible for the nucleus to split into two pieces of roughly equal size, a process known as fission. Note that in addition to the two large fragments, there is a spray of individual neutrons. In a nuclear fission bomb or a nuclear fission reactor, some of these

---

1 Alpha decay is more common because an alpha particle happens to be a very stable arrangement of protons and neutrons.
neutrons fly off and hit other nuclei, causing them to undergo fission as well. The result is a chain reaction.

When a nucleus is able to undergo one of these processes, it is said to be radioactive, and to undergo radioactive decay. Some of the naturally occurring nuclei on earth are radioactive. The term “radioactive” comes from Becquerel’s image of rays radiating out from something, not from radio waves, which are a whole different phenomenon. The term “decay” can also be a little misleading, since it implies that the nucleus turns to dust or simply disappears – actually it is splitting into two new nuclei with the same total number of neutrons and protons, so the term “radioactive transformation” would have been more appropriate. Although the original atom’s electrons are mere spectators in the process of weak radioactive decay, we often speak loosely of “radioactive atoms” rather than “radioactive nuclei.”

**Randomness in physics**

How does an atom decide when to decay? We might imagine that it is like a termite-infested house that gets weaker and weaker, until finally it reaches the day on which it is destined to fall apart. Experiments, however, have not succeeded in detecting such “ticking clock” hidden below the surface; the evidence is that all atoms of a given isotope are absolutely identical. Why, then, would one uranium atom decay today while another lives for another million years? The answer appears to be that it is entirely random. We can make general statements about the average time required for a certain isotope to decay, or how long it will take for half the atoms in a sample to decay (its half-life), but we can never predict the behavior of a particular atom.

This is the first example we have encountered of an inescapable randomness in the laws of physics. If this kind of randomness makes you uneasy, you’re in good company. Einstein’s famous quote is “...I am convinced that He [God] does not play dice.” Einstein’s distaste for randomness, and his association of determinism with divinity, goes back to the Enlightenment conception of the universe as a gigantic piece of clockwork that only had to be set in motion initially by the Builder. Physics had to be entirely rebuilt in the 20th century to incorporate the fundamental randomness of physics, and this modern revolution is the topic of chapters 33-26. In particular, we will delay the mathematical development of the half-life concept until then.
The weak nuclear force; beta decay

All the nuclear processes we’ve discussed so far have involved re-arrangements of neutrons and protons, with no change in the total number of neutrons or the total number of protons. Now consider the proportions of neutrons and protons in your body and in the planet earth: neutrons and protons are roughly equally numerous in your body’s carbon and oxygen nuclei, and also in the nickel and iron that make up most of the earth. The proportions are about 50-50. But, as discussed in more detail on p. 791, the only chemical elements produced in any significant quantities by the big bang were hydrogen (about 90%) and helium (about 10%). If the early universe was almost nothing but hydrogen atoms, whose nuclei are protons, where did all those neutrons come from?

The answer is that there is another nuclear force, the weak nuclear force, that is capable of transforming neutrons into protons and vice-versa. Two possible reactions are

\[ n \rightarrow p + e^- + \bar{\nu} \quad \text{[electron decay]} \]

and

\[ p \rightarrow n + e^+ + \nu. \quad \text{[positron decay]} \]

(There is also a third type called electron capture, in which a proton grabs one of the atom’s electrons and they produce a neutron and a neutrino.)

Whereas alpha decay and fission are just a redivision of the previously existing particles, these reactions involve the destruction of one particle and the creation of three new particles that did not exist before.

There are three new particles here that you have never previously encountered. The symbol \( e^+ \) stands for an antielectron, which is a particle just like the electron in every way, except that its electric charge is positive rather than negative. Antielectrons are also known as positrons. Nobody knows why electrons are so common in the universe and antielectrons are scarce. When an antielectron encounters an electron, they annihilate each other, producing gamma rays, and this is the fate of all the antielectrons that are produced by natural radioactivity on earth. Antielectrons are an example of antimatter. A complete atom of antimatter would consist of antiprotons, antielectrons, and antineutrons. Although individual particles of antimatter occur commonly in nature due to natural radioactivity and cosmic rays, only a few complete atoms of antihydrogen have ever been produced artificially.

The notation \( \nu \) stands for a particle called a neutrino, and \( \bar{\nu} \) means an antineutrino. Neutrinos and antineutrinos have no electric charge (hence the name).

\(^2\)The evidence for the big bang theory of the origin of the universe was discussed on p. 511.
We can now list all four of the known fundamental forces of physics:

- gravity
- electromagnetism
- strong nuclear force
- weak nuclear force

The other forces we have learned about, such as friction and the normal force, all arise from electromagnetic interactions between atoms, and therefore are not considered to be fundamental forces of physics.

\textit{Decay of }^{212}\textit{Pb} \\
\textit{example 5} \\
As an example, consider the radioactive isotope of lead \(^{212}\text{Pb}\). It contains 82 protons and 130 neutrons. It decays by the process \(n \rightarrow p + e^- + \bar{\nu}\). The newly created proton is held inside the nucleus by the strong nuclear force, so the new nucleus contains 83 protons and 129 neutrons. Having 83 protons makes it the element bismuth, so it will be an atom of \(^{212}\text{Bi}\).

In a reaction like this one, the electron flies off at high speed (typically close to the speed of light), and the escaping electrons are the things that make large amounts of this type of radioactivity dangerous. The outgoing electron was the first thing that tipped off scientists in the early 1900s to the existence of this type of radioactivity. Since they didn’t know that the outgoing particles were electrons, they called them beta particles, and this type of radioactive decay was therefore known as beta decay. A clearer but less common terminology is to call the two processes electron decay and positron decay.

The neutrino or antineutrino emitted in such a reaction pretty much ignores all matter, because its lack of charge makes it immune to electrical forces, and it also remains aloof from strong nuclear interactions. Even if it happens to fly off going straight down, it is almost certain to make it through the entire earth without interacting with any atoms in any way. It ends up flying through outer space forever. The neutrino’s behavior makes it exceedingly difficult to detect, and when beta decay was first discovered nobody realized that neutrinos even existed. We now know that the neutrino carries off some of the energy produced in the reaction, but at the time it seemed that the total energy afterwards (not counting the unsuspected neutrino’s energy) was greater than the total energy before the reaction, violating conservation of energy. Physicists were getting ready to throw conservation of energy out the window as a basic law of physics when indirect evidence led them to the conclusion that neutrinos existed.
Discussion questions

A In the reactions $n \rightarrow p + e^- + \bar{\nu}$ and $p \rightarrow n + e^+ + \nu$, verify that charge is conserved. In beta decay, when one of these reactions happens to a neutron or proton within a nucleus, one or more gamma rays may also be emitted. Does this affect conservation of charge? Would it be possible for some extra electrons to be released without violating charge conservation?

B When an antielectron and an electron annihilate each other, they produce two gamma rays. Is charge conserved in this reaction?

Fusion

As we have seen, heavy nuclei tend to fly apart because each proton is being repelled by every other proton in the nucleus, but is only attracted by its nearest neighbors. The nucleus splits up into two parts, and as soon as those two parts are more than about 1 fm apart, the strong nuclear force no longer causes the two fragments to attract each other. The electrical repulsion then accelerates them, causing them to gain a large amount of kinetic energy. This release of kinetic energy is what powers nuclear reactors and fission bombs.

It might seem, then, that the lightest nuclei would be the most stable, but that is not the case. Let’s compare an extremely light nucleus like $^4$He with a somewhat heavier one, $^{16}$O. A neutron or proton in $^4$He can be attracted by the three others, but in $^{16}$O, it might have five or six neighbors attracting it. The $^{16}$O nucleus is therefore more stable.

It turns out that the most stable nuclei of all are those around nickel and iron, having about 30 protons and 30 neutrons. Just as a nucleus that is too heavy to be stable can release energy by splitting apart into pieces that are closer to the most stable size, light nuclei can release energy if you stick them together to make bigger nuclei that are closer to the most stable size. Fusing one nucleus with another is called nuclear fusion. Nuclear fusion is what powers our sun and other stars.
The first release of energy by nuclear fusion through human technology was the 1952 Ivy Mike test at the Enewetak Atoll. This array of gamma-ray detectors is called GAMMASPHERE. During operation, the array is closed up, and a beam of ions produced by a particle accelerator strikes a target at its center, producing nuclear fusion reactions. The gamma rays can be studied for information about the structure of the fused nuclei, which are typically varieties not found in nature. Nuclear fusion promises to be a clean, inexhaustible source of energy. However, the goal of commercially viable nuclear fusion power has remained elusive, due to the engineering difficulties involved in magnetically containing a plasma (ionized gas) at a sufficiently high temperature and density. This photo shows the experimental JET reactor, with the device opened up on the left, and in action on the right.

Nuclear energy and binding energies

In the same way that chemical reactions can be classified as exothermic (releasing energy) or endothermic (requiring energy to react), so nuclear reactions may either release or use up energy. The energies involved in nuclear reactions are greater by a huge factor. Thousands of tons of coal would have to be burned to produce as much
energy as would be produced in a nuclear power plant by one kg of fuel.

Although nuclear reactions that use up energy (endothermic reactions) can be initiated in accelerators, where one nucleus is rammed into another at high speed, they do not occur in nature, not even in the sun. The amount of kinetic energy required is simply not available.

To find the amount of energy consumed or released in a nuclear reaction, you need to know how much nuclear interaction energy, \( U_{\text{nuc}} \), was stored or released. Experimentalists have determined the amount of nuclear energy stored in the nucleus of every stable element, as well as many unstable elements. This is the amount of mechanical work that would be required to pull the nucleus apart into its individual neutrons and protons, and is known as the nuclear binding energy.

\[ ^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + \gamma \]

The sun produces its energy through a series of nuclear fusion reactions. One of the reactions is

The excess energy is almost all carried off by the gamma ray (not by the kinetic energy of the helium-3 atom). The binding energies in units of pJ (picojoules) are:

\[ ^1\text{H} \; 0 \text{ J} \]
\[ ^2\text{H} \; 0.35593 \text{ pJ} \]
\[ ^3\text{He} \; 1.23489 \text{ pJ} \]

The total initial nuclear energy is 0 pJ+0.35593 pJ, and the final nuclear energy is 1.23489 pJ, so by conservation of energy, the gamma ray must carry off 0.87896 pJ of energy. The gamma ray is then absorbed by the sun and converted to heat.

**self-check C**

Why is the binding energy of \(^1\text{H}\) exactly equal to zero?  

\[ \text{Answer, p. 1042} \]

Figure ac is a compact way of showing the vast variety of the nuclei. Each box represents a particular number of neutrons and protons. The black boxes are nuclei that are stable, i.e., that would require an input of energy in order to change into another. The gray boxes show all the unstable nuclei that have been studied experimentally. Some of these last for billions of years on the average before decaying and are found in nature, but most have much shorter average lifetimes, and can only be created and studied in the laboratory.

The curve along which the stable nuclei lie is called the line of stability. Nuclei along this line have the most stable proportion of...
ac / A map showing levels of radiation near the site of the Chernobyl nuclear accident.

rad / The known nuclei, represented on a chart of proton number versus neutron number. Note the two nuclei in the bottom row with zero protons.

neutrons to protons. For light nuclei the most stable mixture is about 50-50, but we can see that stable heavy nuclei have two or three times more neutrons than protons. This is because the electrical repulsions of all the protons in a heavy nucleus add up to a powerful force that would tend to tear it apart. The presence of a large number of neutrons increases the distances among the protons, and also increases the number of attractions due to the strong nuclear force.

**Biological effects of ionizing radiation**

*Units used to measure exposure*

As a science educator, I find it frustrating that nowhere in the massive amount of journalism devoted to nuclear safety does one ever find any numerical statements about the amount of radiation to which people have been exposed. Anyone capable of understanding sports statistics or weather reports ought to be able to understand such measurements, as long as something like the following explanatory text was inserted somewhere in the article:

Radiation exposure is measured in units of Sieverts (Sv). The aver-
In this classic zombie flick, a newscaster speculates that the dead have been reanimated due to radiation brought back to earth by a space probe. Radiation doesn’t mutate entire multicellular organisms. An average person is exposed to about 2000 µSv (microSieverts) each year from natural background sources.

With this context, people would be able to come to informed conclusions. For example, figure ae shows a scary-looking map of the levels of radiation in the area surrounding the 1986 nuclear accident at Chernobyl, Ukraine, the most serious that has ever occurred. At the boundary of the most highly contaminated (bright red) areas, people would be exposed to about 13,000 µSv per year, or about four times the natural background level. In the pink areas, which are still densely populated, the exposure is comparable to the natural level found in a high-altitude city such as Denver.

What is a Sievert? It measures the amount of energy per kilogram deposited in the body by ionizing radiation, multiplied by a “quality factor” to account for the different health hazards posed by alphas, betas, gammas, neutrons, and other types of radiation. Only ionizing radiation is counted, since nonionizing radiation simply heats one’s body rather than killing cells or altering DNA. For instance, alpha particles are typically moving so fast that their kinetic energy is sufficient to ionize thousands of atoms, but it is possible for an alpha particle to be moving so slowly that it would not have enough kinetic energy to ionize even one atom.

Unfortunately, most people don’t know much about radiation and tend to react to it based on unscientific cultural notions. These may, as in figure ae, be based on fictional tropes silly enough to require the suspension of disbelief by the audience, but they can also be more subtle. People of my kids’ generation are more familiar with the 2011 Fukushima nuclear accident than with the much more serious Chernobyl accident. The news coverage of Fukushima showed scary scenes of devastated landscapes and distraught evacuees, implying that people had been killed and displaced by the release of radiation from the reaction. In fact, there were no deaths at all due to the radiation released at Fukushima, and no excess cancer deaths are statistically predicted in the future. The devastation and the death toll of 16,000 were caused by the earthquake and tsunami, which were also what damaged the plant.

Effects of exposure

Notwithstanding the pop culture images like figure af, it is not possible for a multicellular animal to become “mutated” as a whole. In most cases, a particle of ionizing radiation will not even hit the DNA, and even if it does, it will only affect the DNA of a single cell, not every cell in the animal’s body. Typically, that cell is simply killed, because the DNA becomes unable to function properly. Once in a while, however, the DNA may be altered so as to make that cell cancerous. For instance, skin cancer can be caused by UV light hitting a single skin cell in the body of a sunbather. If that cell becomes cancerous and begins reproducing uncontrollably, she will...
A typical example of radiation hormesis: the health of mice is improved by low levels of radiation. In this study, young mice were exposed to fairly high levels of x-rays, while a control group of mice was not exposed. The mice were weighed, and their rate of growth was taken as a measure of their health. At levels below about 50,000 µSv, the radiation had a beneficial effect on the health of the mice, presumably by activating cellular damage control mechanisms. The two highest data points are statistically significant at the 99% level. The curve is a fit to a theoretical model. Redrawn from T.D. Luckey, *Hormesis with Ionizing Radiation*, CRC Press, 1980.

Effects of high doses of radiation

A whole-body exposure of 5,000,000 µSv will kill a person within a week or so. Luckily, only a small number of humans have ever been exposed to such levels: one scientist working on the Manhattan Project, some victims of the Nagasaki and Hiroshima explosions, and 31 workers at Chernobyl. Death occurs by massive killing of cells, especially in the blood-producing cells of the bone marrow.

Effects of low doses radiation

Lower levels, on the order of 1,000,000 µSv, were inflicted on some people at Nagasaki and Hiroshima. No acute symptoms result from this level of exposure, but certain types of cancer are significantly more common among these people. It was originally expected that the radiation would cause many mutations resulting in birth defects, but very few such inherited effects have been observed.

A great deal of time has been spent debating the effects of very low levels of ionizing radiation. The following table gives some sample figures.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Radiation Dose (µSv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum beneficial dose per day</td>
<td>~ 10,000 µSv</td>
</tr>
<tr>
<td>CT scan</td>
<td>~ 10,000 µSv</td>
</tr>
<tr>
<td>natural background per year</td>
<td>2,000-7,000 µSv</td>
</tr>
<tr>
<td>health guidelines for exposure to a fetus</td>
<td>1,000 µSv</td>
</tr>
<tr>
<td>flying from New York to Tokyo</td>
<td>150 µSv</td>
</tr>
<tr>
<td>chest x-ray</td>
<td>50 µSv</td>
</tr>
</tbody>
</table>

Note that the largest number, on the first line of the table, is the maximum beneficial dose. The most useful evidence comes from experiments in animals, which can intentionally be exposed to significant and well measured doses of radiation under controlled conditions. Experiments show that low levels of radiation activate cellular damage control mechanisms, increasing the health of the organism. For example, exposure to radiation up to a certain level makes mice grow faster; makes guinea pigs' immune systems function better against diptheria; increases fertility in trout and mice; improves fetal mice’s resistance to disease; increases the life-spans of flour beetles and mice; and reduces mortality from cancer in mice. This type of effect is called radiation hormesis.

There is also some evidence that in humans, small doses of radiation end up with a tumor twenty years later.

Other than cancer, the only other dramatic effect that can result from altering a single cell’s DNA is if that cell happens to be a sperm or ovum, which can result in nonviable or mutated offspring. Men are relatively immune to reproductive harm from radiation, because their sperm cells are replaced frequently. Women are more vulnerable because they keep the same set of ova as long as they live.
increase fertility, reduce genetic abnormalities, and reduce mortality from cancer. The human data, however, tend to be very poor compared to the animal data. Due to ethical issues, one cannot do controlled experiments in humans. For example, one of the best sources of information has been from the survivors of the Hiroshima and Nagasaki bomb blasts, but these people were also exposed to high levels of carcinogenic chemicals in the smoke from their burning cities; for comparison, firefighters have a heightened risk of cancer, and there are also significant concerns about cancer from the 9/11 attacks in New York. The direct empirical evidence about radiation hormesis in humans is therefore not good enough to tell us anything unambiguous, and the most scientifically reasonable approach is to assume that the results in animals also hold for humans: small doses of radiation in humans are beneficial, rather than harmful. However, a variety of cultural and historical factors have led to a situation in which public health policy is based on the assumption, known as “linear no-threshold” (LNT), that even tiny doses of radiation are harmful, and that the risk they carry is proportional to the dose. In other words, law and policy are made based on the assumption that the effects of radiation on humans are dramatically different than its effects on mice and guinea pigs. Even with the unrealistic assumption of LNT, one can still evaluate risks by comparing with natural background radiation. For example, we can see that the effect of a chest x-ray is about a hundred times smaller than the effect of spending a year in Colorado, where the level of natural background radiation from cosmic rays is higher than average, due to the high altitude. Dropping the implausible LNT assumption, we can see that the impact on one’s health of spending a year in Colorado is likely to be positive, because the excess radiation is below the maximum beneficial level.

The green case for nuclear power

In the late twentieth century, antinuclear activists largely succeeded in bringing construction of new nuclear power plants to a halt in the U.S. Ironically, we now know that the burning of fossil fuels, which leads to global warming, is a far more grave threat to the environment than even the Chernobyl disaster. A team of biologists writes: “During recent visits to Chernobyl, we experienced numerous sightings of moose (Alces alces), roe deer (Capreol capreolus), Russian wild boar (Sus scrofa), foxes (Vulpes vulpes), river otter (Lutra canadensis), and rabbits (Lepus europaeus) ... Diversity of flowers and other plants in the highly radioactive regions is impressive and equals that observed in protected habitats outside the zone.

... The observation that typical human activity (industrialization, farming, cattle raising, collection of firewood, hunting, etc.) is more devastating to biodiversity and abundance of local flora and fauna than is the worst nuclear power plant disaster validates the negative impact the exponential growth of human populations has on wildlife."\(^4\)

Nuclear power is the only source of energy that is sufficient to replace any significant percentage of energy from fossil fuels on the rapid schedule demanded by the speed at which global warming is progressing. People worried about the downside of nuclear energy might be better off putting their energy into issues related to nuclear weapons: the poor stewardship of the former Soviet Union’s warheads; nuclear proliferation in unstable states such as Pakistan; and the poor safety and environmental history of the superpowers’ nuclear weapons programs, including the loss of several warheads in plane crashes, and the environmental disaster at the Hanford, Washington, weapons plant.

**Protection from radiation**

People do sometimes work with strong enough radioactivity that there is a serious health risk. Typically the scariest sources are those used in cancer treatment and in medical and biological research. Also, a dental technician, for example, needs to take precautions to avoid accumulating a large radiation dose from giving dental x-rays to many patients. There are three general ways to reduce exposure: time, distance, and shielding. This is why a dental technician doing x-rays wears a lead apron (shielding) and steps outside of the x-ray room while running an exposure (distance). Reducing the time of exposure dictates, for example, that a person working with a hot cancer-therapy source would minimize the amount of time spent near it.

Shielding against alpha and beta particles is trivial to accomplish. (Alphas can’t even penetrate the skin.) Gammas and x-rays interact most strongly with materials that are dense and have high atomic numbers, which is why lead is so commonly used. But other materials will also work. For example, the reason that bones show up so clearly on x-ray images is that they are dense and contain plenty of calcium, which has a higher atomic number than the elements found in most other body tissues, which are mostly made of water.

Neutrons are difficult to shield against. Because they are electrically neutral, they don’t interact intensely with matter in the same way.

\(^4\)Baker and Chesser, Env. Toxicology and Chem. 19 (1231) 2000. Similar effects have been seen at the Bikini Atoll, the site of a 1954 hydrogen bomb test. Although some species have disappeared from the area, the coral reef is in many ways healthier than similar reefs elsewhere, because humans have tended to stay away for fear of radiation (Richards et al., Marine Pollution Bulletin 56 (2008) 503).
The Crab Nebula is a remnant of a supernova explosion. Almost all the elements our planet is made of originated in such explosions.

Construction of the UNILAC accelerator in Germany, one of whose uses is for experiments to create very heavy artificial elements. In such an experiment, fusion products recoil through a device called SHIP (not shown) that separates them based on their charge-to-mass ratios — it is essentially just a scaled-up version of Thomson’s apparatus. A typical experiment runs for several months, and out of the billions of fusion reactions induced during this time, only one or two may result in the production of superheavy atoms. In all the rest, the fused nucleus breaks up immediately. SHIP is used to identify the small number of “good” reactions and separate them from this intense background.

The evidence for the big bang theory of the origin of the universe was discussed in subsection 19.5.1.
such as a black hole or neutron star.

The second generation of stars, of which our own sun is an example, condensed out of clouds of gas that had been enriched in heavy elements due to supernova explosions. It is those heavy elements that make up our planet and our bodies.

**Discussion questions**

A. Should the quality factor for neutrinos be very small, because they mostly don't interact with your body?

B. Would an alpha source be likely to cause different types of cancer depending on whether the source was external to the body or swallowed in contaminated food? What about a gamma source?

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### 26.5 Relativistic mass and energy

The radioactive decay processes described in this chapter do not conserve mass. For example, you’re probably reading this in a building that has smoke detectors containing the isotope $^{241}\text{Am}$, an alpha emitter. The decay products are a helium atom plus an atom of $^{237}\text{Np}$. In units of $10^{-27}$ kg, the masses involved are:

- $^{241}\text{Am} \quad 400.28421$
- $^{237}\text{Np} \quad 393.62768$
- $^4\text{He} \quad 6.64647$

The final state has a total mass of 400.27415 of these units, meaning a loss of 0.01006. This is an example of Einstein’s famous $E = mc^2$ at work: some mass has been converted into energy. In fact this type of mass-energy conversion is not just a property of nuclear decay. It is an example of a much wider set of phenomena in relativity. Let’s see how quantities like mass, force, momentum, and energy behave relativistically.

**Momentum**

Consider the following scheme for traveling faster than the speed of light. The basic idea can be demonstrated by dropping a ping-pong ball and a baseball stacked on top of each other like a snowman. They separate slightly in mid-air, and the baseball therefore has time to hit the floor and rebound before it collides with the ping-pong ball, which is still on the way down. The result is a surprise if you haven’t seen it before: the ping-pong ball flies off at high speed and hits the ceiling! A similar fact is known to people who investigate the scenes of accidents involving pedestrians. If a car moving at 90 kilometers per hour hits a pedestrian, the pedestrian flies off at nearly double that speed, 180 kilometers per hour. Now suppose the car was moving at 90 percent of the speed of light. Would the pedestrian fly off at 180% of $c$?
To see why not, we have to back up a little and think about where this speed-doubling result comes from. For any collision, there is a special frame of reference, the center-of-mass frame, in which the two colliding objects approach each other, collide, and rebound with their velocities reversed. In the center-of-mass frame, the total momentum of the objects is zero both before and after the collision.

Figure al/1 shows such a frame of reference for objects of very unequal mass. Before the collision, the large ball is moving relatively slowly toward the top of the page, but because of its greater mass, its momentum cancels the momentum of the smaller ball, which is moving rapidly in the opposite direction. The total momentum is zero. After the collision, the two balls just reverse their directions of motion. We know that this is the right result for the outcome of the collision because it conserves both momentum and kinetic energy, and everything not forbidden is compulsory, i.e., in any experiment, there is only one possible outcome, which is the one that obeys all the conservation laws.

**self-check D**

How do we know that momentum and kinetic energy are conserved in figure al/1?

Let’s make up some numbers as an example. Say the small ball is 1 kg, the big one 6 kg. In frame 1, let’s make the velocities as follows:

<table>
<thead>
<tr>
<th></th>
<th>before the collision</th>
<th>after the collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>⬤</td>
<td>-0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>○</td>
<td>0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
Figure al/2 shows the same collision in a frame of reference where the small ball was initially at rest. To find all the velocities in this frame, we just add 0.6 to all the ones in the previous table.

<table>
<thead>
<tr>
<th>before the collision</th>
<th>after the collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>⋄ 0</td>
<td>1.2</td>
</tr>
<tr>
<td>○ 0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In this frame, as expected, the small ball flies off with a velocity, 1.2, that is almost twice the initial velocity of the big ball, 0.7. In this example the ratio of the two balls’ masses was 6, but if the ratio of the masses is made larger and larger, the ratio of the velocities gets closer and closer to 2.

If all those velocities were in meters per second, then that’s exactly what would happen. But what if all these velocities were in units of the speed of light? Now it’s no longer a good approximation just to add velocities. We need to combine them according to the relativistic rules. For instance, in problem 1 on p. 720 you showed that combining a velocity of 0.6 times the speed of light with another velocity of 0.6 results in 0.88, not 1.2. The results are very different:

<table>
<thead>
<tr>
<th>before the collision</th>
<th>after the collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>⋄ 0</td>
<td>0.88</td>
</tr>
<tr>
<td>○ 0.67</td>
<td>0.51</td>
</tr>
</tbody>
</table>

A 6-kg ball moving at 88% of the speed of light hits a 1-kg ball. The balls appear foreshortened due to the relativistic distortion of space.

We can interpret this as follows. Figure al/1 is one in which the big ball is moving fairly slowly. This is very nearly the way the scene would be seen by an ant standing on the big ball. According to an observer in frame am, however, both balls are moving at nearly the speed of light after the collision. Because of this, the balls appear foreshortened, but the distance between the two balls is also shortened. To this observer, it seems that the small ball isn’t pulling away from the big ball very fast.

Now here’s what’s interesting about all this. The outcome shown in figure al/2 was supposed to be the only one possible, the only one that satisfied both conservation of energy and conservation of momentum. So how can the different result shown in figure am be possible? The answer is that relativistically, momentum must not equal \(mv\). The old, familiar definition is only an approximation
that’s valid at low speeds. If we observe the behavior of the small ball in figure am, it looks as though it somehow had some extra inertia. It’s as though a football player tried to knock another player down without realizing that the other guy had a three-hundred-pound bag full of lead shot hidden under his uniform — he just doesn’t seem to react to the collision as much as he should. This extra inertia is described\(^6\) by redefining momentum as

\[
p = m\gamma v.
\]

At very low velocities, \(\gamma\) is close to 1, and the result is very nearly \(mv\), as demanded by the correspondence principle. But at very high velocities, \(\gamma\) gets very big — the small ball in figure am has a \(\gamma\) of 2.1, and therefore has more than double the inertia that we would expect nonrelativistically.

This also explains the answer to another paradox often posed by beginners at relativity. Suppose you keep on applying a steady force to an object that’s already moving at 0.9999\(c\). Why doesn’t it just keep on speeding up past \(c\)? The answer is that force is the rate of change of momentum. At 0.9999\(c\), an object already has a \(\gamma\) of 71, and therefore has already sucked up 71 times the momentum you’d expect at that speed. As its velocity gets closer and closer to \(c\), its \(\gamma\) approaches infinity. To move at \(c\), it would need an infinite momentum, which could only be caused by an infinite force.

\(^6\)See p. 803 for a proof.

\(^7\)Newton’s second law gives \(a_N = \frac{F}{m} = \frac{eE}{m}\). The constant-acceleration equation \(\Delta x = \frac{1}{2}at^2\) then gives \(v_f = \sqrt{2m\Delta x/eE}\).

\(^8\)To make the low-energy portion of the graph legible, Bertozzi’s highest-energy data point is omitted.

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Two early high-precision tests of the relativistic equation \( p = mGv \) for momentum. Graphing \( p/m \) rather than \( p \) allows the data for electrons and protons to be placed on the same graph. Natural units are used, so that the horizontal axis is the velocity in units of \( c \), and the vertical axis is the unitless quantity \( p/mc \). The very small error bars for the data point from Zrelov are represented by the height of the black rectangle.

Figure ao shows experimental data confirming the relativistic equation for momentum.

**Equivalence of mass and energy**

Now we’re ready to see why mass and energy must be equivalent as claimed in the famous \( E = mc^2 \). So far we’ve only considered collisions in which none of the kinetic energy is converted into any other form of energy, such as heat or sound. Let’s consider what happens if a blob of putty moving at velocity \( v \) hits another blob that is initially at rest, sticking to it. The nonrelativistic result is that to obey conservation of momentum the two blobs must fly off together at \( v/2 \). Half of the initial kinetic energy has been converted to heat.¹

Relativistically, however, an interesting thing happens. A hot object has more momentum than a cold object! This is because the relativistically correct expression for momentum is \( mGv \), and the more rapidly moving atoms in the hot object have higher values of \( G \). In our collision, the final combined blob must therefore be moving a little more slowly than the expected \( v/2 \), since otherwise the final momentum would have been a little greater than the initial momentum. To an observer who believes in conservation of momentum and knows only about the overall motion of the objects and not about their heat content, the low velocity after the collision would seem to be the result of a magical change in the mass, as if the mass of two combined, hot blobs of putty was more than the sum of their individual masses.

We know that the masses of all the atoms in the blobs must be the same as they always were. The change is due to the change in \( G \) with heating, not to a change in mass. The heat energy, however, seems to be acting as if it was equivalent to some extra mass. If the quantity of heat is \( E \), then it turns out that the extra mass \( m \) is such that \( E = mc^2 \) (proof, p. 802).

But this whole argument was based on the fact that heat is a form of kinetic energy at the atomic level. Would \( E = mc^2 \) apply to other forms of energy as well? Suppose a rocket ship contains some electrical energy stored in a battery. If we believed that \( E = mc^2 \) applied to forms of kinetic energy but not to electrical energy, then we would have to believe that the pilot of the rocket could slow

---

¹A double-mass object moving at half the speed does not have the same kinetic energy. Kinetic energy depends on the square of the velocity, so cutting the velocity in half reduces the energy by a factor of 1/4, which, multiplied by the doubled mass, makes 1/2 the original energy.
the ship down by using the battery to run a heater! This would not only be strange, but it would violate the principle of relativity, because the result of the experiment would be different depending on whether the ship was at rest or not. The only logical conclusion is that all forms of energy are equivalent to mass. Running the heater then has no effect on the motion of the ship, because the total energy in the ship was unchanged; one form of energy (electrical) was simply converted to another (heat).

The equation $E = mc^2$ tells us how much energy is equivalent to how much mass: the conversion factor is the square of the speed of light, $c$. Since $c$ a big number, you get a really really big number when you multiply it by itself to get $c^2$. This means that even a small amount of mass is equivalent to a very large amount of energy.

Gravity bending light

Gravity is a universal attraction between things that have mass, and since the energy in a beam of light is equivalent to some very small amount of mass, we expect that light will be affected by gravity, although the effect should be very small. The first important experimental confirmation of relativity came in 1919 when stars next to the sun during a solar eclipse were observed to have shifted a little from their ordinary position. (If there was no eclipse, the glare of the sun would prevent the stars from being observed.) Starlight had been deflected by the sun's gravity. Figure aq is a photographic negative, so the circle that appears bright is actually
the dark face of the moon, and the dark area is really the bright corona of the sun. The stars, marked by lines above and below them, appeared at positions slightly different than their normal ones.

Black holes example 9
A star with sufficiently strong gravity can prevent light from leaving. Quite a few black holes have been detected via their gravitational forces on neighboring stars or clouds of gas and dust.

You’ve learned about conservation of mass and conservation of energy, but now we see that they’re not even separate conservation laws. As a consequence of the theory of relativity, mass and energy are equivalent, and are not separately conserved — one can be converted into the other. Imagine that a magician waves his wand, and changes a bowl of dirt into a bowl of lettuce. You’d be impressed, because you were expecting that both dirt and lettuce would be conserved quantities. Neither one can be made to vanish, or to appear out of thin air. However, there are processes that can change one into the other. A farmer changes dirt into lettuce, and a compost heap changes lettuce into dirt. At the most fundamental level, lettuce and dirt aren’t really different things at all; they’re just collections of the same kinds of atoms — carbon, hydrogen, and so on. Because mass and energy are like two different sides of the same coin, we may speak of mass-energy, a single conserved quantity, found by adding up all the mass and energy, with the appropriate conversion factor: \( E + mc^2 \).

A rusting nail example 10
\( \triangleright \) An iron nail is left in a cup of water until it turns entirely to rust. The energy released is about 0.5 MJ. In theory, would a sufficiently precise scale register a change in mass? If so, how much?

\( \triangleright \) The energy will appear as heat, which will be lost to the environment. The total mass-energy of the cup, water, and iron will indeed be lessened by 0.5 MJ. (If it had been perfectly insulated, there would have been no change, since the heat energy would have been trapped in the cup.) The speed of light is \( c = 3 \times 10^8 \) meters per second, so converting to mass units, we have

\[
m = \frac{E}{c^2} = \frac{0.5 \times 10^6 \text{ J}}{(3 \times 10^8 \text{ m/s})^2} = 6 \times 10^{-12} \text{ kilograms.}
\]

The change in mass is too small to measure with any practical technique. This is because the square of the speed of light is such a large number.

Electron-positron annihilation example 11
Natural radioactivity in the earth produces positrons, which are
like electrons but have the opposite charge. A form of antimatter, positrons annihilate with electrons to produce gamma rays, a form of high-frequency light. Such a process would have been considered impossible before Einstein, because conservation of mass and energy were believed to be separate principles, and this process eliminates 100% of the original mass. The amount of energy produced by annihilating 1 kg of matter with 1 kg of antimatter is

\[ E = mc^2 \]

\[ = (2 \text{ kg}) \left( 3.0 \times 10^8 \text{ m/s} \right)^2 \]

\[ = 2 \times 10^{17} \text{ J}, \]

which is on the same order of magnitude as a day’s energy consumption for the entire world’s population!

Positron annihilation forms the basis for the medical imaging technique called a PET (positron emission tomography) scan, in which a positron-emitting chemical is injected into the patient and mapped by the emission of gamma rays from the parts of the body where it accumulates.

One commonly hears some misinterpretations of \( E = mc^2 \), one being that the equation tells us how much kinetic energy an object would have if it was moving at the speed of light. This wouldn’t make much sense, both because the equation for kinetic energy has 1/2 in it, \( KE = (1/2)mv^2 \), and because a material object can’t be made to move at the speed of light. However, this naturally leads to the question of just how much mass-energy a moving object has. We know that when the object is at rest, it has no kinetic energy, so its mass-energy is simply equal to the energy-equivalent of its mass, \( mc^2 \),

\[ \mathcal{E} = mc^2 \text{ when } v = 0, \]

where the symbol \( \mathcal{E} \) (cursive “E”) stands for mass-energy. The point of using the new symbol is simply to remind ourselves that we’re talking about relativity, so an object at rest has \( \mathcal{E} = mc^2 \), not \( E = 0 \) as we’d assume in classical physics.

Suppose we start accelerating the object with a constant force. A constant force means a constant rate of transfer of momentum, but \( p = m\dot{v}v \) approaches infinity as \( v \) approaches \( c \), so the object will only get closer and closer to the speed of light, but never reach it. Now what about the work being done by the force? The force keeps doing work and doing work, which means that we keep on using up energy. Mass-energy is conserved, so the energy being expended must equal the increase in the object’s mass-energy. We can continue this process for as long as we like, and the amount of mass-energy will increase without limit. We therefore conclude that an object’s mass-energy approaches infinity as its speed approaches the speed of light.
Now that we have some idea what to expect, what is the actual equation for the mass-energy? As proved in my book Simple Nature, it is

$$\mathcal{E} = m\gamma c^2.$$  

**self-check E**

Verify that this equation has the two properties we wanted. Answer, p. 1042

1. **KE compared to mc² at low speeds** example 12

▷ An object is moving at ordinary nonrelativistic speeds. Compare its kinetic energy to the energy $mc^2$ it has purely because of its mass.

▷ The speed of light is a very big number, so $mc^2$ is a huge number of joules. The object has a gigantic amount of energy because of its mass, and only a relatively small amount of additional kinetic energy because of its motion.

Another way of seeing this is that at low speeds, $\gamma$ is only a tiny bit greater than 1, so $\mathcal{E}$ is only a tiny bit greater than $mc^2$.

2. **The correspondence principle for mass-energy** example 13

▷ Show that the equation $\mathcal{E} = m\gamma c^2$ obeys the correspondence principle.

▷ As we accelerate an object from rest, its mass-energy becomes greater than its resting value. We interpret this excess mass-energy as the object's kinetic energy,

$$KE = \mathcal{E}(v) - \mathcal{E}(v = 0)$$
$$= m\gamma c^2 - mc^2$$
$$= m(\gamma - 1)c^2.$$  

In example 4 on page 681, we found $\gamma \approx 1 + v^2/2c^2$, so

$$KE \approx m(1 + \frac{v^2}{2c^2} - 1)c^2$$
$$= \frac{1}{2}mv^2,$$

which is the nonrelativistic expression. As demanded by the correspondence principle, relativity agrees with nonrelativistic physics at speeds that are small compared to the speed of light.

## 26.6 *Proofs*

In section 26.5 I gave physical arguments to the effect that relativistic momentum should be greater than $mv$ and that an energy $E$