The main variables that relate to the value of $g$ on Earth are latitude and elevation. Although you have not yet learned how $g$ would be calculated based on any deeper theory of gravity, it is not too hard to guess why $g$ depends on elevation. Gravity is an attraction between things that have mass, and the attraction gets weaker with increasing distance. As you ascend from the seaport of Guayaquil to the nearby top of Mt. Cotopaxi, you are distancing yourself from the mass of the planet. The dependence on latitude occurs because we are measuring the acceleration of gravity relative to the earth’s surface, but the earth’s rotation causes the earth’s surface to fall out from under you. (We will discuss both gravity and rotation in more detail later in the course.)

<table>
<thead>
<tr>
<th>location</th>
<th>latitude</th>
<th>elevation (m)</th>
<th>$g$ (m/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>north pole</td>
<td>90°N</td>
<td>0</td>
<td>9.8322</td>
</tr>
<tr>
<td>Reykjavik, Iceland</td>
<td>64°N</td>
<td>0</td>
<td>9.8225</td>
</tr>
<tr>
<td>Guayaquil, Ecuador</td>
<td>2°S</td>
<td>0</td>
<td>9.7806</td>
</tr>
<tr>
<td>Mt. Cotopaxi, Ecuador</td>
<td>1°S</td>
<td>5896</td>
<td>9.7624</td>
</tr>
<tr>
<td>Mt. Everest</td>
<td>28°N</td>
<td>8848</td>
<td>9.7643</td>
</tr>
</tbody>
</table>

This false-color map shows variations in the strength of the earth’s gravity. Purple areas have the strongest gravity, yellow the weakest. The overall trend toward weaker gravity at the equator and stronger gravity at the poles has been artificially removed to allow the weaker local variations to show up. The map covers only the oceans because of the technique used to make it: satellites look for bulges and depressions in the surface of the ocean. A very slight bulge will occur over an undersea mountain, for instance, because the mountain’s gravitational attraction pulls water toward it. The US government originally began collecting data like these for military use, to correct for the deviations in the paths of missiles. The data have recently been released for scientific and commercial use (e.g., searching for sites for off-shore oil wells).

Much more spectacular differences in the strength of gravity can be observed away from the Earth’s surface:
<table>
<thead>
<tr>
<th>location</th>
<th>g (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>asteroid Vesta (surface)</td>
<td>0.3</td>
</tr>
<tr>
<td>Earth’s moon (surface)</td>
<td>1.6</td>
</tr>
<tr>
<td>Mars (surface)</td>
<td>3.7</td>
</tr>
<tr>
<td>Earth (surface)</td>
<td>9.8</td>
</tr>
<tr>
<td>Jupiter (cloud-tops)</td>
<td>26</td>
</tr>
<tr>
<td>Sun (visible surface)</td>
<td>270</td>
</tr>
<tr>
<td>typical neutron star (surface)</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>black hole (center)</td>
<td>infinite according to some theories, on the order of $10^{52}$ according to others</td>
</tr>
</tbody>
</table>

A typical neutron star is not so different in size from a large asteroid, but is orders of magnitude more massive, so the mass of a body definitely correlates with the $g$ it creates. On the other hand, a neutron star has about the same mass as our Sun, so why is its $g$ billions of times greater? If you had the misfortune of being on the surface of a neutron star, you’d be within a few thousand miles of all its mass, whereas on the surface of the Sun, you’d still be millions of miles from most of its mass.

**Discussion questions**

**A** What is wrong with the following definitions of $g$?

1. “$g$ is gravity.”
2. “$g$ is the speed of a falling object.”
3. “$g$ is how hard gravity pulls on things.”

**B** When advertisers specify how much acceleration a car is capable of, they do not give an acceleration as defined in physics. Instead, they usually specify how many seconds are required for the car to go from rest to 60 miles/hour. Suppose we use the notation “$a$” for the acceleration as defined in physics, and “$a_{\text{car ad}}$” for the quantity used in advertisements for cars. In the US’s non-metric system of units, what would be the units of $a$ and $a_{\text{car ad}}$? How would the use and interpretation of large and small, positive and negative values be different for $a$ as opposed to $a_{\text{car ad}}$?

**C** Two people stand on the edge of a cliff. As they lean over the edge, one person throws a rock down, while the other throws one straight up with an exactly opposite initial velocity. Compare the speeds of the rocks on impact at the bottom of the cliff.

### 3.3 Positive and negative acceleration

Gravity always pulls down, but that does not mean it always speeds things up. If you throw a ball straight up, gravity will first slow it down to $v = 0$ and then begin increasing its speed. When I took physics in high school, I got the impression that positive signs of acceleration indicated speeding up, while negative accelerations represented slowing down, i.e., deceleration. Such a definition would be inconvenient, however, because we would then have to say that the same downward tug of gravity could produce either a positive
or a negative acceleration. As we will see in the following example, such a definition also would not be the same as the slope of the $v-t$ graph.

Let’s study the example of the rising and falling ball. In the example of the person falling from a bridge, I assumed positive velocity values without calling attention to it, which meant I was assuming a coordinate system whose $x$ axis pointed down. In this example, where the ball is reversing direction, it is not possible to avoid negative velocities by a tricky choice of axis, so let’s make the more natural choice of an axis pointing up. The ball’s velocity will initially be a positive number, because it is heading up, in the same direction as the $x$ axis, but on the way back down, it will be a negative number. As shown in the figure, the $v-t$ graph does not do anything special at the top of the ball’s flight, where $v$ equals 0. Its slope is always negative. In the left half of the graph, there is a negative slope because the positive velocity is getting closer to zero. On the right side, the negative slope is due to a negative velocity that is getting farther from zero, so we say that the ball is speeding up, but its velocity is decreasing!

To summarize, what makes the most sense is to stick with the original definition of acceleration as the slope of the $v-t$ graph, $\frac{\Delta v}{\Delta t}$. By this definition, it just isn’t necessarily true that things speeding up have positive acceleration while things slowing down have negative acceleration. The word “deceleration” is not used much by physicists, and the word “acceleration” is used unblushingly to refer to slowing down as well as speeding up: “There was a red light, and we accelerated to a stop.”

1Numerical calculation of a negative acceleration example 4

In figure i, what happens if you calculate the acceleration between $t = 1.0$ and $1.5$ s?

Reading from the graph, it looks like the velocity is about $-1$ m/s at $t = 1.0$ s, and around $-6$ m/s at $t = 1.5$ s. The acceleration, figured between these two points, is

$$a = \frac{\Delta v}{\Delta t} = \frac{(-6 \text{ m/s}) - (-1 \text{ m/s})}{(1.5 \text{ s}) - (1.0 \text{ s})} = -10 \text{ m/s}^2.$$

Even though the ball is speeding up, it has a negative acceleration.

Another way of convincing you that this way of handling the plus and minus signs makes sense is to think of a device that measures acceleration. After all, physics is supposed to use operational definitions, ones that relate to the results you get with actual measuring devices. Consider an air freshener hanging from the rear-view mirror of your car. When you speed up, the air freshener swings backward. Suppose we define this as a positive reading. When you slow down, the air freshener swings forward, so we’ll call this a negative reading.
on our accelerometer. But what if you put the car in reverse and start speeding up backwards? Even though you’re speeding up, the accelerometer responds in the same way as it did when you were going forward and slowing down. There are four possible cases:

<table>
<thead>
<tr>
<th>motion of car</th>
<th>accelerometer slope of v-t graph</th>
<th>direction of force acting on car</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward, speeding up</td>
<td>backward</td>
<td>+ forward</td>
</tr>
<tr>
<td>forward, slowing down</td>
<td>forward</td>
<td>− backward</td>
</tr>
<tr>
<td>backward, speeding up</td>
<td>forward</td>
<td>− backward</td>
</tr>
<tr>
<td>backward, slowing down</td>
<td>backward</td>
<td>+ forward</td>
</tr>
</tbody>
</table>

Note the consistency of the three right-hand columns — nature is trying to tell us that this is the right system of classification, not the left-hand column.

Because the positive and negative signs of acceleration depend on the choice of a coordinate system, the acceleration of an object under the influence of gravity can be either positive or negative. Rather than having to write things like “$g = 9.8 \text{ m/s}^2$ or $-9.8 \text{ m/s}^2$” every time we want to discuss $g$’s numerical value, we simply define $g$ as the absolute value of the acceleration of objects moving under the influence of gravity. We consistently let $g = 9.8 \text{ m/s}^2$, but we may have either $a = g$ or $a = -g$, depending on our choice of a coordinate system.

**Acceleration with a change in direction of motion**

Example 5

▷ A person kicks a ball, which rolls up a sloping street, comes to a halt, and rolls back down again. The ball has constant acceleration. The ball is initially moving at a velocity of 4.0 m/s, and after 10.0 s it has returned to where it started. At the end, it has sped back up to the same speed it had initially, but in the opposite direction. What was its acceleration?

▷ By giving a positive number for the initial velocity, the statement of the question implies a coordinate axis that points up the slope of the hill. The “same” speed in the opposite direction should therefore be represented by a negative number, -4.0 m/s. The acceleration is

$$ a = \Delta v / \Delta t $$

$$ = (v_f - v_o)/10.0 \text{ s} $$

$$ = [(-4.0 \text{ m/s}) - (4.0 \text{ m/s})]/10.0\text{s} $$

$$ = -0.80 \text{ m/s}^2. $$

The acceleration was no different during the upward part of the roll than on the downward part of the roll.

Incorrect solution: Acceleration is $\Delta v / \Delta t$, and at the end it’s not moving any faster or slower than when it started, so $\Delta v=0$ and
\[ a = 0. \]

\( x \) The velocity does change, from a positive number to a negative number.

Discussion questions

A  A child repeatedly jumps up and down on a trampoline. Discuss the sign and magnitude of his acceleration, including both the time when he is in the air and the time when his feet are in contact with the trampoline.

B  The figure shows a refugee from a Picasso painting blowing on a rolling water bottle. In some cases the person's blowing is speeding the bottle up, but in others it is slowing it down. The arrow inside the bottle shows which direction it is going, and a coordinate system is shown at the bottom of each figure. In each case, figure out the plus or minus signs of the velocity and acceleration. It may be helpful to draw a \( v - t \) graph in each case.

C  Sally is on an amusement park ride which begins with her chair being hoisted straight up a tower at a constant speed of 60 miles/hour. Despite stern warnings from her father that he'll take her home the next time she misbehaves, she decides that as a scientific experiment she really needs to release her corndog over the side as she's on the way up. She does not throw it. She simply sticks it out of the car, lets it go, and watches it against the background of the sky, with no trees or buildings as reference points. What does the corndog's motion look like as observed by Sally? Does its speed ever appear to her to be zero? What acceleration does she observe it to have: is it ever positive? negative? zero? What would her enraged father answer if asked for a similar description of its motion as it appears to him, standing on the ground?

D  Can an object maintain a constant acceleration, but meanwhile reverse the direction of its velocity?

E  Can an object have a velocity that is positive and increasing at the same time that its acceleration is decreasing?
3.4 Varying acceleration

So far we have only been discussing examples of motion for which the \( v - t \) graph is linear. If we wish to generalize our definition to \( v-t \) graphs that are more complex curves, the best way to proceed is similar to how we defined velocity for curved \( x-t \) graphs:

**definition of acceleration**

The acceleration of an object at any instant is the slope of the tangent line passing through its \( v\)-versus-\( t \) graph at the relevant point.

**A skydiver example 6**

The graphs in figure k show the results of a fairly realistic computer simulation of the motion of a skydiver, including the effects of air friction. The \( x \) axis has been chosen pointing down, so \( x \) is increasing as she falls. Find (a) the skydiver’s acceleration at \( t = 3.0 \) s, and also (b) at \( t = 7.0 \) s.

(a) To find the slope of the tangent line, I pick two points on the line (not necessarily on the actual curve): (3.0 s, 28 m/s) and (5.0 s, 42 m/s). The slope of the tangent line is \((42 \text{ m/s} - 28 \text{ m/s})/(5.0 \text{ s} - 3.0 \text{ s}) = 7.0 \text{ m/s}^2\).

(b) Two points on this tangent line are (7.0 s, 47 m/s) and (9.0 s, 52 m/s). The slope of the tangent line is \((52 \text{ m/s} - 47 \text{ m/s})/(9.0 \text{ s} - 7.0 \text{ s}) = 2.5 \text{ m/s}^2\).

Physically, what’s happening is that at \( t = 3.0 \) s, the skydiver is not yet going very fast, so air friction is not yet very strong. She therefore has an acceleration almost as great as \( g \). At \( t = 7.0 \) s, she is moving almost twice as fast (about 100 miles per hour), and air friction is extremely strong, resulting in a significant departure from the idealized case of no air friction.

In example 6, the \( x-t \) graph was not even used in the solution of the problem, since the definition of acceleration refers to the slope of the \( v-t \) graph. It is possible, however, to interpret an \( x-t \) graph to find out something about the acceleration. An object with zero acceleration, i.e., constant velocity, has an \( x-t \) graph that is a straight line. A straight line has no curvature. A change in velocity requires a change in the slope of the \( x-t \) graph, which means that it is a curve rather than a line. Thus acceleration relates to the curvature of the \( x-t \) graph. Figure m shows some examples.
In example 6, the $x - t$ graph was more strongly curved at the beginning, and became nearly straight at the end. If the $x - t$ graph is nearly straight, then its slope, the velocity, is nearly constant, and the acceleration is therefore small. We can thus interpret the acceleration as representing the curvature of the $x - t$ graph, as shown in figure m. If the “cup” of the curve points up, the acceleration is positive, and if it points down, the acceleration is negative.

m / Acceleration relates to the curvature of the $x - t$ graph.
Since the relationship between $a$ and $v$ is analogous to the relationship between $v$ and $x$, we can also make graphs of acceleration as a function of time, as shown in figure n.

Examples of graphs of $x$, $v$, and $a$ versus $t$. 1. An object in free fall, with no friction. 2. A continuation of example 6, the skydiver.

- **Solved problem:** Drawing a $v - t$ graph. page 119, problem 14
- **Solved problem:** Drawing $v - t$ and $a - t$ graphs. page 120, problem 20

Figure o summarizes the relationships among the three types of graphs.

**Discussion questions**

A. Describe in words how the changes in the $a - t$ graph in figure n/2 relate to the behavior of the $v - t$ graph.
Explain how each set of graphs contains inconsistencies, and fix them.

In each case, pick a coordinate system and draw $x - t$, $v - t$, and $a - t$ graphs. Picking a coordinate system means picking where you want $x = 0$ to be, and also picking a direction for the positive $x$ axis.

1. An ocean liner is cruising in a straight line at constant speed.
2. You drop a ball. Draw two different sets of graphs (a total of 6), with one set's positive $x$ axis pointing in the opposite direction compared to the other's.
3. You're driving down the street looking for a house you've never been to before. You realize you've passed the address, so you slow down, put the car in reverse, back up, and stop in front of the house.

3.5 The area under the velocity-time graph

A natural question to ask about falling objects is how fast they fall, but Galileo showed that the question has no answer. The physical law that he discovered connects a cause (the attraction of the planet Earth’s mass) to an effect, but the effect is predicted in terms of an acceleration rather than a velocity. In fact, no physical law predicts a definite velocity as a result of a specific phenomenon, because velocity cannot be measured in absolute terms, and only changes in velocity relate directly to physical phenomena.

The unfortunate thing about this situation is that the definitions of velocity and acceleration are stated in terms of the tangent-line technique, which lets you go from $x$ to $v$ to $a$, but not the other way around. Without a technique to go backwards from $a$ to $v$ to $x$, we cannot say anything quantitative, for instance, about the $x - t$ graph of a falling object. Such a technique does exist, and I used it to make the $x - t$ graphs in all the examples above.
First let’s concentrate on how to get \( x \) information out of a \( v - t \) graph. In example p/1, an object moves at a speed of 20 m/s for a period of 4.0 s. The distance covered is \( \Delta x = v \Delta t = (20 \text{ m/s}) \times (4.0 \text{ s}) = 80 \text{ m} \). Notice that the quantities being multiplied are the width and the height of the shaded rectangle — or, strictly speaking, the time represented by its width and the velocity represented by its height. The distance of \( \Delta x = 80 \text{ m} \) thus corresponds to the area of the shaded part of the graph.

The next step in sophistication is an example like p/2, where the object moves at a constant speed of 10 m/s for two seconds, then for two seconds at a different constant speed of 20 m/s. The shaded region can be split into a small rectangle on the left, with an area representing \( \Delta x = 20 \text{ m} \), and a taller one on the right, corresponding to another 40 m of motion. The total distance is thus 60 m, which corresponds to the total area under the graph.

An example like p/3 is now just a trivial generalization; there is simply a large number of skinny rectangular areas to add up. But notice that graph p/3 is quite a good approximation to the smooth curve p/4. Even though we have no formula for the area of a funny shape like p/4, we can approximate its area by dividing it up into smaller areas like rectangles, whose area is easier to calculate. If someone hands you a graph like p/4 and asks you to find the area under it, the simplest approach is just to count up the little rectangles on the underlying graph paper, making rough estimates of fractional rectangles as you go along.

That’s what I’ve done in figure q. Each rectangle on the graph paper is 1.0 s wide and 2 m/s tall, so it represents 2 m. Adding up all the numbers gives \( \Delta x = 41 \text{ m} \). If you needed better accuracy, you could use graph paper with smaller rectangles.

It’s important to realize that this technique gives you \( \Delta x \), not \( x \). The \( v - t \) graph has no information about where the object was when it started.

The following are important points to keep in mind when applying this technique:

- If the range of \( v \) values on your graph does not extend down to zero, then you will get the wrong answer unless you compensate by adding in the area that is not shown.
- As in the example, one rectangle on the graph paper does not necessarily correspond to one meter of distance.
- Negative velocity values represent motion in the opposite direction, so as suggested by figure r, area under the \( t \) axis should be subtracted, i.e., counted as “negative area.”
Since the result is a \( \Delta x \) value, it only tells you \( x_{after} - x_{before} \), which may be less than the actual distance traveled. For instance, the object could come back to its original position at the end, which would correspond to \( \Delta x = 0 \), even though it had actually moved a nonzero distance.

Finally, note that one can find \( \Delta v \) from an \( a-t \) graph using an entirely analogous method. Each rectangle on the \( a-t \) graph represents a certain amount of velocity change.

**Discussion question**
A Roughly what would a pendulum’s \( v-t \) graph look like? What would happen when you applied the area-under-the-curve technique to find the pendulum’s \( \Delta x \) for a time period covering many swings?

### 3.6 Algebraic results for constant acceleration

Although the area-under-the-curve technique can be applied to any graph, no matter how complicated, it may be laborious to carry out, and if fractions of rectangles must be estimated the result will only be approximate. In the special case of motion with constant acceleration, it is possible to find a convenient shortcut which produces exact results. When the acceleration is constant, the \( v-t \) graph
The shaded area tells us how far an object moves while accelerating at a constant rate. The area under the curve can be divided into a triangle plus a rectangle, both of whose areas can be calculated exactly: \( A = bh \) for a rectangle and \( A = bh/2 \) for a triangle. The height of the rectangle is the initial velocity, \( v_0 \), and the height of the triangle is the change in velocity from beginning to end, \( \Delta v \). The object’s \( \Delta x \) is therefore given by the equation \( \Delta x = v_0 \Delta t + \Delta v \Delta t/2 \). This can be simplified a little by using the definition of acceleration, \( a = \Delta v/\Delta t \), to eliminate \( \Delta v \), giving

\[
\Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2. 
\]

[motion with constant acceleration]

Since this is a second-order polynomial in \( \Delta t \), the graph of \( \Delta x \) versus \( \Delta t \) is a parabola, and the same is true of a graph of \( x \) versus \( t \) — the two graphs differ only by shifting along the two axes. Although I have derived the equation using a figure that shows a positive \( v_0 \), positive \( a \), and so on, it still turns out to be true regardless of what plus and minus signs are involved.

Another useful equation can be derived if one wants to relate the change in velocity to the distance traveled. This is useful, for instance, for finding the distance needed by a car to come to a stop. For simplicity, we start by deriving the equation for the special case of \( v_0 = 0 \), in which the final velocity \( v_f \) is a synonym for \( \Delta v \). Since velocity and distance are the variables of interest, not time, we take the equation \( \Delta x = \frac{1}{2} a \Delta t^2 \) and use \( \Delta t = \Delta v/a \) to eliminate \( \Delta t \). This gives \( \Delta x = (\Delta v)^2/2a \), which can be rewritten as

\[
v_f^2 = 2a \Delta x. \quad \text{[motion with constant acceleration, } v_0 = 0]\n\]

For the more general case where \( v_0 \neq 0 \), we skip the tedious algebra leading to the more general equation,

\[
v_f^2 = v_0^2 + 2a \Delta x. \quad \text{[motion with constant acceleration]}\n\]
To help get this all organized in your head, first let’s categorize the variables as follows:

Variables that change during motion with constant acceleration:

\[ x, v, t \]

Variable that doesn’t change:

\[ a \]

If you know one of the changing variables and want to find another, there is always an equation that relates those two:

\[ \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \]

\[ v_f^2 = v_0^2 + 2a \Delta x \]

\[ a = \frac{\Delta v}{\Delta t} \]

The symmetry among the three variables is imperfect only because the equation relating \( x \) and \( t \) includes the initial velocity.

There are two main difficulties encountered by students in applying these equations:

- The equations apply only to motion with constant acceleration. You can’t apply them if the acceleration is changing.
- Students are often unsure of which equation to use, or may cause themselves unnecessary work by taking the longer path around the triangle in the chart above. Organize your thoughts by listing the variables you are given, the ones you want to find, and the ones you aren’t given and don’t care about.

**Saving an old lady example 7**

\[ \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \]

\[ v_f^2 = v_0^2 + 2a \Delta x \]

\[ a = \frac{\Delta v}{\Delta t} \]

\[ \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \]

You are trying to pull an old lady out of the way of an oncoming truck. You are able to give her an acceleration of 20 m/s². Starting from rest, how much time is required in order to move her 2 m?

First we organize our thoughts:

Variables given: \( \Delta x \), \( a \), \( v_0 \)

Variables desired: \( \Delta t \)

Irrelevant variables: \( v_f \)

Consulting the triangular chart above, the equation we need is clearly \( \Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \), since it has the four variables of interest and omits the irrelevant one. Eliminating the \( v_0 \) term and solving for \( \Delta t \) gives

\[ \Delta t = \sqrt{\frac{2x}{a}} = 0.4 \text{ s}. \]
Discussion questions

A In chapter 1, I gave examples of correct and incorrect reasoning about proportionality, using questions about the scaling of area and volume. Try to translate the incorrect modes of reasoning shown there into mistakes about the following question: If the acceleration of gravity on Mars is 1/3 that on Earth, how many times longer does it take for a rock to drop the same distance on Mars?

B Check that the units make sense in the three equations derived in this section.

3.7 ★ A test of the principle of inertia

Historically, the first quantitative and well documented experimental test of the principle of inertia (p. 80) was performed by Galileo around 1590 and published decades later when he managed to find a publisher in the Netherlands that was beyond the reach of the Roman Inquisition.¹ It was ingenious but somewhat indirect, and required a layer of interpretation and extrapolation on top of the actual observations. As described on p. 97, he established that objects rolling on inclined planes moved according to mathematical laws that we would today describe as in section 3.6. He knew that his rolling balls were subject to friction, as well as random errors due to the limited precision of the water clock that he used, but he took the approximate agreement of his equations with experiment to indicate that they gave the results that would be exact in the absence of friction. He also showed, purely empirically, that when a ball went up or down a ramp inclined at an angle \( \theta \), its acceleration was proportional to \( \sin \theta \). Again, this required extrapolation to idealized conditions of zero friction. He then reasoned that if a ball was rolled on a horizontal ramp, with \( \theta = 0 \), its acceleration would be zero. This is exactly what is required by the principle of inertia: in the absence of friction, motion continues indefinitely.

¹Galileo, Discourses and Mathematical Demonstrations Relating to Two New Sciences, 1638. The experiments are described in the Third Day, and their support for the principle of inertia is discussed in the Scholium following Theorems I-XIV. Another experiment involving a ship is described in Galileo’s 1624 reply to a letter from Fr. Ingoli, but although Galileo vigorously asserts that he really did carry it out, no detailed description or quantitative results are given.
Applications of calculus

In section 2.7, I discussed how the slope-of-the-tangent-line idea related to the calculus concept of a derivative, and the branch of calculus known as differential calculus. The other main branch of calculus, integral calculus, has to do with the area-under-the-curve concept discussed in section 3.5. Again there is a concept, a notation, and a bag of tricks for doing things symbolically rather than graphically. In calculus, the area under the $v - t$ graph between $t = t_1$ and $t = t_2$ is notated like this:

$$\text{area under curve} = \Delta x = \int_{t_1}^{t_2} v \, dt.$$  

The expression on the right is called an integral, and the s-shaped symbol, the integral sign, is read as “integral of . . .”

Integral calculus and differential calculus are closely related. For instance, if you take the derivative of the function $x(t)$, you get the function $v(t)$, and if you integrate the function $v(t)$, you get $x(t)$ back again. In other words, integration and differentiation are inverse operations. This is known as the fundamental theorem of calculus.

On an unrelated topic, there is a special notation for taking the derivative of a function twice. The acceleration, for instance, is the second (i.e., double) derivative of the position, because differentiating $x$ once gives $v$, and then differentiating $v$ gives $a$. This is written as

$$a = \frac{d^2 x}{dt^2}.$$  

The seemingly inconsistent placement of the twos on the top and bottom confuses all beginning calculus students. The motivation for this funny notation is that acceleration has units of m/s$^2$, and the notation correctly suggests that: the top looks like it has units of meters, the bottom seconds$^2$. The notation is not meant, however, to suggest that $t$ is really squared.
Summary

Selected vocabulary

gravity . . . . . . A general term for the phenomenon of attraction between things having mass. The attraction between our planet and a human-sized object causes the object to fall.

acceleration . . . The rate of change of velocity; the slope of the tangent line on a $v - t$ graph.

Notation

$v_o$ . . . . . . . . initial velocity
$v_f$ . . . . . . . . final velocity
$a$ . . . . . . . . acceleration
$g$ . . . . . . . . the acceleration of objects in free fall; the strength of the local gravitational field

Summary

Galileo showed that when air resistance is negligible all falling bodies have the same motion regardless of mass. Moreover, their $v - t$ graphs are straight lines. We therefore define a quantity called acceleration as the slope, $\Delta v/\Delta t$, of an object’s $v - t$ graph. In cases other than free fall, the $v - t$ graph may be curved, in which case the definition is generalized as the slope of a tangent line on the $v - t$ graph. The acceleration of objects in free fall varies slightly across the surface of the earth, and greatly on other planets.

Positive and negative signs of acceleration are defined according to whether the $v - t$ graph slopes up or down. This definition has the advantage that a force with a given sign, representing its direction, always produces an acceleration with the same sign.

The area under the $v - t$ graph gives $\Delta x$, and analogously the area under the $a - t$ graph gives $\Delta v$.

For motion with constant acceleration, the following three equations hold:

$$\Delta x = v_o \Delta t + \frac{1}{2} a \Delta t^2$$

$$v_f^2 = v_o^2 + 2a \Delta x$$

$$a = \frac{\Delta v}{\Delta t}$$

They are not valid if the acceleration is changing.
Problems

Key
✓ A computerized answer check is available online.
∫ A problem that requires calculus.
★ A difficult problem.

1 The graph represents the velocity of a bee along a straight line. At \( t = 0 \), the bee is at the hive. (a) When is the bee farthest from the hive? (b) How far is the bee at its farthest point from the hive? (c) At \( t = 13 \) s, how far is the bee from the hive? ✓

2 A rock is dropped into a pond. Draw plots of its position versus time, velocity versus time, and acceleration versus time. Include its whole motion, starting from the moment it is dropped, and continuing while it falls through the air, passes through the water, and ends up at rest on the bottom of the pond. Do your work on a photocopy or a printout of page 125.

3 In an 18th-century naval battle, a cannon ball is shot horizontally, passes through the side of an enemy ship’s hull, flies across the galley, and lodges in a bulkhead. Draw plots of its horizontal position, velocity, and acceleration as functions of time, starting while it is inside the cannon and has not yet been fired, and ending when it comes to rest. There is not any significant amount of friction from the air. Although the ball may rise and fall, you are only concerned with its horizontal motion, as seen from above. Do your work on a photocopy or a printout of page 125.

4 Draw graphs of position, velocity, and acceleration as functions of time for a person bunjee jumping. (In bunjee jumping, a person has a stretchy elastic cord tied to his/her ankles, and jumps off of a high platform. At the bottom of the fall, the cord brings the person up short. Presumably the person bounces up a little.) Do your work on a photocopy or a printout of page 125.
5 A ball rolls down the ramp shown in the figure, consisting of a curved knee, a straight slope, and a curved bottom. For each part of the ramp, tell whether the ball’s velocity is increasing, decreasing, or constant, and also whether the ball’s acceleration is increasing, decreasing, or constant. Explain your answers. Assume there is no air friction or rolling resistance. Hint: Try problem 20 first. [Based on a problem by Hewitt.]

6 A toy car is released on one side of a piece of track that is bent into an upright $U$ shape. The car goes back and forth. When the car reaches the limit of its motion on one side, its velocity is zero. Is its acceleration also zero? Explain using a $v-t$ graph. [Based on a problem by Serway and Faughn.]

7 What is the acceleration of a car that moves at a steady velocity of 100 km/h for 100 seconds? Explain your answer. [Based on a problem by Hewitt.]

8 A physics homework question asks, “If you start from rest and accelerate at $1.54 \, \text{m/s}^2$ for $3.29 \, \text{s}$, how far do you travel by the end of that time?” A student answers as follows:

$$1.54 \times 3.29 = 5.07 \, \text{m}$$

His Aunt Wanda is good with numbers, but has never taken physics. She doesn’t know the formula for the distance traveled under constant acceleration over a given amount of time, but she tells her nephew his answer cannot be right. How does she know?

9 You are looking into a deep well. It is dark, and you cannot see the bottom. You want to find out how deep it is, so you drop a rock in, and you hear a splash 3.0 seconds later. How deep is the well?

$$\sqrt{10}$$

10 You take a trip in your spaceship to another star. Setting off, you increase your speed at a constant acceleration. Once you get half-way there, you start decelerating, at the same rate, so that by the time you get there, you have slowed down to zero speed. You see the tourist attractions, and then head home by the same method.

(a) Find a formula for the time, $T$, required for the round trip, in terms of $d$, the distance from our sun to the star, and $a$, the magnitude of the acceleration. Note that the acceleration is not constant over the whole trip, but the trip can be broken up into constant-acceleration parts.

(b) The nearest star to the Earth (other than our own sun) is Proxima Centauri, at a distance of $d = 4 \times 10^{16} \, \text{m}$. Suppose you use an acceleration of $a = 10 \, \text{m/s}^2$, just enough to compensate for the lack of true gravity and make you feel comfortable. How long does the round trip take, in years?

(c) Using the same numbers for $d$ and $a$, find your maximum speed. Compare this to the speed of light, which is $3.0 \times 10^8 \, \text{m/s}$. (Later in this course, you will learn that there are some new things going
on in physics when one gets close to the speed of light, and that it is impossible to exceed the speed of light. For now, though, just use the simpler ideas you’ve learned so far.

11 You climb half-way up a tree, and drop a rock. Then you climb to the top, and drop another rock. How many times greater is the velocity of the second rock on impact? Explain. (The answer is not two times greater.)

12 Alice drops a rock off a cliff. Bubba shoots a gun straight down from the edge of the same cliff. Compare the accelerations of the rock and the bullet while they are in the air on the way down. [Based on a problem by Serway and Faughn.]

13 A person is parachute jumping. During the time between when she leaps out of the plane and when she opens her chute, her altitude is given by an equation of the form

\[ y = b - c \left( t + ke^{-t/k} \right), \]

where \( e \) is the base of natural logarithms, and \( b, c, \) and \( k \) are constants. Because of air resistance, her velocity does not increase at a steady rate as it would for an object falling in vacuum.

(a) What units would \( b, c, \) and \( k \) have to have for the equation to make sense?

(b) Find the person’s velocity, \( v, \) as a function of time. [You will need to use the chain rule, and the fact that \( d(e^x)/dx = e^x. \)]

(c) Use your answer from part (b) to get an interpretation of the constant \( c. \) [Hint: \( e^{-x} \) approaches zero for large values of \( x. \)]

(d) Find the person’s acceleration, \( a, \) as a function of time. \( \sqrt{\text{Solution}} \)

(e) Use your answer from part (d) to show that if she waits long enough to open her chute, her acceleration will become very small.

14 The top part of the figure shows the position-versus-time graph for an object moving in one dimension. On the bottom part of the figure, sketch the corresponding \( v \)-versus-\( t \) graph.

15 On New Year’s Eve, a stupid person fires a pistol straight up. The bullet leaves the gun at a speed of 100 m/s. How long does it take before the bullet hits the ground? \( \sqrt{\text{Solution}} \)

16 If the acceleration of gravity on Mars is 1/3 that on Earth, how many times longer does it take for a rock to drop the same distance on Mars? Ignore air resistance. \( \sqrt{\text{Solution}} \)

17 A honeybee’s position as a function of time is given by \( x = 10t - t^3, \) where \( t \) is in seconds and \( x \) in meters. What is its acceleration at \( t = 3.0 \) s? \( \sqrt{\text{Solution}} \)
18 In July 1999, Popular Mechanics carried out tests to find which car sold by a major auto maker could cover a quarter mile (402 meters) in the shortest time, starting from rest. Because the distance is so short, this type of test is designed mainly to favor the car with the greatest acceleration, not the greatest maximum speed (which is irrelevant to the average person). The winner was the Dodge Viper, with a time of 12.08 s. The car’s top (and presumably final) speed was 118.51 miles per hour (52.98 m/s). (a) If a car, starting from rest and moving with constant acceleration, covers a quarter mile in this time interval, what is its acceleration? (b) What would be the final speed of a car that covered a quarter mile with the constant acceleration you found in part a? (c) Based on the discrepancy between your answer in part b and the actual final speed of the Viper, what do you conclude about how its acceleration changed over time?

19 The graph represents the motion of a ball that rolls up a hill and then back down. When does the ball return to the location it had at \( t = 0 \)?

20 (a) The ball is released at the top of the ramp shown in the figure. Friction is negligible. Use physical reasoning to draw \( v-t \) and \( a-t \) graphs. Assume that the ball doesn’t bounce at the point where the ramp changes slope. (b) Do the same for the case where the ball is rolled up the slope from the right side, but doesn’t quite have enough speed to make it over the top.

21 You throw a rubber ball up, and it falls and bounces several times. Draw graphs of position, velocity, and acceleration as functions of time.

22 Starting from rest, a ball rolls down a ramp, traveling a distance \( L \) and picking up a final speed \( v \). How much of the distance did the ball have to cover before achieving a speed of \( v/2 \)? [Based on a problem by Arnold Arons.]

23 The graph shows the acceleration of a chipmunk in a TV cartoon. It consists of two circular arcs and two line segments. At \( t = 0.00 \) s, the chipmunk’s velocity is \(-3.10 \) m/s. What is its velocity at \( t = 10.00 \) s?

24 Find the error in the following calculation. A student wants to find the distance traveled by a car that accelerates from rest for 5.0 s with an acceleration of 2.0 m/s². First he solves \( a = \Delta v/\Delta t \) for \( \Delta v = 10 \) m/s. Then he multiplies to find \((10 \) m/s)(5.0 s) = 50 m. Do not just recalculate the result by a different method; if that was all you did, you’d have no way of knowing which calculation was correct, yours or his.
25 Acceleration could be defined either as $\Delta v/\Delta t$ or as the slope of the tangent line on the $v - t$ graph. Is either one superior as a definition, or are they equivalent? If you say one is better, give an example of a situation where it makes a difference which one you use.

26 If an object starts accelerating from rest, we have $v^2 = 2a\Delta x$ for its speed after it has traveled a distance $\Delta x$. Explain in words why it makes sense that the equation has velocity squared, but distance only to the first power. Don’t recapitulate the derivation in the book, or give a justification based on units. The point is to explain what this feature of the equation tells us about the way speed increases as more distance is covered.

27 The figure shows a practical, simple experiment for determining $g$ to high precision. Two steel balls are suspended from electromagnets, and are released simultaneously when the electric current is shut off. They fall through unequal heights $\Delta x_1$ and $\Delta x_2$. A computer records the sounds through a microphone as first one ball and then the other strikes the floor. From this recording, we can accurately determine the quantity $T$ defined as $T = \Delta t_2 - \Delta t_1$, i.e., the time lag between the first and second impacts. Note that since the balls do not make any sound when they are released, we have no way of measuring the individual times $\Delta t_2$ and $\Delta t_1$.

(a) Find an equation for $g$ in terms of the measured quantities $T$, $\Delta x_1$ and $\Delta x_2$.
(b) Check the units of your equation.
(c) Check that your equation gives the correct result in the case where $\Delta x_1$ is very close to zero. However, is this case realistic?
(d) What happens when $\Delta x_1 = \Delta x_2$? Discuss this both mathematically and physically.

28 The speed required for a low-earth orbit is $7.9 \times 10^3$ m/s. When a rocket is launched into orbit, it goes up a little at first to get above almost all of the atmosphere, but then tips over horizontally to build up to orbital speed. Suppose the horizontal acceleration is limited to $3g$ to keep from damaging the cargo (or hurting the crew, for a crewed flight). (a) What is the minimum distance the rocket must travel downrange before it reaches orbital speed? How much does it matter whether you take into account the initial eastward velocity due to the rotation of the earth? (b) Rather than a rocket ship, it might be advantageous to use a railgun design, in which the craft would be accelerated to orbital speeds along a railroad track. This has the advantage that it isn’t necessary to lift a large mass of fuel, since the energy source is external. Based on your answer to part a, comment on the feasibility of this design for crewed launches from the earth’s surface.
Problem 29. This spectacular series of photos from a 2011 paper by Burrows and Sutton ("Biomechanics of jumping in the flea," J. Exp. Biology 214:836) shows the flea jumping at about a 45-degree angle, but for the sake of this estimate just consider the case of a flea jumping vertically.

29 Some fleas can jump as high as 30 cm. The flea only has a short time to build up speed — the time during which its center of mass is accelerating upward but its feet are still in contact with the ground. Make an order-of-magnitude estimate of the acceleration the flea needs to have while straightening its legs, and state your answer in units of \( g \), i.e., how many “g’s it pulls.” (For comparison, fighter pilots black out or die if they exceed about 5 or 10 g’s.)

30 Consider the following passage from Alice in Wonderland, in which Alice has been falling for a long time down a rabbit hole:

Down, down, down. Would the fall never come to an end? “I wonder how many miles I’ve fallen by this time?” she said aloud. “I must be getting somewhere near the center of the earth. Let me see: that would be four thousand miles down, I think” (for, you see, Alice had learned several things of this sort in her lessons in the schoolroom, and though this was not a very good opportunity for showing off her knowledge, as there was no one to listen to her, still it was good practice to say it over)...

Alice doesn’t know much physics, but let’s try to calculate the amount of time it would take to fall four thousand miles, starting from rest with an acceleration of 10 m/s\(^2\). This is really only a lower limit; if there really was a hole that deep, the fall would actually take a longer time than the one you calculate, both because there is air friction and because gravity gets weaker as you get deeper (at the center of the earth, \( g \) is zero, because the earth is pulling you equally in every direction at once).

31 The photo shows Apollo 16 astronaut John Young jumping on the moon and saluting at the top of his jump. The video footage of the jump shows him staying aloft for 1.45 seconds. Gravity on the moon is 1/6 as strong as on the earth. Compute the height of the jump.
32 Most people don’t know that *Spinosaurus aegyptiacus*, not *Tyrannosaurus rex*, was the biggest theropod dinosaur. We can’t put a dinosaur on a track and time it in the 100 meter dash, so we can only infer from physical models how fast it could have run. When an animal walks at a normal pace, typically its legs swing more or less like pendulums of the same length $\ell$. As a further simplification of this model, let’s imagine that the leg simply moves at a fixed acceleration as it falls to the ground. That is, we model the time for a quarter of a stride cycle as being the same as the time required for free fall from a height $\ell$. $S. aegyptiacus$ had legs about four times longer than those of a human. (a) Compare the time required for a human’s stride cycle to that for $S. aegyptiacus$. √
(b) Compare their running speeds.

33 Engineering professor Qingming Li used sensors and video cameras to study punches delivered in the lab by British former welterweight boxing champion Ricky “the Hitman” Hatton. For comparison, Li also let a TV sports reporter put on the gloves and throw punches. The time it took for Hatton’s best punch to arrive, i.e., the time his opponent would have had to react, was about 0.47 of that for the reporter. Let’s assume that the fist starts from rest and moves with constant acceleration all the way up until impact, at some fixed distance (arm’s length). Compare Hatton’s acceleration to the reporter’s. √

34 Aircraft carriers originated in World War I, and the first landing on a carrier was performed by E.H. Dunning in a Sopwith Pup biplane, landing on HMS Furious. (Dunning was killed the second time he attempted the feat.) In such a landing, the pilot slows down to just above the plane’s stall speed, which is the minimum speed at which the plane can fly without stalling. The plane then lands and is caught by cables and decelerated as it travels the length of the flight deck. Comparing a modern US F-14 fighter jet landing on an Enterprise-class carrier to Dunning’s original exploit, the stall speed is greater by a factor of 4.8, and to accommodate this, the length of the flight deck is greater by a factor of 1.9. Which deceleration is greater, and by what factor? √

35 In college-level women’s softball in the U.S., typically a pitcher is expected to be at least 1.75 m tall, but Virginia Tech pitcher Jasmin Harrell is 1.62 m. Although a pitcher actually throws by stepping forward and swinging her arm in a circle, let’s make a simplified physical model to estimate how much of a disadvantage Harrell has had to overcome due to her height. We’ll pretend that the pitcher gives the ball a constant acceleration in a straight line, and that the length of this line is proportional to the pitcher’s height. Compare the acceleration Harrell would have to supply with the acceleration that would suffice for a pitcher of the nominal minimum height, if both were to throw a pitch at the same speed. √
When the police engage in a high-speed chase on city streets, it can be extremely dangerous both to the police and to other motorists and pedestrians. Suppose that the police car must travel at a speed that is limited by the need to be able to stop before hitting a baby carriage, and that the distance at which the driver first sees the baby carriage is fixed. Tests show that in a panic stop from high speed, a police car based on a Chevy Impala has a deceleration 9% greater than that of a Dodge Intrepid. Compare the maximum safe speeds for the two cars.

For each of the two graphs, find the change in position $\Delta x$ from beginning to end, using the technique described in section 3.5.

You shove a box with initial velocity 2.0 m/s, and it stops after sliding 1.3 m. What is the magnitude of the deceleration, assuming it is constant?

You’re an astronaut, and you’ve arrived on planet X, which is airless. You drop a hammer from a height of 1.00 m and find that it takes 350 ms to fall to the ground. What is the acceleration due to gravity on planet X?

A naughty child drops a golf ball from the roof of your apartment building, and you see it drop past your window. It takes the ball time $T$ to traverse the window’s height $H$. Find the initial speed of the ball when it first came into view.
Chapter 4

Force and Motion

If I have seen farther than others, it is because I have stood on the shoulders of giants.

*Newton, referring to Galileo*

Even as great and skeptical a genius as Galileo was unable to make much progress on the causes of motion. It was not until a generation later that Isaac Newton (1642-1727) was able to attack the problem successfully. In many ways, Newton’s personality was the opposite of Galileo’s. Where Galileo aggressively publicized his ideas,
Newton had to be coaxed by his friends into publishing a book on his physical discoveries. Where Galileo’s writing had been popular and dramatic, Newton originated the stilted, impersonal style that most people think is standard for scientific writing. (Scientific journals today encourage a less ponderous style, and papers are often written in the first person.) Galileo’s talent for arousing animosity among the rich and powerful was matched by Newton’s skill at making himself a popular visitor at court. Galileo narrowly escaped being burned at the stake, while Newton had the good fortune of being on the winning side of the revolution that replaced King James II with William and Mary of Orange, leading to a lucrative post running the English royal mint.

Newton discovered the relationship between force and motion, and revolutionized our view of the universe by showing that the same physical laws applied to all matter, whether living or nonliving, on or off of our planet’s surface. His book on force and motion, the Mathematical Principles of Natural Philosophy, was uncontradicted by experiment for 200 years, but his other main work, Optics, was on the wrong track, asserting that light was composed of particles rather than waves. Newton was also an avid alchemist, a fact that modern scientists would like to forget.

4.1 Force

We need only explain changes in motion, not motion itself.

So far you’ve studied the measurement of motion in some detail, but not the reasons why a certain object would move in a certain way. This chapter deals with the “why” questions. Aristotle’s ideas about the causes of motion were completely wrong, just like all his other ideas about physical science, but it will be instructive to start with them, because they amount to a road map of modern students’ incorrect preconceptions.

Aristotle thought he needed to explain both why motion occurs and why motion might change. Newton inherited from Galileo the important counter-Aristotelian idea that motion needs no explanation, that it is only changes in motion that require a physical cause. Aristotle’s needlessly complex system gave three reasons for motion:

Natural motion, such as falling, came from the tendency of objects to go to their “natural” place, on the ground, and come to rest.

Voluntary motion was the type of motion exhibited by animals, which moved because they chose to.

Forced motion occurred when an object was acted on by some other object that made it move.
**Motion changes due to an interaction between two objects.**

In the Aristotelian theory, natural motion and voluntary motion are one-sided phenomena: the object causes its own motion. Forced motion is supposed to be a two-sided phenomenon, because one object imposes its “commands” on another. Where Aristotle conceived of some of the phenomena of motion as one-sided and others as two-sided, Newton realized that a change in motion was always a two-sided relationship of a force acting between two physical objects.

The one-sided “natural motion” description of falling makes a crucial omission. The acceleration of a falling object is not caused by its own “natural” tendencies but by an attractive force between it and the planet Earth. Moon rocks brought back to our planet do not “want” to fly back up to the moon because the moon is their “natural” place. They fall to the floor when you drop them, just like our homegrown rocks. As we’ll discuss in more detail later in this course, gravitational forces are simply an attraction that occurs between any two physical objects. Minute gravitational forces can even be measured between human-scale objects in the laboratory.

The idea of natural motion also explains incorrectly why things come to rest. A basketball rolling across a beach slows to a stop because it is interacting with the sand via a frictional force, not because of its own desire to be at rest. If it was on a frictionless surface, it would never slow down. Many of Aristotle’s mistakes stemmed from his failure to recognize friction as a force.

The concept of voluntary motion is equally flawed. You may have been a little uneasy about it from the start, because it assumes a clear distinction between living and nonliving things. Today, however, we are used to having the human body likened to a complex machine. In the modern world-view, the border between the living and the inanimate is a fuzzy no-man’s land inhabited by viruses, prions, and silicon chips. Furthermore, Aristotle’s statement that you can take a step forward “because you choose to” inappropriately mixes two levels of explanation. At the physical level of explanation, the reason your body steps forward is because of a frictional force acting between your foot and the floor. If the floor was covered with a puddle of oil, no amount of “choosing to” would enable you to take a graceful stride forward.

**Forces can all be measured on the same numerical scale.**

In the Aristotelian-scholastic tradition, the description of motion as natural, voluntary, or forced was only the broadest level of classification, like splitting animals into birds, reptiles, mammals, and amphibians. There might be thousands of types of motion, each of which would follow its own rules. Newton’s realization that all changes in motion were caused by two-sided interactions made
it seem that the phenomena might have more in common than had been apparent. In the Newtonian description, there is only one cause for a change in motion, which we call force. Forces may be of different types, but they all produce changes in motion according to the same rules. Any acceleration that can be produced by a magnetic force can equally well be produced by an appropriately controlled stream of water. We can speak of two forces as being equal if they produce the same change in motion when applied in the same situation, which means that they pushed or pulled equally hard in the same direction.

The idea of a numerical scale of force and the newton unit were introduced in chapter 0. To recapitulate briefly, a force is when a pair of objects push or pull on each other, and one newton is the force required to accelerate a 1-kg object from rest to a speed of 1 m/s in 1 second.

**More than one force on an object**

As if we hadn’t kicked poor Aristotle around sufficiently, his theory has another important flaw, which is important to discuss because it corresponds to an extremely common student misconception. Aristotle conceived of forced motion as a relationship in which one object was the boss and the other “followed orders.” It therefore would only make sense for an object to experience one force at a time, because an object couldn’t follow orders from two sources at once. In the Newtonian theory, forces are numbers, not orders, and if more than one force acts on an object at once, the result is found by adding up all the forces. It is unfortunate that the use of the English word “force” has become standard, because to many people it suggests that you are “forcing” an object to do something. The force of the earth’s gravity cannot “force” a boat to sink, because there are other forces acting on the boat. Adding them up gives a total of zero, so the boat accelerates neither up nor down.

**Objects can exert forces on each other at a distance.**

Aristotle declared that forces could only act between objects that were touching, probably because he wished to avoid the type of occult speculation that attributed physical phenomena to the influence of a distant and invisible pantheon of gods. He was wrong, however, as you can observe when a magnet leaps onto your refrigerator or when the planet earth exerts gravitational forces on objects that are in the air. Some types of forces, such as friction, only operate between objects in contact, and are called contact forces. Magnetism, on the other hand, is an example of a noncontact force. Although the magnetic force gets stronger when the magnet is closer to your refrigerator, touching is not required.
Weight

In physics, an object’s weight, $F_W$, is defined as the earth’s gravitational force on it. The SI unit of weight is therefore the Newton. People commonly refer to the kilogram as a unit of weight, but the kilogram is a unit of mass, not weight. Note that an object’s weight is not a fixed property of that object. Objects weigh more in some places than in others, depending on the local strength of gravity. It is their mass that always stays the same. A baseball pitcher who can throw a 90-mile-per-hour fastball on earth would not be able to throw any faster on the moon, because the ball’s inertia would still be the same.

Positive and negative signs of force

We’ll start by considering only cases of one-dimensional center-of-mass motion in which all the forces are parallel to the direction of motion, i.e., either directly forward or backward. In one dimension, plus and minus signs can be used to indicate directions of forces, as shown in figure c. We can then refer generically to addition of forces, rather than having to speak sometimes of addition and sometimes of subtraction. We add the forces shown in the figure and get 11 N. In general, we should choose a one-dimensional coordinate system with its $x$ axis parallel the direction of motion. Forces that point along the positive $x$ axis are positive, and forces in the opposite direction are negative. Forces that are not directly along the $x$ axis cannot be immediately incorporated into this scheme, but that’s OK, because we’re avoiding those cases for now.

Discussion questions

A. In chapter 0, I defined 1 N as the force that would accelerate a 1-kg mass from rest to 1 m/s in 1 s. Anticipating the following section, you might guess that 2 N could be defined as the force that would accelerate the same mass to twice the speed, or twice the mass to the same speed. Is there an easier way to define 2 N based on the definition of 1 N?

4.2 Newton’s first law

We are now prepared to make a more powerful restatement of the principle of inertia.\(^1\)

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\( ^1\)Page 81 lists places in this book where we describe experimental tests of the principle of inertia and Newton’s first law.
remains in motion with the same velocity in the same direction. The converse of Newton’s first law is also true: if we observe an object moving with constant velocity along a straight line, then the total force on it must be zero.

In a future physics course or in another textbook, you may encounter the term “net force,” which is simply a synonym for total force.

What happens if the total force on an object is not zero? It accelerates. Numerical prediction of the resulting acceleration is the topic of Newton’s second law, which we’ll discuss in the following section.

This is the first of Newton’s three laws of motion. It is not important to memorize which of Newton’s three laws are numbers one, two, and three. If a future physics teacher asks you something like, “Which of Newton’s laws are you thinking of?,” a perfectly acceptable answer is “The one about constant velocity when there’s zero total force.” The concepts are more important than any specific formulation of them. Newton wrote in Latin, and I am not aware of any modern textbook that uses a verbatim translation of his statement of the laws of motion. Clear writing was not in vogue in Newton’s day, and he formulated his three laws in terms of a concept now called momentum, only later relating it to the concept of force. Nearly all modern texts, including this one, start with force and do momentum later.

An elevator

An elevator has a weight of 5000 N. Compare the forces that the cable must exert to raise it at constant velocity, lower it at constant velocity, and just keep it hanging.

In all three cases the cable must pull up with a force of exactly 5000 N. Most people think you’d need at least a little more than 5000 N to make it go up, and a little less than 5000 N to let it down, but that’s incorrect. Extra force from the cable is only necessary for speeding the car up when it starts going up or slowing it down when it finishes going down. Decreased force is needed to speed the car up when it gets going down and to slow it down when it finishes going up. But when the elevator is cruising at constant velocity, Newton’s first law says that you just need to cancel the force of the earth’s gravity.

To many students, the statement in the example that the cable’s upward force “cancels” the earth’s downward gravitational force implies that there has been a contest, and the cable’s force has won, vanquishing the earth’s gravitational force and making it disappear. That is incorrect. Both forces continue to exist, but because they add up numerically to zero, the elevator has no center-of-mass acceleration. We know that both forces continue to exist because they both have side-effects other than their effects on the car’s center-of-
mass motion. The force acting between the cable and the car continues to produce tension in the cable and keep the cable taut. The earth’s gravitational force continues to keep the passengers (whom we are considering as part of the elevator-object) stuck to the floor and to produce internal stresses in the walls of the car, which must hold up the floor.

**Terminal velocity for falling objects**

An object like a feather that is not dense or streamlined does not fall with constant acceleration, because air resistance is nonnegligible. In fact, its acceleration tapers off to nearly zero within a fraction of a second, and the feather finishes dropping at constant speed (known as its terminal velocity). Why does this happen?

> Newton’s first law tells us that the total force on the feather must have been reduced to nearly zero after a short time. There are two forces acting on the feather: a downward gravitational force from the planet earth, and an upward frictional force from the air. As the feather speeds up, the air friction becomes stronger and stronger, and eventually it cancels out the earth’s gravitational force, so the feather just continues with constant velocity without speeding up any more.

The situation for a skydiver is exactly analogous. It’s just that the skydiver experiences perhaps a million times more gravitational force than the feather, and it is not until she is falling very fast that the force of air friction becomes as strong as the gravitational force. It takes her several seconds to reach terminal velocity, which is on the order of a hundred miles per hour.

**More general combinations of forces**

It is too constraining to restrict our attention to cases where all the forces lie along the line of the center of mass’s motion. For one thing, we can’t analyze any case of horizontal motion, since any object on earth will be subject to a vertical gravitational force! For instance, when you are driving your car down a straight road, there are both horizontal forces and vertical forces. However, the vertical forces have no effect on the center of mass motion, because the road’s upward force simply counteracts the earth’s downward gravitational force and keeps the car from sinking into the ground.

Later in the book we’ll deal with the most general case of many forces acting on an object at any angles, using the mathematical technique of vector addition, but the following slight generalization of Newton’s first law allows us to analyze a great many cases of interest:

Suppose that an object has two sets of forces acting on it, one set along the line of the object’s initial motion and another set perpendicular to the first set. If both sets of forces cancel, then the object’s center of mass continues in the same state of motion.
A passenger riding the subway  

Describe the forces acting on a person standing in a subway train that is cruising at constant velocity.

No force is necessary to keep the person moving relative to the ground. He will not be swept to the back of the train if the floor is slippery. There are two vertical forces on him, the earth's downward gravitational force and the floor's upward force, which cancel. There are no horizontal forces on him at all, so of course the total horizontal force is zero.

Forces on a sailboat

If a sailboat is cruising at constant velocity with the wind coming from directly behind it, what must be true about the forces acting on it?

The forces acting on the boat must be canceling each other out. The boat is not sinking or leaping into the air, so evidently the vertical forces are canceling out. The vertical forces are the downward gravitational force exerted by the planet earth and an upward force from the water.

The air is making a forward force on the sail, and if the boat is not accelerating horizontally then the water's backward frictional force must be canceling it out.

Contrary to Aristotle, more force is not needed in order to maintain a higher speed. Zero total force is always needed to maintain constant velocity. Consider the following made-up numbers:

<table>
<thead>
<tr>
<th></th>
<th>boat moving at a low, constant velocity</th>
<th>boat moving at a high, constant velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward force of the wind on the sail . . .</td>
<td>10,000 N</td>
<td>20,000 N</td>
</tr>
<tr>
<td>backward force of the water on the hull . . .</td>
<td>−10,000 N</td>
<td>−20,000 N</td>
</tr>
<tr>
<td>total force on the boat . . .</td>
<td>0 N</td>
<td>0 N</td>
</tr>
</tbody>
</table>

The faster boat still has zero total force on it. The forward force on it is greater, and the backward force smaller (more negative), but that's irrelevant because Newton's first law has to do with the total force, not the individual forces.

This example is quite analogous to the one about terminal velocity of falling objects, since there is a frictional force that increases with speed. After casting off from the dock and raising the sail, the boat will accelerate briefly, and then reach its terminal velocity, at which the water's frictional force has become as great as the wind's force on the sail.
A car crash example 5

▷ If you drive your car into a brick wall, what is the mysterious force that slams your face into the steering wheel?

▷ Your surgeon has taken physics, so she is not going to believe your claim that a mysterious force is to blame. She knows that your face was just following Newton’s first law. Immediately after your car hit the wall, the only forces acting on your head were the same canceling-out forces that had existed previously: the earth’s downward gravitational force and the upward force from your neck. There were no forward or backward forces on your head, but the car did experience a backward force from the wall, so the car slowed down and your face caught up.

Discussion questions

A Newton said that objects continue moving if no forces are acting on them, but his predecessor Aristotle said that a force was necessary to keep an object moving. Why does Aristotle’s theory seem more plausible, even though we now believe it to be wrong? What insight was Aristotle missing about the reason why things seem to slow down naturally? Give an example.

B In the figure what would have to be true about the saxophone’s initial motion if the forces shown were to result in continued one-dimensional motion of its center of mass?

C This figure requires an ever further generalization of the preceding discussion. After studying the forces, what does your physical intuition tell you will happen? Can you state in words how to generalize the conditions for one-dimensional motion to include situations like this one?

4.3 Newton’s second law

What about cases where the total force on an object is not zero, so that Newton’s first law doesn’t apply? The object will have an acceleration. The way we’ve defined positive and negative signs of force and acceleration guarantees that positive forces produce positive accelerations, and likewise for negative values. How much acceleration will it have? It will clearly depend on both the object’s mass and on the amount of force.

Experiments with any particular object show that its acceleration is directly proportional to the total force applied to it. This may seem wrong, since we know of many cases where small amounts of force fail to move an object at all, and larger forces get it going. This apparent failure of proportionality actually results from forgetting that there is a frictional force in addition to the force we apply to move the object. The object’s acceleration is exactly proportional to the total force on it, not to any individual force on it. In the absence of friction, even a very tiny force can slowly change the velocity of a very massive object.
Experiments (e.g., the one described in example 11 on p. 139) also show that the acceleration is inversely proportional to the object’s mass, and combining these two proportionalities gives the following way of predicting the acceleration of any object:

**Newton’s second law**

\[ a = \frac{F_{\text{total}}}{m}, \]

where

- \( m \) is an object’s mass, a measure of its resistance to changes in its motion
- \( F_{\text{total}} \) is the sum of the forces acting on it, and
- \( a \) is the acceleration of the object’s center of mass.

We are presently restricted to the case where the forces of interest are parallel to the direction of motion.

We have already encountered the SI unit of force, which is the newton (N). It is designed so that the units in Newton’s second law all work out if we use SI units: \( \text{m/s}^2 \) for acceleration and kg (not grams!) for mass.

---

**Rocket science example 6**

The Falcon 9 launch vehicle, built and operated by the private company SpaceX, has mass \( m = 5.1 \times 10^5 \text{ kg} \). At launch, it has two forces acting on it: an upward thrust \( F_t = 5.9 \times 10^6 \text{ N} \) and a downward gravitational force of \( F_g = 5.0 \times 10^6 \text{ N} \). Find its acceleration.

Let’s choose our coordinate system such that positive is up. Then the downward force of gravity is considered negative. Using Newton’s second law,

\[
\begin{align*}
    a &= \frac{F_{\text{total}}}{m} \\
    &= \frac{F_t - F_g}{m} \\
    &= \frac{(5.9 \times 10^6 \text{ N}) - (5.0 \times 10^6 \text{ N})}{5.1 \times 10^5 \text{ kg}} \\
    &= 1.6 \text{ m/s}^2,
\end{align*}
\]

where as noted above, units of N/kg (newtons per kilogram) are the same as \( \text{m/s}^2 \).
An accelerating bus  
A VW bus with a mass of 2000 kg accelerates from 0 to 25 m/s (freeway speed) in 34 s. Assuming the acceleration is constant, what is the total force on the bus?

We solve Newton’s second law for $F_{total} = ma$, and substitute $\Delta v/\Delta t$ for $a$, giving

$$F_{total} = m\Delta v/\Delta t$$
$$= (2000 \text{ kg})(25 \text{ m/s} - 0 \text{ m/s})/(34 \text{ s})$$
$$= 1.5 \text{ kN}.$$  

A generalization

As with the first law, the second law can be easily generalized to include a much larger class of interesting situations:

Suppose an object is being acted on by two sets of forces, one set lying parallel to the object’s initial direction of motion and another set acting along a perpendicular line. If the forces perpendicular to the initial direction of motion cancel out, then the object accelerates along its original line of motion according to $a = F_{\parallel}/m$, where $F_{\parallel}$ is the sum of the forces parallel to the line.

A coin sliding across a table  
Suppose a coin is sliding to the right across a table, and let’s choose a positive $x$ axis that points to the right. The coin’s velocity is positive, and we expect based on experience that it will slow down, i.e., its acceleration should be negative.

Although the coin’s motion is purely horizontal, it feels both vertical and horizontal forces. The Earth exerts a downward gravitational force $F_2$ on it, and the table makes an upward force $F_3$ that prevents the coin from sinking into the wood. In fact, without these vertical forces the horizontal frictional force wouldn’t exist: surfaces don’t exert friction against one another unless they are being pressed together.

Although $F_2$ and $F_3$ contribute to the physics, they do so only indirectly. The only thing that directly relates to the acceleration along the horizontal direction is the horizontal force: $a = F_1/m$.  

Section 4.3  Newton’s second law  137
The relationship between mass and weight

Mass is different from weight, but they’re related. An apple’s mass tells us how hard it is to change its motion. Its weight measures the strength of the gravitational attraction between the apple and the planet earth. The apple’s weight is less on the moon, but its mass is the same. Astronauts assembling the International Space Station in zero gravity couldn’t just pitch massive modules back and forth with their bare hands; the modules were weightless, but not massless.

We have already seen the experimental evidence that when weight (the force of the earth’s gravity) is the only force acting on an object, its acceleration equals the constant \( g \), and \( g \) depends on where you are on the surface of the earth, but not on the mass of the object. Applying Newton’s second law then allows us to calculate the magnitude of the gravitational force on any object in terms of its mass:

\[
|F_W| = mg.
\]

(The equation only gives the magnitude, i.e. the absolute value, of \( F_W \), because we’re defining \( g \) as a positive number, so it equals the absolute value of a falling object’s acceleration.)

**Solved problem: Decelerating a car**

Let’s start with the single kilogram. It’s not accelerating, so evidently the total force on it is zero: the spring scale’s upward force on it is canceling out the earth’s downward gravitational force. The spring scale tells us how much force it is being obliged to supply, but since the two forces are equal in strength, the spring scale’s reading can also be interpreted as measuring the strength of the gravitational force, i.e., the weight of the one-kilogram mass. The weight of a one-kilogram mass should be

\[
F_W = mg
\]

\[
= (1.0 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ N},
\]

and that’s indeed the reading on the spring scale.

Similarly for the two-kilogram mass, we have

\[
F_W = mg
\]

\[
= (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 19.6 \text{ N}.
\]
Calculating terminal velocity example 10

Experiments show that the force of air friction on a falling object such as a skydiver or a feather can be approximated fairly well with the equation $|F_{\text{air}}| = c \rho A v^2$, where $c$ is a constant, $\rho$ is the density of the air, $A$ is the cross-sectional area of the object as seen from below, and $v$ is the object’s velocity. Predict the object’s terminal velocity, i.e., the final velocity it reaches after a long time.

As the object accelerates, its greater $v$ causes the upward force of the air to increase until finally the gravitational force and the force of air friction cancel out, after which the object continues at constant velocity. We choose a coordinate system in which positive is up, so that the gravitational force is negative and the force of air friction is positive. We want to find the velocity at which

$$F_{\text{air}} + F_W = 0, \quad \text{i.e.,} \quad c \rho A v^2 - mg = 0.$$ 

Solving for $v$ gives

$$v_{\text{terminal}} = \sqrt{\frac{mg}{c \rho A}}$$

self-check A

It is important to get into the habit of interpreting equations. This may be difficult at first, but eventually you will get used to this kind of reasoning.

(1) Interpret the equation $v_{\text{terminal}} = \sqrt{\frac{mg}{c \rho A}}$ in the case of $\rho = 0$.

(2) How would the terminal velocity of a 4-cm steel ball compare to that of a 1-cm ball?

(3) In addition to teasing out the mathematical meaning of an equation, we also have to be able to place it in its physical context. How generally important is this equation? Answer, p. 562

A test of the second law example 11

Because the force $mg$ of gravity on an object of mass $m$ is proportional to $m$, the acceleration predicted by Newton’s second law is $a = \frac{F}{m} = \frac{mg}{m} = g$, in which the mass cancels out. It is therefore an ironclad prediction of Newton’s laws of motion that free fall is universal: in the absence of other forces such as air resistance, heavier objects do not fall with a greater acceleration than lighter ones. The experiment by Galileo at the Leaning Tower of Pisa (p. 96) is therefore consistent with Newton’s second law. Since Galileo’s time, experimental methods have had several centuries in which to improve, and the second law has been subjected to similar tests with exponentially improving precision. For such an experiment in 1993, physicists at the University of Pisa

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(1) built a metal disk out of copper and tungsten semicircles joined together at their flat edges. They evacuated the air from a vertical shaft and dropped the disk down it 142 times, using lasers to measure any tiny rotation that would result if the accelerations of the copper and tungsten were very slightly different. The results were statistically consistent with zero rotation, and put an upper limit of $1 \times 10^{-9}$ on the fractional difference in acceleration $|g_{\text{copper}} - g_{\text{tungsten}}|/g$. A more recent experiment using test masses in orbit has refined this bound to $10^{-14}$.

Discussion questions

A  Show that the Newton can be reexpressed in terms of the three basic mks units as the combination $\text{kg} \cdot \text{m/s}^2$.

B  What is wrong with the following statements?

(1) “$g$ is the force of gravity.”

(2) “Mass is a measure of how much space something takes up.”

C  Criticize the following incorrect statement:

“If an object is at rest and the total force on it is zero, it stays at rest. There can also be cases where an object is moving and keeps on moving without having any total force on it, but that can only happen when there’s no friction, like in outer space.”

D  Table j gives laser timing data for Ben Johnson’s 100 m dash at the 1987 World Championship in Rome. (His world record was later revoked because he tested positive for steroids.) How does the total force on him change over the duration of the race?

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$t$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.84</td>
</tr>
<tr>
<td>20</td>
<td>2.86</td>
</tr>
<tr>
<td>30</td>
<td>3.80</td>
</tr>
<tr>
<td>40</td>
<td>4.67</td>
</tr>
<tr>
<td>50</td>
<td>5.53</td>
</tr>
<tr>
<td>60</td>
<td>6.38</td>
</tr>
<tr>
<td>70</td>
<td>7.23</td>
</tr>
<tr>
<td>80</td>
<td>8.10</td>
</tr>
<tr>
<td>90</td>
<td>8.96</td>
</tr>
<tr>
<td>100</td>
<td>9.83</td>
</tr>
</tbody>
</table>

$j/$ Discussion question D.

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4.4 What force is not

Violin teachers have to endure their beginning students’ screeching. A frown appears on the woodwind teacher’s face as she watches her student take a breath with an expansion of his ribcage but none in his belly. What makes physics teachers cringe is their students’ verbal statements about forces. Below I have listed six dicta about what force is not.

1. Force is not a property of one object.

A great many of students’ incorrect descriptions of forces could be cured by keeping in mind that a force is an interaction of two objects, not a property of one object.

Incorrect statement: “That magnet has a lot of force.”

X If the magnet is one millimeter away from a steel ball bearing, they may exert a very strong attraction on each other, but if they were a meter apart, the force would be virtually undetectable. The magnet’s strength can be rated using certain electrical units (ampere − meters$^2$), but not in units of force.

2. Force is not a measure of an object’s motion.

If force is not a property of a single object, then it cannot be used as a measure of the object’s motion.

Incorrect statement: “The freight train rumbled down the tracks with awesome force.”

X Force is not a measure of motion. If the freight train collides with a stalled cement truck, then some awesome forces will occur, but if it hits a fly the force will be small.

3. Force is not energy.

There are two main approaches to understanding the motion of objects, one based on force and one on a different concept, called energy. The SI unit of energy is the Joule, but you are probably more familiar with the calorie, used for measuring food’s energy, and the kilowatt-hour, the unit the electric company uses for billing you. Physics students’ previous familiarity with calories and kilowatt-hours is matched by their universal unfamiliarity with measuring forces in units of Newtons, but the precise operational definitions of the energy concepts are more complex than those of the force concepts, and textbooks, including this one, almost universally place the force description of physics before the energy description. During the long period after the introduction of force and before the careful definition of energy, students are therefore vulnerable to situations in which, without realizing it, they are imputing the properties of energy to phenomena of force.

Incorrect statement: “How can my chair be making an upward force on my rear end? It has no power!”

X Power is a concept related to energy, e.g., a 100-watt lightbulb uses
up 100 joules per second of energy. When you sit in a chair, no energy is used up, so forces can exist between you and the chair without any need for a source of power.

4. Force is not stored or used up.

Because energy can be stored and used up, people think force also can be stored or used up.

*Incorrect statement:* “If you don’t fill up your tank with gas, you’ll run out of force.”

*X* Energy is what you’ll run out of, not force.

5. Forces need not be exerted by living things or machines.

Transforming energy from one form into another usually requires some kind of living or mechanical mechanism. The concept is not applicable to forces, which are an interaction between objects, not a thing to be transferred or transformed.

*Incorrect statement:* “How can a wooden bench be making an upward force on my rear end? It doesn’t have any springs or anything inside it.”

*X* No springs or other internal mechanisms are required. If the bench didn’t make any force on you, you would obey Newton’s second law and fall through it. Evidently it does make a force on you!

6. A force is the direct cause of a change in motion.

I can click a remote control to make my garage door change from being at rest to being in motion. My finger’s force on the button, however, was not the force that acted on the door. When we speak of a force on an object in physics, we are talking about a force that acts directly. Similarly, when you pull a reluctant dog along by its leash, the leash and the dog are making forces on each other, not your hand and the dog. The dog is not even touching your hand.

**self-check B**

Which of the following things can be correctly described in terms of force?

1. A nuclear submarine is charging ahead at full steam.
2. A nuclear submarine’s propellers spin in the water.
3. A nuclear submarine needs to refuel its reactor periodically.

Answer, p. 563

**Discussion questions**

A. Criticize the following incorrect statement: “If you shove a book across a table, friction takes away more and more of its force, until finally it stops.”

B. You hit a tennis ball against a wall. Explain any and all incorrect ideas in the following description of the physics involved: “The ball gets some force from you when you hit it, and when it hits the wall, it loses part of that force, so it doesn’t bounce back as fast. The muscles in your arm are the only things that a force can come from.”

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4.5 Inertial and noninertial frames of reference

One day, you're driving down the street in your pickup truck, on your way to deliver a bowling ball. The ball is in the back of the truck, enjoying its little jaunt and taking in the fresh air and sunshine. Then you have to slow down because a stop sign is coming up. As you brake, you glance in your rearview mirror, and see your trusty companion accelerating toward you. Did some mysterious force push it forward? No, it only seems that way because you and the car are slowing down. The ball is faithfully obeying Newton’s first law, and as it continues at constant velocity it gets ahead relative to the slowing truck. No forces are acting on it (other than the same canceling-out vertical forces that were always acting on it).\(^4\) The ball only appeared to violate Newton’s first law because there was something wrong with your frame of reference, which was based on the truck.

How, then, are we to tell in which frames of reference Newton’s laws are valid? It’s no good to say that we should avoid moving frames of reference, because there is no such thing as absolute rest or absolute motion. All frames can be considered as being either at rest or in motion. According to an observer in India, the strip mall that constituted the frame of reference in panel (b) of the figure was moving along with the earth’s rotation at hundreds of miles per hour.

The reason why Newton’s laws fail in the truck’s frame of reference that moves with the truck, the bowling ball appears to violate Newton’s first law by accelerating despite having no horizontal forces on it. 2. In an inertial frame of reference, which the surface of the earth approximately is, the bowling ball obeys Newton’s first law. It moves equal distances in equal time intervals, i.e., maintains constant velocity. In this frame of reference, it is the truck that appears to have a change in velocity, which makes sense, since the road is making a horizontal force on it.

\(^4\)Let’s assume for simplicity that there is no friction.
ence is not because the truck is moving but because it is accelerating. (Recall that physicists use the word to refer either to speeding up or slowing down.) Newton’s laws were working just fine in the moving truck’s frame of reference as long as the truck was moving at constant velocity. It was only when its speed changed that there was a problem. How, then, are we to tell which frames are accelerating and which are not? What if you claim that your truck is not accelerating, and the sidewalk, the asphalt, and the Burger King are accelerating? The way to settle such a dispute is to examine the motion of some object, such as the bowling ball, which we know has zero total force on it. Any frame of reference in which the ball appears to obey Newton’s first law is then a valid frame of reference, and to an observer in that frame, Mr. Newton assures us that all the other objects in the universe will obey his laws of motion, not just the ball.

Valid frames of reference, in which Newton’s laws are obeyed, are called inertial frames of reference. Frames of reference that are not inertial are called noninertial frames. In those frames, objects violate the principle of inertia and Newton’s first law. While the truck was moving at constant velocity, both it and the sidewalk were valid inertial frames. The truck became an invalid frame of reference when it began changing its velocity.

You usually assume the ground under your feet is a perfectly inertial frame of reference, and we made that assumption above. It isn’t perfectly inertial, however. Its motion through space is quite complicated, being composed of a part due to the earth’s daily rotation around its own axis, the monthly wobble of the planet caused by the moon’s gravity, and the rotation of the earth around the sun. Since the accelerations involved are numerically small, the earth is approximately a valid inertial frame.

Noninertial frames are avoided whenever possible, and we will seldom, if ever, have occasion to use them in this course. Sometimes, however, a noninertial frame can be convenient. Naval gunners, for instance, get all their data from radars, human eyeballs, and other detection systems that are moving along with the earth’s surface. Since their guns have ranges of many miles, the small discrepancies between their shells’ actual accelerations and the accelerations predicted by Newton’s second law can have effects that accumulate and become significant. In order to kill the people they want to kill, they have to add small corrections onto the equation \( a = \frac{F_{\text{total}}}{m} \). Doing their calculations in an inertial frame would allow them to use the usual form of Newton’s second law, but they would have to convert all their data into a different frame of reference, which would require cumbersome calculations.
Discussion question

A If an object has a linear $x - t$ graph in a certain inertial frame, what is the effect on the graph if we change to a coordinate system with a different origin? What is the effect if we keep the same origin but reverse the positive direction of the $x$ axis? How about an inertial frame moving alongside the object? What if we describe the object’s motion in a noninertial frame?
Summary

Selected vocabulary

- weight . . . . . . . . the force of gravity on an object, equal to $mg$
- inertial frame . . a frame of reference that is not accelerating, one in which Newton’s first law is true
- noninertial frame an accelerating frame of reference, in which Newton’s first law is violated

Notation

- $F_W$ . . . . . . weight

Other terminology and notation

- net force . . . . another way of saying “total force”

Summary

Newton’s first law of motion states that if all the forces acting on an object cancel each other out, then the object continues in the same state of motion. This is essentially a more refined version of Galileo’s principle of inertia, which did not refer to a numerical scale of force.

Newton’s second law of motion allows the prediction of an object’s acceleration given its mass and the total force on it, $a_{em} = F_{\text{total}}/m$. This is only the one-dimensional version of the law; the full-three dimensional treatment will come in chapter 8, Vectors. Without the vector techniques, we can still say that the situation remains unchanged by including an additional set of vectors that cancel among themselves, even if they are not in the direction of motion.

Newton’s laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.
Problems

Key
✓ A computerized answer check is available online.
∫ A problem that requires calculus.
★ A difficult problem.

1 An object is observed to be moving at constant speed in a certain direction. Can you conclude that no forces are acting on it? Explain. [Based on a problem by Serway and Faughn.]

2 At low speeds, every car’s acceleration is limited by traction, not by the engine’s power. Suppose that at low speeds, a certain car is normally capable of an acceleration of 3 m/s². If it is towing a trailer with half as much mass as the car itself, what acceleration can it achieve? [Based on a problem from PSSC Physics.]

3 (a) Let $T$ be the maximum tension that an elevator’s cable can withstand without breaking, i.e., the maximum force it can exert. If the motor is programmed to give the car an acceleration $a$ ($a > 0$ is upward), what is the maximum mass that the car can have, including passengers, if the cable is not to break? ★
(b) Interpret the equation you derived in the special cases of $a = 0$ and of a downward acceleration of magnitude $g$. (“Interpret” means to analyze the behavior of the equation, and connect that to reality, as in the self-check on page 139.)

4 A helicopter of mass $m$ is taking off vertically. The only forces acting on it are the earth’s gravitational force and the force, $F_{air}$, of the air pushing up on the propeller blades.
(a) If the helicopter lifts off at $t = 0$, what is its vertical speed at time $t$?
(b) Check that the units of your answer to part a make sense.
(c) Discuss how your answer to part a depends on all three variables, and show that it makes sense. That is, for each variable, discuss what would happen to the result if you changed it while keeping the other two variables constant. Would a bigger value give a smaller result, or a bigger result? Once you’ve figured out this mathematical relationship, show that it makes sense physically.
(d) Plug numbers into your equation from part a, using $m = 2300$ kg, $F_{air} = 27000$ N, and $t = 4.0$ s.

5 In the 1964 Olympics in Tokyo, the best men’s high jump was 2.18 m. Four years later in Mexico City, the gold medal in the same event was for a jump of 2.24 m. Because of Mexico City’s altitude (2400 m), the acceleration of gravity there is lower than that in Tokyo by about 0.01 m/s². Suppose a high-jumper has a mass of 72 kg.
(a) Compare his mass and weight in the two locations.
(b) Assume that he is able to jump with the same initial vertical
velocity in both locations, and that all other conditions are the same except for gravity. How much higher should he be able to jump in Mexico City?

(Actually, the reason for the big change between '64 and '68 was the introduction of the “Fosbury flop.”)

6 A blimp is initially at rest, hovering, when at \( t = 0 \) the pilot turns on the engine driving the propeller. The engine cannot instantly get the propeller going, but the propeller speeds up steadily. The steadily increasing force between the air and the propeller is given by the equation \( F = kt \), where \( k \) is a constant. If the mass of the blimp is \( m \), find its position as a function of time. (Assume that during the period of time you’re dealing with, the blimp is not yet moving fast enough to cause a significant backward force due to air resistance.)

7 A car is accelerating forward along a straight road. If the force of the road on the car’s wheels, pushing it forward, is a constant 3.0 kN, and the car’s mass is 1000 kg, then how long will the car take to go from 20 m/s to 50 m/s?

8 Some garden shears are like a pair of scissors: one sharp blade slices past another. In the “anvil” type, however, a sharp blade presses against a flat one rather than going past it. A gardening book says that for people who are not very physically strong, the anvil type can make it easier to cut tough branches, because it concentrates the force on one side. Evaluate this claim based on Newton’s laws. [Hint: Consider the forces acting on the branch, and the motion of the branch.]

9 A uranium atom deep in the earth spits out an alpha particle. An alpha particle is a fragment of an atom. This alpha particle has initial speed \( v \), and travels a distance \( d \) before stopping in the earth.

(a) Find the force, \( F \), from the dirt that stopped the particle, in terms of \( v, d \), and its mass, \( m \). Don’t plug in any numbers yet. Assume that the force was constant.

(b) Show that your answer has the right units.

(c) Discuss how your answer to part a depends on all three variables, and show that it makes sense. That is, for each variable, discuss what would happen to the result if you changed it while keeping the other two variables constant. Would a bigger value give a smaller result, or a bigger result? Once you’ve figured out this mathematical relationship, show that it makes sense physically.

(d) Evaluate your result for \( m = 6.7 \times 10^{-27} \text{ kg} \), \( v = 2.0 \times 10^4 \text{ km/s} \), and \( d = 0.71 \text{ mm} \).
10 You are given a large sealed box, and are not allowed to open it. Which of the following experiments measure its mass, and which measure its weight? [Hint: Which experiments would give different results on the moon?]
(a) Put it on a frozen lake, throw a rock at it, and see how fast it scoots away after being hit.
(b) Drop it from a third-floor balcony, and measure how loud the sound is when it hits the ground.
(c) As shown in the figure, connect it with a spring to the wall, and watch it vibrate.

Solution, p. 549

11 While escaping from the palace of the evil Martian emperor, Sally Spacehound jumps from a tower of height $h$ down to the ground. Ordinarily the fall would be fatal, but she fires her blaster rifle straight down, producing an upward force of magnitude $F_B$. This force is insufficient to levitate her, but it does cancel out some of the force of gravity. During the time $t$ that she is falling, Sally is unfortunately exposed to fire from the emperor’s minions, and can’t dodge their shots. Let $m$ be her mass, and $g$ the strength of gravity on Mars.
(a) Find the time $t$ in terms of the other variables.
(b) Check the units of your answer to part a.
(c) For sufficiently large values of $F_B$, your answer to part a becomes nonsense — explain what’s going on.

12 When I cook rice, some of the dry grains always stick to the measuring cup. To get them out, I turn the measuring cup upside-down and hit the “roof” with my hand so that the grains come off of the “ceiling.” (a) Explain why static friction is irrelevant here. (b) Explain why gravity is negligible. (c) Explain why hitting the cup works, and why its success depends on hitting the cup hard enough.

13 At the turn of the 20th century, Samuel Langley engaged in a bitter rivalry with the Wright brothers to develop human flight. Langley’s design used a catapult for launching. For safety, the catapult was built on the roof of a houseboat, so that any crash would be into the water. This design required reaching cruising speed within a fixed, short distance, so large accelerations were required, and the forces frequently damaged the craft, causing dangerous and embarrassing accidents. Langley achieved several uncrewed, unguided flights, but never succeeded with a human pilot. If the force of the catapult is fixed by the structural strength of the plane, and the distance for acceleration by the size of the houseboat, by what factor is the launch velocity reduced when the plane’s 340 kg is augmented by the 60 kg mass of a small man?
14. The tires used in Formula 1 race cars can generate traction (i.e., force from the road) that is as much as 1.9 times greater than with the tires typically used in a passenger car. Suppose that we’re trying to see how fast a car can cover a fixed distance starting from rest, and traction is the limiting factor. By what factor is this time reduced when switching from ordinary tires to Formula 1 tires?

15. In the figure, the rock climber has finished the climb, and his partner is lowering him back down to the ground at approximately constant speed. The following is a student’s analysis of the forces acting on the climber. The arrows give the directions of the forces.

- force of the earth’s gravity, ↓
- force from the partner’s hands, ↑
- force from the rope, ↑

The student says that since the climber is moving down, the sum of the two upward forces must be slightly less than the downward force of gravity.

Correct all mistakes in the above analysis. → Solution, p. 549

16. A bullet of mass \( m \) is fired from a pistol, accelerating from rest to a speed \( v \) in the barrel’s length \( L \).
   (a) What is the force on the bullet? (Assume this force is constant.)
   (b) Check that the units of your answer to part a make sense.
   (c) Check that the dependence of your answer on each of the three variables makes sense.
   [problem by B. Shotwell]

17. Blocks of mass \( M_1 \), \( M_2 \), and \( M_3 \) are stacked on a table as shown in the figure. Let the upward direction be positive.
   (a) What is the force on block 2 from block 3?
   (b) What is the force on block 2 from block 1?
   [problem by B. Shotwell]