

# Lab Manual for Physics 222

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# 1 Electricity

## Apparatus

scotch tape  
rubber rod  
heat lamp  
fur  
bits of paper  
rods and strips of various materials  
30-50 cm rods, and angle brackets, for hanging charged rods  
power supply (Thornton), in lab benches ..1/group  
multimeter (PRO-100), in lab benches .... 1/group  
alligator clips  
flashlight bulbs  
spare fuses for multimeters — Let students replace fuses themselves.

## Goals

Determine the qualitative rules governing electrical charge and forces.

Light up a lightbulb, and measure the current through it and the voltage difference across it.

## Introduction

Newton's law of gravity gave a mathematical formula for the gravitational force, but his theory also made several important non-mathematical statements about gravity:

Every mass in the universe attracts every other mass in the universe.

Gravity works the same for earthly objects as for heavenly bodies.

The force acts at a distance, without any need for physical contact.

Mass is always positive, and gravity is always attractive, not repulsive.

The last statement is interesting, especially because it would be fun and useful to have access to some

negative mass, which would fall up instead of down (like the “upsydaisium” of Rocky and Bullwinkle fame).

Although it has never been found, there is no theoretical reason why a second, negative type of mass can't exist. Indeed, it is believed that the nuclear force, which holds quarks together to form protons and neutrons, involves three qualities analogous to mass. These are facetiously referred to as “red,” “green,” and “blue,” although they have nothing to do with the actual colors. The force between two of the same “colors” is repulsive: red repels red, green repels green, and blue repels blue. The force between two different “colors” is attractive: red and green attract each other, as do green and blue, and red and blue.

When your freshly laundered socks cling together, that is an example of an electrical force. If the gravitational force involves one type of mass, and the nuclear force involves three colors, how many types of electrical “stuff” are there? In the days of Benjamin Franklin, some scientists thought there were two types of electrical “charge” or “fluid,” while others thought there was only a single type. In the first part of this lab, you will try to find out experimentally how many types of electrical charge there are.

The unit of charge is the coulomb, C; one coulomb is defined as the amount of charge such that if two objects, each with a charge of one coulomb, are one meter apart, the magnitude of the electrical force between them is  $9 \times 10^9$  N. Practical applications of electricity usually involve an electric circuit, in which charge is sent around and around in a circle and recycled. Electric current,  $I$ , measures how many coulombs per second flow past a given point; a shorthand for units of C/s is the ampere, A. Voltage,  $V$ , measures the electrical potential energy per unit charge; its units of J/C can be abbreviated as volts, V. Making the analogy between electrical interactions and gravitational ones, voltage is like height. Just as water loses gravitational potential energy by going over a waterfall, electrically charged particles lose electrical potential energy as they flow through a circuit. The second part of this lab involves building an electric circuit to light up a lightbulb, and measuring both the current that flows through the bulb and the voltage difference across it.

## Observations

### A Inferring the rules of electrical repulsion and attraction

Stick a piece of scotch tape on a table, and then lay another piece on top of it. Pull both pieces off the table, and then separate them. If you now bring them close together, you will observe them exerting a force on each other. Electrical effects can also be created by rubbing the fur against the rubber rod.

Your job in this lab is to use these techniques to test various hypotheses about electric charge. The most common difficulty students encounter is that the charge tends to leak off, especially if the weather is humid. If you have charged an object up, you should not wait any longer than necessary before making your measurements. It helps if you keep your hands dry.

To keep this lab from being too long, the class will pool its data for part A. Your instructor will organize the results on the whiteboard.

#### *i. Repulsion and/or attraction*

Test the following hypotheses. Note that they are mutually exclusive, i.e., only one of them can be true.

A) Electrical forces are always attractive.

R) Electrical forces are always repulsive.

AR) Electrical forces are sometimes attractive and sometimes repulsive.

Interpretation: Once the class has tested these hypotheses thoroughly, we will discuss what this implies about how many different types of charge there might be.

#### *ii. Are there forces on objects that have not been specially prepared?*

So far, special preparations have been necessary in order to get objects to exhibit electrical forces. These preparations involved either rubbing objects against each other (against resistance from friction) or pulling objects apart (e.g. overcoming the sticky force that holds the tape together). In everyday life, we do not seem to notice electrical forces in objects that have not been prepared this way.

Now try to test the following hypotheses. Bits of paper are a good thing to use as unprepared objects, since they are light and therefore would be easily moved by any force. *Do not* use tape as an uncharged object, since it can become charged a little bit just by pulling it off the roll.

U0) Objects that have not been specially prepared are immune to electrical forces.

UA) Unprepared objects can participate in electrical forces with prepared objects, and the forces involved are always attractive.

UR) Unprepared objects can participate in electrical forces with prepared objects, and the forces involved are always repulsive.

UAR) Unprepared objects can participate in electrical forces with prepared objects, and the forces involved can be either repulsive or attractive.

These four hypotheses are mutually exclusive.

Once the class has tested these hypotheses thoroughly, we will discuss what practical implications this has for planning the observations for part iii.

#### *iii. Rules of repulsion and/or attraction and the number of types of charge*

Test the following mutually exclusive hypotheses:

1A) There is only one type of electric charge, and the force is always attractive.

1R) There is only one type of electric charge, and the force is always repulsive.

2LR) There are two types of electric charge, call them X and Y. Like charges repel (X repels X and Y repels Y) and opposite charges attract (X and Y attract each other).

2LA) There are two types of electric charge. Like charges attract and opposite charges repel.

3LR) There are three types of electric charge, X, Y and Z. Like charges repel and unlike charges attract.

On the whiteboard, we will make a square table, in which the rows and columns correspond to the different objects you're testing against each other for attraction and repulsion. To test hypotheses 1A through 3LR, you'll need to see if you can successfully explain your whole table by labeling the objects with only one label, X, or whether you need two or three.

Some of the equipment may look identical, but not be identical. In particular, some of the clear rods have higher density than others, which may be because they're made of different types of plastic, or glass. This could affect your conclusions, so you may want to check, for example, whether two rods with the same diameter, that you think are made of the same material, actually weigh the same.

In general, you will find that some materials, and

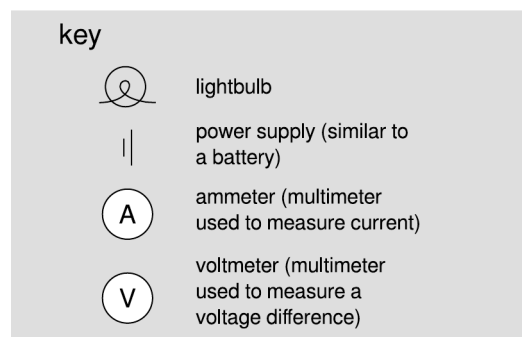
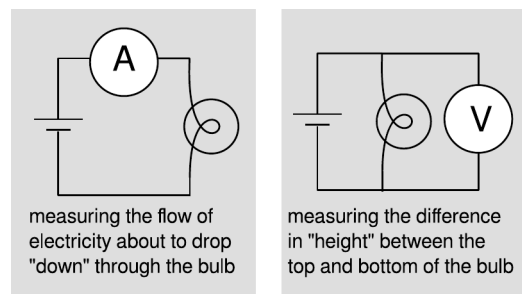
some combinations of materials, are more easily charged than others. For example, if you find that the mahogany rod rubbed with the weasel fur doesn't charge well, then don't keep using it! The white plastic strips tend to work well, so don't neglect them.

Once we have enough data in the table to reach a definite conclusion, we will summarize the results from part A and then discuss the following examples of incorrect reasoning about this lab.

- (1) "The first piece of tape exerted a force on the second, but the second didn't exert one on the first."
- (2) "The first piece of tape repelled the second, and the second attracted the first."
- (3) "We observed three types of charge: two that exert forces, and a third, neutral type."
- (4) "The piece of tape that came from the top was positive, and the bottom was negative."
- (5) "One piece of tape had electrons on it, and the other had protons on it."
- (6) "We know there were two types of charge, not three, because we observed two types of interactions, attraction and repulsion."

## B Measuring current and voltage

As shown in the figure, measuring current and voltage requires hooking the meter into the circuit in two completely different ways.



The arrangement for the ammeter is called a series circuit, because every charged particle that travels the circuit has to go through each component in a row, one after another. The series circuit is arranged

like beads on a necklace.

The setup for the voltmeter is an example of a parallel circuit. A charged particle flowing, say, clockwise around the circuit passes through the power supply and then reaches a fork in the road, where it has a choice of which way to go. Some particles will pass through the bulb, others (not as many) through the meter; all of them are reunited when they reach the junction on the right.

Students tend to have a mental block against setting up the ammeter correctly in series, because it involves breaking the circuit apart in order to insert the meter. To drive home this point, we will act out the process using students to represent the circuit components. If you hook up the ammeter incorrectly, in parallel rather than in series, the meter provides an easy path for the flow of current, so a large amount of current will flow. To protect the meter from this surge, there is a fuse inside, which will blow, and the meter will stop working. This is not a huge tragedy; just ask your instructor for a replacement fuse and open up the meter to replace it.

Unscrew your lightbulb from its holder and look closely at it. Note that it has two separate electrical contacts: one at its tip and one at the metal screw threads.

Turn the power supply's off-on switch to the off position, and turn its (uncalibrated) knob to zero. Set up the basic lightbulb circuit without any meter in it. There is a rack of cables in the back of the room with banana-plug connectors on the end, and most of your equipment accepts these plugs. To connect to the two brass screws on the lightbulb's base, you'll need to stick alligator clips on the banana plugs.

Check your basic circuit with your instructor, then turn on the power switch and *slowly* turn up the knob until the bulb lights. The knob is uncalibrated and highly nonlinear; as you turn it up, the voltage it produces goes zerozerozerozerozerozero*six!* To light the bulb without burning it out, you will need to find a position for the knob in the narrow range where it rapidly ramps up from 0 to 6 V.

Once you have your bulb lit, do not mess with the knob on the power supply anymore. You do not even need to switch the power supply off while rearranging the circuit for the two measurements with the meter; the voltage that lights the bulb is only about a volt or a volt and a half (similar to a battery), so it can't hurt you.

We have a single meter that plays both the role of



the voltmeter and the role of the ammeter in this lab. Because it can do both these things, it is referred to as a multimeter. Multimeters are highly standardized, and the following instructions are generic ones that will work with whatever meters you happen to be using in this lab.

#### *Voltage difference*

Two wires connect the meter to the circuit. At the places where three wires come together at one point, you can plug a banana plug into the back of another banana plug. At the meter, make one connection at the “common” socket (“COM”) and the other at the socket labeled “V” for volts. The common plug is called that because it is used for every measurement, not just for voltage.

Many multimeters have more than one scale for measuring a given thing. For instance, a meter may have a millivolt scale and a volt scale. One is used for measuring small voltage differences and the other for large ones. You may not be sure in advance what scale is appropriate, but that’s not a big problem — once everything is hooked up, you can try different scales and see what’s appropriate. Use the switch or buttons on the front to select one of the voltage scales. By trial and error, find the most precise scale that doesn’t cause the meter to display an error message about being overloaded.

Write down your measurement, with the units of volts, and stop for a moment to think about what it is that you’ve measured. Imagine holding your breath and trying to make your eyeballs pop out with the pressure. Intuitively, the voltage difference is like the pressure difference between the inside and outside of your body.

What do you think will happen if you unscrew the bulb, leaving an air gap, while the power supply and the voltmeter are still going? Try it. Interpret your observation in terms of the breath-holding metaphor.

#### *Current*

The procedure for measuring the current differs only because you have to hook the meter up in series and because you have to use the “A” (amps) plug on the meter and select a current scale.

In the breath-holding metaphor, the number you’re measuring now is like the rate at which air flows through your lips as you let it hiss out. Based on this metaphor, what do you think will happen to the reading when you unscrew the bulb? Try it.

Discuss with your group and check with your in-

structor:

(1) What *goes through* the wires? Current? Voltage? Both?

(2) Using the breath-holding metaphor, explain why the voltmeter needs *two* connections to the circuit, not just one. What about the ammeter?

While waiting for your instructor to come around and discuss these questions with you, you can go on to the next part of the lab.

#### *Resistance*

The ratio of voltage difference to current is called the resistance of the bulb,  $R = \Delta V/I$ . Its units of volts per amp can be abbreviated as ohms,  $\Omega$  (capital Greek letter omega).

Calculate the resistance of your lightbulb. Resistance is the electrical equivalent of kinetic friction. Just as rubbing your hands together heats them up, objects that have electrical resistance produce heat when a current is passed through them. This is why the bulb’s filament gets hot enough to heat up.

When you unscrew the bulb, leaving an air gap, what is the resistance of the air?

Ohm’s law is a generalization about the electrical properties of a variety of materials. It states that the resistance is constant, i.e., that when you increase the voltage difference, the flow of current increases exactly in proportion. If you have time, test whether Ohm’s law holds for your lightbulb, by cutting the voltage to half of what you had before and checking whether the current drops by the same factor. (In this condition, the bulb’s filament doesn’t get hot enough to create enough visible light for your eye to see, but it does emit infrared light.)

#### *List of materials for static electricity*

You don’t have to know anything about what the various materials are in order to do this lab, but here is a list for use by instructors and the lab technician:

- scotch tape (used as two different objects, top and bottom)
- teflon fabric (brown, coarse)
- teflon rods (white, rigid, slippery, skinny)
- PVC pipe
- polyurethane rods (brown, flexible)
- nylon (?) fabric (blue)
- fur

## Notes For Next Week

(1) Next week, when you turn in your writeup for this lab, you also need to turn in a prelab writeup for the next lab. The prelab questions are listed at the end of the description of that lab in the lab manual. Never start a lab without understanding the answers to all the prelab questions; if you turn in partial answers or answers you're unsure of, discuss the questions with your instructor or with other students to make sure you understand what's going on.

(2) You should exchange phone numbers with your lab partners for general convenience throughout the semester. You can also get each other's e-mail addresses by logging in to Spotter and clicking on "e-mail."

## Rules and Organization

Collection of raw data is work you share with your lab partners. Once you're done collecting data, you need to do your own analysis. E.g., it is not okay for two people to turn in the same calculations, or on a lab requiring a graph for the whole group to make one graph and turn in copies.

You'll do some labs as formal writeups, others as informal "check-off" labs. As described in the syllabus, they're worth different numbers of points, and you have to do a certain number of each type by the end of the semester.

The format of formal lab writeups is given in appendix 1 on page 60. The raw data section must be contained in your bound lab notebook. Typically people word-process the abstract section, and any other sections that don't include much math, and stick the printout in the notebook to turn it in. The calculations and reasoning section will usually just consist of hand-written calculations you do in your lab notebook. You need two lab notebooks, because on days when you turn one in, you need your other one to take raw data in for the next lab. You may find it convenient to leave one or both of your notebooks in the cupboard at your lab bench whenever you don't need to have them at home to work on; this eliminates the problem of forgetting to bring your notebook to school.

For a check-off lab, the main thing I'll pay attention to is your abstract. The rest of your work for a check-off lab can be informal, and I may not ask to see it unless I think there's a problem after reading your abstract.



# 2 Electrical Resistance

## Apparatus

- DC power supply (Thornton) .....1/group
- digital multimeters (Fluke and HP) .....2/group
- resistors, various values
- unknown electrical components
- alligator clips
- spare fuses for multimeters — Let students replace fuses themselves.

## Goals

- Measure curves of voltage versus current for three objects: your body and two unknown electrical components.
- Determine whether they are ohmic, and if so, determine their resistances.

## Introduction

Your nervous system depends on electrical currents, and every day you use many devices based on electrical currents without even thinking about it. Despite its ordinariness, the phenomenon of electric currents passing through liquids (e.g., cellular fluids) and solids (e.g., copper wires) is a subtle one. For example, we now know that atoms are composed of smaller, subatomic particles called electrons and nuclei, and that the electrons and nuclei are electrically charged, i.e., matter is electrical. Thus, we now have a picture of these electrically charged particles sitting around in matter, ready to create an electric current by moving in response to an externally applied voltage. Electricity had been used for practical purposes for a hundred years, however, before the electrical nature of matter was proven at the turn of the 20th century.

Another subtle issue involves Ohm’s law,




$$I = \frac{\Delta V}{R} ,$$


where  $\Delta V$  is the voltage difference applied across an object (e.g., a wire), and  $I$  is the current that flows in response. A piece of copper wire, for instance, has a constant value of  $R$  over a wide range of voltages. Such materials are called ohmic. Materials with non-constant are called non-ohmic. The interesting question is why so many materials are ohmic.

Since we know that electrons and nuclei are bound together to form atoms, it would be more reasonable to expect that small voltages, creating small electric fields, would be unable to break the electrons and nuclei away from each other, and no current would flow at all — only with fairly large voltages should the atoms be split up, allowing current to flow. Thus we would expect  $R$  to be infinite for small voltages, and small for large voltages, which would not be ohmic behavior. It is only within the last 50 years that a good explanation has been achieved for the strange observation that nearly all solids and liquids are ohmic.

## Terminology, Schematics, and Resistor Color Codes

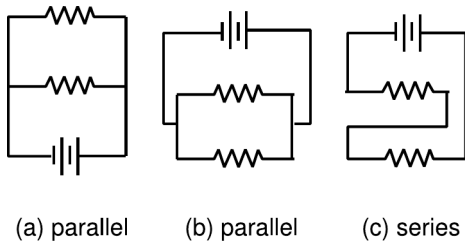
The word “resistor” usually implies a specific type of electrical component, which is a piece of ohmic material with its shape and composition chosen to give a desired value of  $R$ . Any piece of an ohmic substance, however, has a constant value of  $R$ , and therefore in some sense constitutes a “resistor.” The wires in a circuit have electrical resistance, but the resistance is usually negligible (a small fraction of an Ohm for several centimeters of wire).

The usual symbol for a resistor in an electrical schematic is this , but some recent schematics use this . The symbol  represents a fixed

source of voltage such as a battery, while  represents an adjustable voltage source, such as the power supply you will use in this lab.

In a schematic, the lengths and shapes of the lines representing wires are completely irrelevant, and are usually unrelated to the physical lengths and shapes of the wires. The physical behavior of the circuit does not depend on the lengths of the wires (unless the length is so great that the resistance of the wire becomes non-negligible), and the schematic is not meant to give any information other than that needed to understand the circuit’s behavior. All that really matters is what is connected to what.

For instance, the schematics (a) and (b) above are completely equivalent, but (c) is different. In the first two circuits, current heading out from the bat-



tery can “choose” which resistor to enter. Later on, the two currents join back up. Such an arrangement is called a parallel circuit. In the bottom circuit, a series circuit, the current has no “choice” — it must first flow through one resistor and then the other.

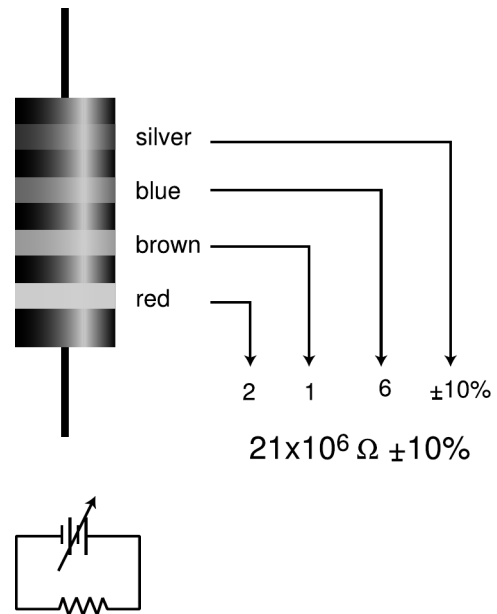
Resistors are usually too small to make it convenient to print numerical resistance values on them, so they are labeled with a color code, as shown in the table and example below.

color	meaning
black	0
brown	1
red	2
orange	3
yellow	4
green	5
blue	6
violet	7
gray	8
white	9
silver	$\pm 10\%$
gold	$\pm 5\%$

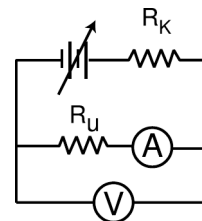
## Setup

Obtain your two unknowns from your instructor. Group 1 will use unknowns 1A and 1B, group 2 will use 2A and 2B, and so on.

Here is a simplified version of the basic circuit you will use for your measurements of  $I$  as a function of  $\Delta V$ . Although I’ve used the symbol for a resistor, the objects you are using are not necessarily resistors, or even ohmic.



Here is the actual circuit, with the meters included. In addition to the unknown resistance  $R_U$ , a known resistor  $R_K$  ( $\sim 1\text{k}\Omega$  is fine) is included to limit the possible current that will flow and keep from blowing fuses or burning out the unknown resistance with too much current. This type of current-limiting application is one of the main uses of resistors.



## Observations

### A Unknown component A

Set up the circuit shown above with unknown component A. Most of your equipment accepts the banana plugs that your cables have on each end, but to connect to  $R_U$  and  $R_K$  you need to stick alligator clips on the banana plugs. See Appendix 6 for information about how to set up and use the two multimeters. Do not use the pointy probes that come with the multimeters, because there is no convenient way to attach them to the circuit — just use the banana plug cables. Note when you need three wires to come together at one point, you can plug a banana plug into the back of another banana plug.

Measure  $I$  as a function of  $\Delta V$ . Make sure to take

measurements for both positive and negative voltages.

Often when we do this lab, it's the first time in several months that the meters have been used. The small hand-held meters have a battery, which may be dead. Check the battery icon on the LCD screen.

## B Unknown component B

Repeat for unknown component B.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

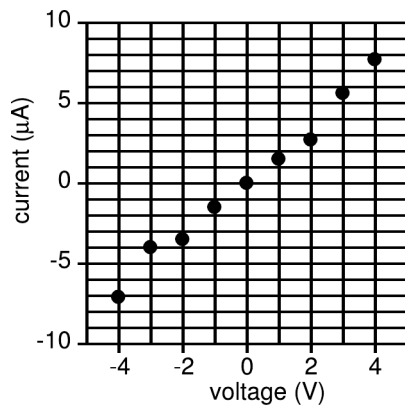
**P1** Check that you understand the interpretations of the following color-coded resistor labels:

blue gray orange silver	= $68 \text{ k}\Omega \pm 10\%$
blue gray orange gold	= $68 \text{ k}\Omega \pm 5\%$
blue gray red silver	= $6.8 \text{ k}\Omega \pm 10\%$
black brown blue silver	= $1 \text{ M}\Omega \pm 10\%$

Now interpret the following color code:

green orange yellow silver = ?

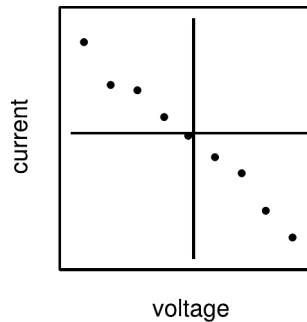
**P2** Fit a line to the following sample data and use the slope to extract the resistance (see Appendix 4).



Your result should be consistent with a resistor color code of green-violet-yellow.

**P3** Plan how you will measure  $I$  versus  $\Delta V$  for both positive and negative values of  $\Delta V$ , since the power supply only supplies positive voltages.

**P4** Would data like these indicate a negative resistance, or did the experimenter just hook something up wrong? If the latter, explain how to fix it.



**P5** Explain why the following statement about the resistor  $R_K$  is incorrect: “You have to make  $R_K$  small compared to  $R_U$ , so it won't affect things too much.”

## Analysis

Graph  $I$  versus  $\Delta V$  for all three unknowns. Decide which ones are ohmic and which are non-ohmic. For the ones that are ohmic, extract a value for the resistance (see appendix 4). Don't bother with analysis of random errors, because the main source of error in this lab is the systematic error in the calibration of the multimeters (and in part C the systematic error from the subject's fidgeting).

## Programmed Introduction to Practical Electrical Circuits

Physics courses in general are compromises between the fundamental and the practical, between exploring the basic principles of the physical universe and developing certain useful technical skills. Although the electricity and magnetism labs in this manual are structured around the sequence of abstract theoretical concepts that make up the backbone of the lecture course, it's important that you develop certain practical skills as you go along. Not only will they come in handy in real life, but the later parts of this lab manual are written with the assumption that you will have developed them.

As you progress in the lab course, you will find that the instructions on how to construct and use circuits become less and less explicit. The goal is not to make you into an electronics technician, but neither should you emerge from this course able only to flip the switches and push the buttons on prepackaged consumer electronics. To use a mechanical analogy, the level of electrical sophistication you're intended to reach is not like the ability to rebuild a car engine but more like being able to check your own oil.

In addition to the physics-based goals stated at the beginning of this section, you should also be developing the following skills in lab this week:

- (1) Be able to translate back and forth between schematics and actual circuits.
- (2) Use a multimeter (discussed in Appendix 6), given an explicit schematic showing how to connect it to a circuit.

Further practical skills will be developed in the following lab.

# 3 The Loop and Junction Rules

## Apparatus

DC power supply (Thornton) ..... 1/group  
multimeter (PRO-100, in lab benches) .... 1/group  
resistors

## Goal

Test the loop and junction rules in two electrical circuits.

## Introduction

If you ask physicists what are the most fundamentally important principles of their science, almost all of them will start talking to you about conservation laws. A conservation law is a statement that a certain measurable quantity cannot be changed. A conservation law that is easy to understand is the conservation of mass. No matter what you do, you cannot create or destroy mass.

The two conservation laws with which we will be concerned in this lab are conservation of energy and conservation of charge. Energy is related to voltage, because voltage is defined as  $V = PE/q$ . Charge is related to current, because current is defined as  $I = \Delta q/\Delta t$ .

Conservation of charge has an important consequence for electrical circuits:

When two or more wires come together at a point in a DC circuit, the total current entering that point equals the total current leaving it.

Such a coming-together of wires in a circuit is called a junction. If the current leaving a junction was, say, greater than the current entering, then the junction would have to be creating electric charge out of nowhere. (Of course, charge could have been stored up at that point and released later, but then it wouldn't be a DC circuit — the flow of current would change over time as the stored charge was used up.)

Conservation of energy can also be applied to an electrical circuit. The charge carriers are typically electrons in copper wires, and an electron has a potential energy equal to  $-eV$ . Suppose the electron sets off on a journey through a circuit made of re-

sistors. Passing through the first resistor, our subatomic protagonist passes through a voltage difference of  $\Delta V_1$ , so its potential energy changes by  $-e\Delta V_1$ . To use a human analogy, this would be like going up a hill of a certain height and gaining some gravitational potential energy. Continuing on, it passes through more voltage differences,  $-e\Delta V_2$ ,  $-e\Delta V_3$ , and so on. Finally, in a moment of religious transcendence, the electron realizes that life is one big circuit — you always end up coming back where you started from. If it passed through  $N$  resistors before getting back to its starting point, then the total change in its potential energy was

$$-e(\Delta V_1 + \dots + \Delta V_N)$$

But just as there is no such thing as a round-trip hike that is all downhill, it is not possible for the electron to have any net change in potential energy after passing through this loop — if so, we would have created some energy out of nothing. Since the total change in the electron's potential energy must be zero, it must be true that  $\Delta V_1 + \dots + \Delta V_N = 0$ . This is the loop rule:

The sum of the voltage differences around any closed loop in a circuit must equal zero.

When you are hiking, there is an important distinction between uphill and downhill, which depends entirely on which direction you happen to be traveling on the trail. Similarly, it is important when applying the loop rule to be consistent about the signs you give to the voltage differences, say positive if the electron sees an increase in voltage and negative if it sees a decrease along its direction of motion.

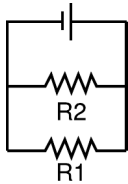
## Observations

### A The junction rule

Construct a circuit like the one in the figure, using the Thornton power supply as your voltage source. To make things more interesting, don't use equal resistors. Use resistors with values in the range of about 1 k $\Omega$  to 10 M $\Omega$ . If they're much higher than that, the currents will be too low for the PRO-100 meters to measure accurately. If they're much smaller than that, you could burn up the resistors, and the multimeter's internal resistance when used as an ammeter might not be negligible in comparison. Insert your multimeter in the circuit to measure all three



currents that you need in order to test the junction rule.



### B The loop rule

Now come up with a circuit to test the loop rule. Since the loop rule is always supposed to be true, it's hard to go wrong here! Make sure that (1) you have at least three resistors in a loop, (2) the whole circuit is not just a single loop, and (3) you hook in the power supply in a way that creates non-zero voltage differences across all the resistors. Measure the voltage differences you need to measure to test the loop rule. Here it is best to use fairly small resistances, so that the multimeter's large internal resistance when used in parallel as a voltmeter will not significantly reduce the resistance of the circuit. Do not use resistances of less than about  $100\ \Omega$ , however, or you may blow a fuse or burn up a resistor.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Draw a schematic showing where you will insert the multimeter in the circuit to measure the currents in part A.

**P2** Invent a circuit for part B, and draw a schematic. You need not indicate actual resistor values, since you will have to choose from among the values actually available in lab.

**P3** Pick a loop from your circuit, and draw a schematic showing how you will attach the multimeter in the circuit to measure the voltage differences in part B.

**P4** Explain why the following statement is incorrect: "We found that the loop rule was not quite true, but the small error could have been because the resistor's value was off by a few percent compared to the color-code value."

## Self-Check

Do the analysis in lab.

## Analysis

Discuss whether you think your observations agree with the loop and junction rules, taking into account systematic and random errors.

## Programmed Introduction to Practical Electrical Circuits

The following practical skills are developed in this lab:

(1) Use a multimeter without being given an explicit schematic showing how to connect it to your circuit. This means connecting it in parallel in order to measure voltages and in series in order to measure currents.

(2) Use your understanding of the loop and junction rules to simplify electrical measurements. These rules often guarantee that you can get the same current or voltage reading by measuring in more than one place in a circuit. In real life, it is often much easier to connect a meter to one place than another, and you can therefore save yourself a lot of trouble using the rules.

# 4 Electric Fields

## Apparatus

board and U-shaped probe  
ruler  
DC power supply (Thornton)  
multimeter  
scissors  
stencils for drawing electrode shapes on paper

## Goals

To be better able to visualize electric fields and understand their meaning.

To examine the electric fields around certain charge distributions.

## Introduction

By definition, the electric field,  $E$ , at a particular point equals the force on a test charge at that point divided by the amount of charge,  $E = F/q$ . We can plot the electric field around any charge distribution by placing a test charge at different locations and making note of the direction and magnitude of the force on it. The direction of the electric field at any point P is the same as the direction of the force on a positive test charge at P. The result would be a page covered with arrows of various lengths and directions, known as a “sea of arrows” diagram..

In practice, Radio Shack does not sell equipment for preparing a known test charge and measuring the force on it, so there is no easy way to measure electric fields. What really is practical to measure at any given point is the voltage,  $V$ , defined as the electrical energy (potential energy) that a test charge would have at that point, divided by the amount of charge ( $E/Q$ ). This quantity would have units of J/C (Joules per Coulomb), but for convenience we normally abbreviate this combination of units as volts. Just as many mechanical phenomena can be described using either the language of force or the language of energy, it may be equally useful to describe electrical phenomena either by their electric fields or by the voltages involved.

Since it is only ever the difference in potential energy (interaction energy) between two points that

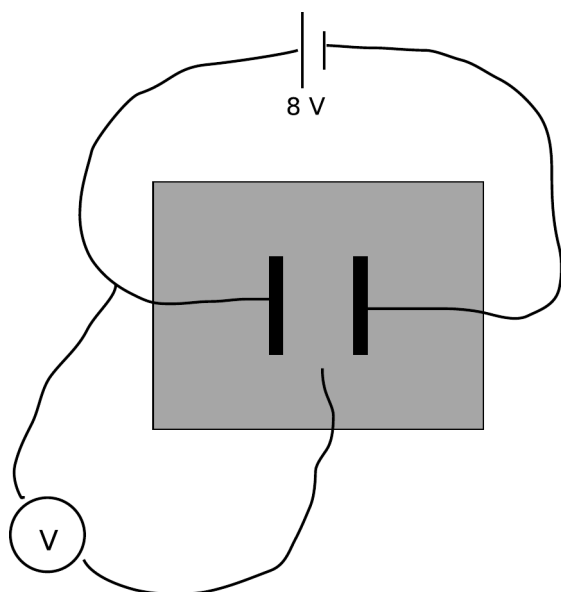
can be defined unambiguously, the same is true for voltages. Every voltmeter has two probes, and the meter tells you the difference in voltage between the two places at which you connect them. Two points have a nonzero voltage difference between them if it takes work (either positive or negative) to move a charge from one place to another. If there is a voltage difference between two points in a conducting substance, charges will move between them just like water will flow if there is a difference in levels. The charge will always flow in the direction of lower potential energy (just like water flows downhill).

All of this can be visualized most easily in terms of maps of constant-voltage curves (also known as equipotentials); you may be familiar with topographical maps, which are very similar. On a topographical map, curves are drawn to connect points having the same height above sea level. For instance, a cone-shaped volcano would be represented by concentric circles. The outermost circle might connect all the points at an altitude of 500 m, and inside it you might have concentric circles showing higher levels such as 600, 700, 800, and 900 m. Now imagine a similar representation of the voltage surrounding an isolated point charge. There is no “sea level” here, so we might just imagine connecting one probe of the voltmeter to a point within the region to be mapped, and the other probe to a fixed reference point very far away. The outermost circle on your map might connect all the points having a voltage of 0.3 V relative to the distant reference point, and within that would lie a 0.4-V circle, a 0.5-V circle, and so on. These curves are referred to as constant-voltage curves, because they connect points of equal voltage. In this lab, you are going to map out constant-voltage curves, but not just for an isolated point charge, which is just a simple example like the idealized example of a conical volcano.

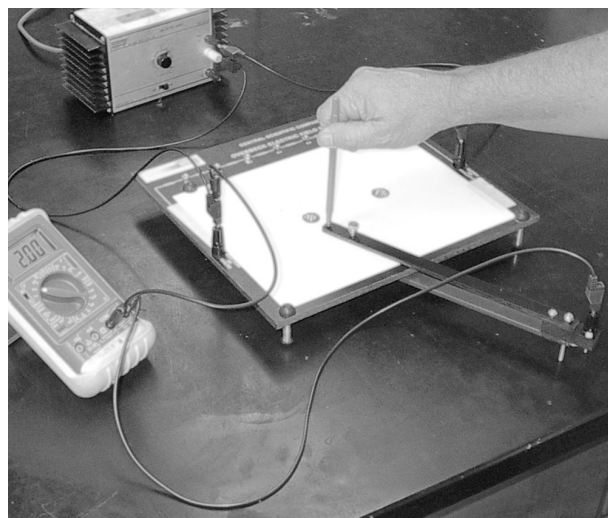
You could move a charge along a constant-voltage curve in either direction without doing any work, because you are not moving it to a place of higher potential energy. If you do not do any work when moving along a constant-voltage curve, there must not be a component of electric force along the surface (or you would be doing work). A metal wire is a constant-voltage curve. We know that electrons in a metal are free to move. If there were a force along the wire, electrons would move because of it. In fact the electrons would move until they were distributed

in such a way that there is no longer any force on them. At that point they would all stay put and then there would be no force along the wire and it would be a constant-voltage curve. (More generally, any flat piece of conductor or any three-dimensional volume consisting of conducting material will be a constant-voltage region.)

There are geometrical and numerical relationships between the electric field and the voltage, so even though the voltage is what you'll measure directly in this lab, you can also relate your data to electric fields. Since there is not any component of electric force parallel to a constant-voltage curve, electric field lines always pass through constant-voltage curves at right angles. (Analogously, a stream flowing straight downhill will cross the lines on a topographical map at right angles.) Also, if you divide the work equation  $(\Delta\text{energy}) = Fd$  by  $q$ , you get  $(\Delta\text{energy})/q = (F/q)d$ , which translates into  $\Delta V = -Ed$ . (The minus sign is because  $V$  goes down when some other form of energy is released.) This means that you can find the electric field strength at a point  $P$  by dividing the voltage difference between the two constant-voltage curves on either side of  $P$  by the distance between them. You can see that units of  $\text{V}/\text{m}$  can be used for the  $E$  field as an alternative to the units of  $\text{N}/\text{C}$  suggested by its definition — the units are completely equivalent.



A simplified schematic of the apparatus, being used with pattern 1 on page 20.



A photo of the apparatus, being used with pattern 3 on page 20.

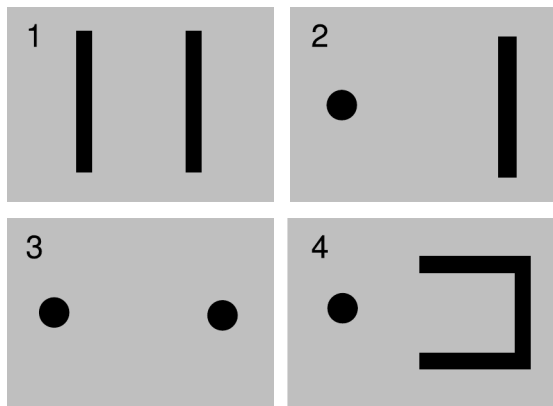
## Method

The first figure shows a simplified schematic of the apparatus. The power supply provides an 8 V voltage difference between the two metal electrodes, drawn in black. A voltmeter measures the voltage difference between an arbitrary reference voltage and a point of interest in the gray area around the electrodes. The result will be somewhere between 0 and 8 V. A voltmeter won't actually work if it's not part of a complete circuit, but the gray area is intentionally made from a material that isn't a very good insulator, so enough current flows to allow the voltmeter to operate.

The photo shows the actual apparatus. The electrodes are painted with silver paint on a detachable board, which goes underneath the big board. What you actually see on top is just a piece of paper on which you'll trace the equipotentials with a pen. The voltmeter is connected to a U-shaped probe with a metal contact that slides underneath the board, and a hole in the top piece for your pen.

Turn your large board upside down. Find the small detachable board with the parallel-plate capacitor pattern (pattern 1 on page 20) on it, and screw it to the underside of the equipotential board, with the silver-painted side facing down toward the tabletop. Use the washers to protect the silver paint so that it doesn't get scraped off when you tighten the screws. Now connect the voltage source (using the provided wires) to the two large screws on either side of the board. Connect the multimeter so that you can mea-

sure the voltage difference across the terminals of the voltage source. Adjust the voltage source to give 8 volts.



If you press down on the board, you can slip the paper between the board and the four buttons you see at the corners of the board. Tape the paper to your board, because the buttons aren't very dependable. There are plastic stencils in some of the envelopes, and you can use these to draw the electrodes accurately onto your paper so you know where they are. The photo, for example, shows pattern 3 traced onto the paper.

Now put the U-probe in place so that the top is above the equipotential board and the bottom of it is below the board. You will first be looking for places on the pattern board where the voltage is one volt — look for places where the meter reads 1.0 and mark them through the hole on the top of your U-probe with a pencil or pen. You should find a whole bunch of places there the voltage equals one volt, so that you can draw a nice constant-voltage curve connecting them. (If the line goes very far or curves strangely, you may have to do more.) You can then repeat the procedure for 2 V, 3 V, and so on. Label each constant-voltage curve. Once you've finished tracing the equipotentials, everyone in your group will need one copy of each of the two patterns you do, so you will need to photocopy them or simply trace them by hand.

If you're using the PRO-100 meters, they will try to outsmart you by automatically choosing a range. Most people find this annoying. To defeat this misfeature, press the RANGE button, and you'll see the AUTO indicator on the screen turn off.

Repeat this procedure with another pattern. Groups 1 and 4 should do patterns 1 and 2; groups 2 and 5 patterns 1 and 3; groups 3, 6, and 7 patterns 1 and 4.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Looking at a plot of constant-voltage curves, how could you tell where the strongest electric fields would be? (Don't just say that the field is strongest when you're close to "the charge," because you may have a complex charge distribution, and we don't have any way to see or measure the charge distribution.)

**P2** What would the constant-voltage curves look like in a region of uniform electric field (i.e., one in which the  $\mathbf{E}$  vectors are all the same strength, and all in the same direction)?

## Self-Check

Calculate at least one numerical electric field value to make sure you understand how to do it.

You have probably found some constant-voltage curves that form closed loops. Do the electric field patterns ever seem to close back on themselves? Make sure you understand why or why not.

Make sure the people in your group all have a copy of each pattern.

## Analysis

On each plot, find the strongest and weakest electric fields, and calculate them.

On top of your plots, draw in electric field vectors. You will then have two different representations of the field superimposed on one another.

As always when drawing vectors, the lengths of the arrows should represent the magnitudes of the vectors, although you don't need to calculate them all numerically or use an actual scale. Remember that electric field vectors are always perpendicular to constant-voltage curves. The electric field lines point from high voltage to low voltage, just as the force on a rolling ball points downhill.



# 5 Magnetism

## Apparatus

- bar magnet (stack of 6 Nd)
- compass
- Hall effect magnetic field probes
- LabPro interfaces, DC power supplies, and USB cables
- 2-meter stick
- Heath solenoids .....2/group
- Mastech power supply .....1/group
- wood blocks .....2/group
- PRO-100 multimeter (in lab bench .....1/group
- another multimeter .....1/group
- D-cell batteries and holders
- Cenco decade resistor box .....1/group

## Goal

Find how the magnetic field of a magnet changes with distance along one of the magnet's lines of symmetry.

## Introduction

### A Variation of Field With Distance: Deflection of a Magnetic Compass

You can infer the strength of the bar magnet's field at a given point by putting the compass there and seeing how much it is deflected from north.

The task can be simplified quite a bit if you restrict yourself to measuring the magnetic field at points along one of the magnet's two lines of symmetry, shown in the top figure on the page three pages after this one.

If the magnet is flipped across the vertical axis, the north and south poles remain just where they were, and the field is unchanged. That means the entire magnetic field is also unchanged, and the field at a point such as point b, along the line of symmetry, must therefore point straight up.

If the magnet is flipped across the horizontal axis, then the north and south poles are swapped, and the field everywhere has to reverse its direction. Thus, the field at points along this axis, e.g., point a, must point straight up or down.

Line up your magnet so it is pointing east-west.

Choose one of the two symmetry axes of your magnet, and measure the deflection of the compass at two points along that axis, as shown in the figure at the end of the lab. As part of your prelab, you will use vector addition to find an equation for  $B_m/B_e$ , the magnet's field in units of the Earth's, in terms of the deflection angle  $\theta$ . For your first point, find the distance  $r$  at which the deflection is 70 degrees; this angle is chosen because it's about as big as it can be without giving very poor relative precision in the determination of the magnetic field. For your second data-point, use twice that distance. By what factor does the field decrease when you double  $r$ ?

The lab benches contain iron or steel parts that distort the magnetic field. You can easily observe this simply by putting a compass on the top of the bench and sliding it around to different places. To work around this problem, lay a 2-meter stick across the space between two lab benches, and carry out the experiment along the line formed by the stick. Even in the air between the lab benches, the magnetic field due to the building materials in the building is significant, and this field varies from place to place. Therefore you should move the magnet while keeping the compass in one place. Then the field from the building becomes a fixed part of the background experienced by the compass, just like the earth's field.

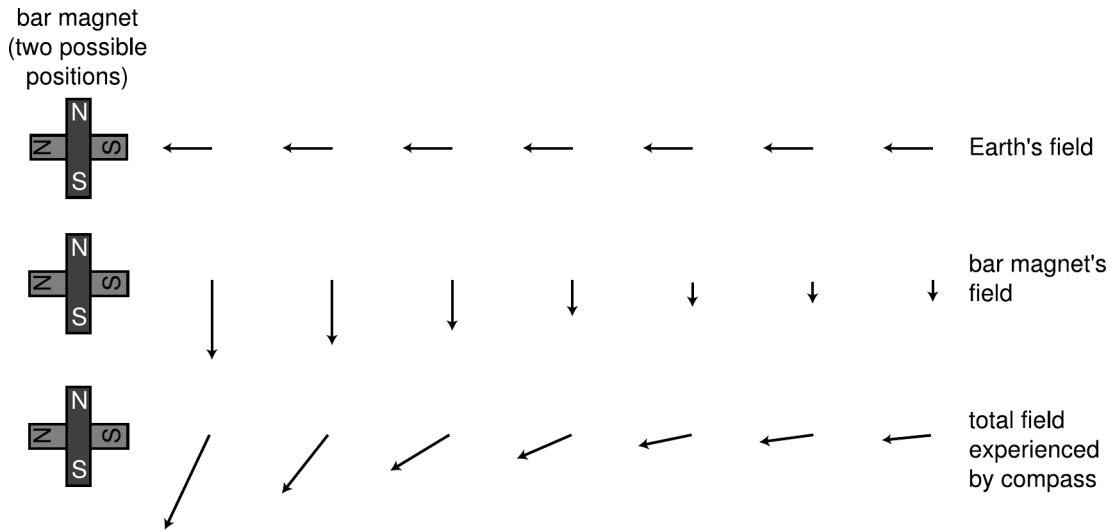
Note that the measurements are very sensitive to the relative position and orientation of the bar magnet and compass.

Based on your two data-points, form a hypothesis about the variation of the magnet's field with distance according to a power law  $B \propto r^p$ .

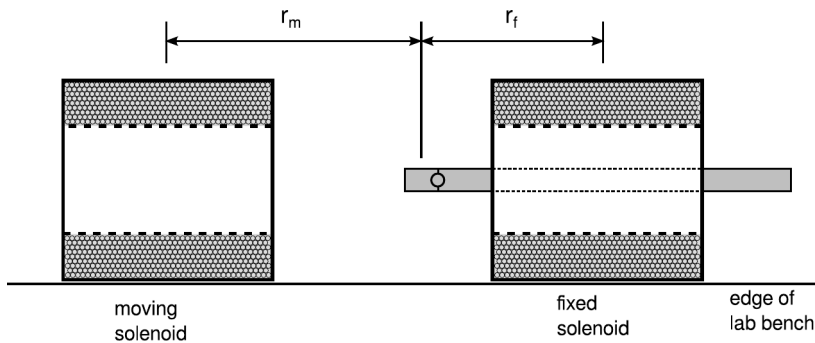
### B Variation of Field With Distance: Hall Effect Magnetometer

In this part of the lab, you will test your hypothesis about the power law relationship  $B \propto r^p$ ; you will find out whether the field really does obey such a law, and if it does, you will determine  $p$  accurately.

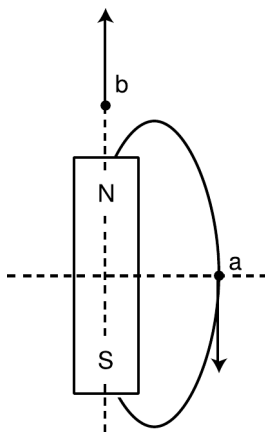
This part of the lab uses a device called a Hall effect magnetometer for measuring magnetic fields. It works by sending an electric current through a substance, and measuring the force exerted on those moving charges by the surrounding magnetic field. The probe only measures the component of the magnetic field vector that is parallel to its own axis. Plug the probe into CH 1 of the LabPro interface, connect



Part A, measuring the variation of the bar magnet's field with respect to distance.



Part B, a different method of measuring the variation of field with distance. The solenoids are shown in cross-section, with empty space on their interiors and their axes running right-left.



the interface to the computer's USB port, and plug the interface's DC power supply in to it. Start up version 3 of Logger Pro, and it will automatically recognize the probe and start displaying magnetic fields on the screen, in units of mT (millitesla). The

probe has two ranges, one that can read fields up to 0.3 mT, and one that goes up to 6.4 mT. Select the more sensitive 0.3 mT scale using the switch on the probe.

The technique is shown in the bottom figure on the last page of the lab. Identical solenoids (cylindrical coils of wire) are positioned with their axes coinciding, by lining up their edges with the edge of the lab bench. When an electrical current passes through a coil, it creates a magnetic field. At distances that are large compared to the size of the solenoid, we expect that this field will have the same universal pattern as with any magnetic dipole. The sensor is positioned on the axis, with wood blocks (not shown) to hold it up. One solenoid is fixed, while the other is moved to different positions along the axis, including positions (more distant than the one shown) at which we expect its contribution to the field at the sensor to be of the universal dipole form.

The key to the high precision of the measurement is that in this configuration, the fields of the two solenoids can be made to cancel at the position of the probe. Because of the solenoids' unequal distances from the probe, this requires unequal currents. Because the fields cancel, the probe can be used on its most sensitive and accurate scale; it can also be zeroed when the circuits are open, so that the effect of any ambient field is removed. For example, suppose that at a certain distance  $r_m$ , the current  $I_m$  through the moving coil has to be five times greater than the current  $I_f$  through the fixed coil at the constant distance  $r_f$ . Then we have determined that the field pattern of these coils is such that increasing the distance along the axis from  $r_f$  to  $r_m$  causes the field to fall off by a factor of five.

It's a good idea to take data all the way down to  $r_m = 0$ , since this makes it possible to see on a graph where the field does and doesn't behave like a dipole. Note that the distances  $r_f$  and  $r_m$  can't be measured directly with good precision.

The Mastech power supply is capable of delivering a large amount of current, so it can be used to provide  $I_m$ , which needs to be high when  $r_m$  is large. The power supply has some strange behavior that makes it not work unless you power it up in exactly the right way. It has four knobs, going from left to right: (1) current regulation, (2) over-voltage protection, (3) fine voltage control, (4) coarse voltage control. Before turning the power supply on, turn knobs 1 and 2 all the way up, and knobs 3 and 4 all the way down. Turn the power supply on. Now use knobs 3 and 4 to control how much current flows.

At large values of  $r_m$ , it can be difficult to get a power supply to give a *small* enough  $I_f$ . Try using a battery, and further reducing the current by placing another resistance in series with the coil. The Cenco decade resistance boxes can be used for this purpose; they are variable resistors whose resistance can be dialed up as desired using decimal knobs. Use the plugs on the resistance box labeled H and L.

For every current measurement, make sure to use the most sensitive possible scale on the meter to get as many sig figs as possible. This is why the ammeter built into the Mastech power supply is not useful here. I found it to be a hassle to measure  $I_m$  with an ammeter, because the currents required were often quite large, and I kept inadvertently blowing the fuse on the milliamp scale. For this reason, you may actually want to measure  $V_m$ , the voltage difference across the moving solenoid. Conceptually, magnetic fields are caused by moving charges, cur-

rent is a measure of moving charge, and therefore current is what is relevant here. But if the DC resistance of the coil is fixed, the current and voltage are proportional to one another, assuming that the voltage is measured directly across the coil and the resistance of the banana-plug connections is either negligible or constant.

As shown in a lecture demonstration, deactivating the electromagnet requires getting rid of the energy stored in the magnetic field, and this can be done in more than one way. If you use your hand to break the circuit by pulling out a banana plug, the energy is dissipated in a spark, and a large value of  $I_m$  is being used the result can be an unpleasant shock. To avoid this, deactivate the moving coil by turning down the knob on the power supply rather than by breaking the circuit.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** In part A, suppose that when the compass is 11.0 cm from the magnet, it is 45 degrees away from north. What is the strength of the bar magnet's field at this location in space, in units of the Earth's field?

**P2** Find  $B_m/B_e$  in terms of the deflection angle  $\theta$  measured in part A. As a special case, you should be able to recover your answer to P1.

## Analysis

Determine the variation of the solenoid's magnetic field with distance. Look for a power-law relationship using the log-log graphing technique described in appendix 5. Does the power law hold for all the distances you investigated, or only at large distances? No error analysis is required.





# 6 The Oscilloscope

## Apparatus

oscilloscope (Tektronix TDS 1001B) ..... 1/group  
microphone (RS 33-1067) ..... for 6 groups  
microphone (Shure C606) ..... for 1 group  
PI-9587C sine wave generator ..... 1/group  
various tuning forks, mounted on wooden boxes

If there's an equipment conflict with respect to the sine wave generators, the HP200CD sine wave generators can be used instead.

## Goals

- Learn to use an oscilloscope.
- Observe sound waves on an oscilloscope.

## Introduction

One of the main differences you will notice between your second semester of physics and the first is that many of the phenomena you will learn about are not directly accessible to your senses. For example, electric fields, the flow of electrons in wires, and the inner workings of the atom are all invisible. The oscilloscope is a versatile laboratory instrument that can indirectly help you to see what's going on.

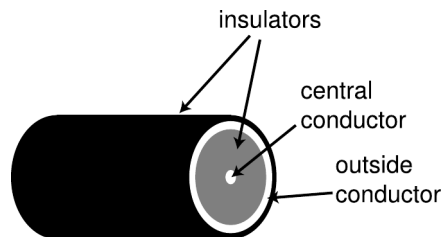
### *The Oscilloscope*

An oscilloscope graphs an electrical signal that varies as a function of time. The graph is drawn from left to right across the screen, being painted in real time as the input signal varies. In this lab, you will be using the signal from a microphone as an input, allowing you to see sound waves.

The input signal is supplied in the form of a voltage. You are already familiar with the term "voltage" from common speech, but you may not have learned the formal definition yet in the lecture course. Voltage, measured in metric units of volts (V), is defined as the electrical potential energy per unit charge. For instance if 2 nC of charge flows from one terminal of a 9-volt battery to the other terminal, the potential energy consumed equals 18 nJ. To use a mechanical analogy, when you blow air out between your lips, the flowing air is like an electrical current,

and the difference in pressure between your mouth and the room is like the difference in voltage. For the purposes of this lab, it is not really necessary for you to work with the fundamental definition of voltage.

The input connector on the front of the oscilloscope accepts a type of cable known as a BNC cable. A BNC cable is a specific example of coaxial cable ("coax"), which is also used in cable TV, radio, and computer networks. The electric current flows in one direction through the central conductor, and returns in the opposite direction through the outside conductor, completing the circuit. The outside conductor is normally kept at ground, and also serves as shielding against radio interference. The advantage of coaxial cable is that it is capable of transmitting rapidly varying signals without distortion.

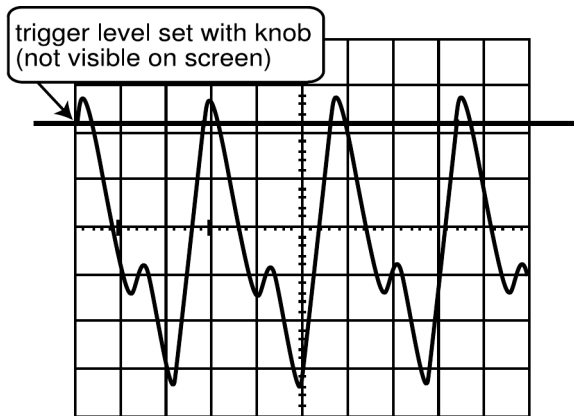


Most of the voltages we wish to measure are not big enough to use directly for the vertical deflection voltage, so the oscilloscope actually amplifies the input voltage, i.e., the small input voltage is used to control a much larger voltage generated internally. The amount of amplification is controlled with a knob on the front of the scope. For instance, setting the knob on 1 mV selects an amplification such that 1 mV at the input deflects the electron beam by one square of the 1-cm grid. Each 1-cm division is referred to as a "division."

### *The Time Base and Triggering*

Since the X axis represents time, there also has to be a way to control the time scale, i.e., how fast the imaginary "penpoint" sweeps across the screen. For instance, setting the knob on 10 ms causes it to sweep across one square in 10 ms. This is known as the time base.

In the figure, suppose the time base is 10 ms. The scope has 10 divisions, so the total time required for



the beam to sweep from left to right would be 100 ms. This is far too short a time to allow the user to examine the graph. The oscilloscope has a built-in method of overcoming this problem, which works well for periodic (repeating) signals. The amount of time required for a periodic signal to perform its pattern once is called the period. With a periodic signal, all you really care about seeing what one period or a few periods in a row look like — once you've seen one, you've seen them all. The scope displays one screenful of the signal, and then keeps on overlaying more and more copies of the wave on top of the original one. Each trace is erased when the next one starts, but is being overwritten continually by later, identical copies of the wave form. You simply see one persistent trace.

How does the scope know when to start a new trace? If the time for one sweep across the screen just happened to be exactly equal to, say, four periods of the signal, there would be no problem. But this is unlikely to happen in real life — normally the second trace would start from a different point in the waveform, producing an offset copy of the wave. Thousands of traces per second would be superimposed on the screen, each shifted horizontally by a different amount, and you would only see a blurry band of light.

To make sure that each trace starts from the same point in the waveform, the scope has a triggering circuit. You use a knob to set a certain voltage level, the trigger level, at which you want to start each trace. The scope waits for the input to move across the trigger level, and then begins a trace. Once that trace is complete, it pauses until the input crosses the trigger level again. To make extra sure that it is really starting over again from the same point in the waveform, you can also specify whether you want to start on an increasing voltage or a decreasing voltage — otherwise there would always be at least two

points in a period where the voltage crossed your trigger level.

## Setup

To start with, we'll use a sine wave generator, which makes a voltage that varies sinusoidally with time. This gives you a convenient signal to work with while you get the scope working. Use the black and white outputs on the PI-9587C.

The figure on the last page is a simplified drawing of the front panel of a digital oscilloscope, showing only the most important controls you'll need for this lab. When you turn on the oscilloscope, it will take a while to start up.

### *Preliminaries:*

Press **DEFAULT SETUP**.

Use the **SEC/DIV** knob to put the time base on something reasonable compared to the period of the signal you're looking at. The time base is displayed on the screen, e.g., 10 ms/div, or 1 s/div.

Use the **VOLTS/DIV** knob to put the voltage scale (Y axis) on a reasonable scale compared to the amplitude of the signal you're looking at.

The scope has two channels, i.e., it can accept input through two BNC connectors and display both or either. You'll only be using channel 1, which is the only one represented in the simplified drawing. By default, the oscilloscope draws graphs of both channels' inputs; to get rid of ch. 2, hold down the **CH 2 MENU** button (not shown in the diagram) for a couple of seconds. You also want to make sure that the scope is triggering on CH 1, rather than CH 2. To do that, press the **TRIG MENU** button, and use an option button to select CH 1 as the source. Set the triggering mode to normal, which is the mode in which the triggering works as I've described above. If the trigger level is set to a level that the signal never actually reaches, you can play with the knob that sets the trigger level until you get something. A quick and easy way to do this without trial and error is to use the **SET TO 50%** button, which automatically sets the trigger level to midway between the top and bottom peaks of the signal.

You want to select AC, not DC or GND, on the channel you're using. You are looking at a voltage that is alternating, creating an alternating current, "AC." The "DC" setting is only necessary when dealing with constant or very slowly varying voltages. The "GND" simply draws a graph using  $y = 0$ , which is only useful in certain situations, such as when you can't find the trace. To select AC, press the CH 1 MENU button, and select AC coupling.

Observe the effect of changing the voltage scale and time base on the scope. Try changing the frequency and amplitude on the sine wave generator.

You can freeze the display by pressing RUN/STOP, and then unfreeze it by pressing the button again.

## Preliminary Observations

Now try observing signals from the microphone.

Notes for the group that uses the Shure mic: As with the Radio Shack mics, polarity matters. The tip of the phono plug connector is the live connection, and the part farther back from the tip is the grounded part. You can connect onto the phono plug with alligator clips.

Once you have your setup working, try measuring the period and frequency of the sound from a tuning fork, and make sure your result for the frequency is the same as what's written on the tuning fork.

## Observations

### A Periodic and nonperiodic speech sounds

Try making various speech sounds that you can sustain continuously: vowels or certain consonants such as "sh," "r," "f" and so on. Which are periodic and which are not?

Note that the names we give to the letters of the alphabet in English are not the same as the speech sounds represented by the letter. For instance, the English name for "f" is "ef," which contains a vowel, "e," and a consonant, "f." We are interested in the basic speech sounds, not the names of the letters. Also, a single letter is often used in the English writing system to represent two sounds. For example, the word "I" really has two vowels in it, "aaah" plus "eee."

### B Loud and soft

What differentiates a loud "aaah" sound from a soft one?

### C High and low pitch

Try singing a vowel, and then singing a higher note with the same vowel. What changes?

### D Differences among vowel sounds

What differentiates the different vowel sounds?

### E Lowest and highest notes you can sing

What is the lowest frequency you can sing, and what is the highest?

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** In the sample oscilloscope trace shown on page 31, what is the period of the waveform? What is its frequency? The time base is 10 ms.

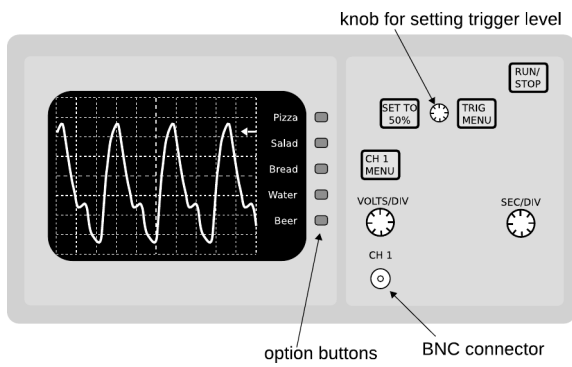
**P2** In the same example, again assume the time base is 10 ms/division. The voltage scale is 2 mV/division. Assume the zero voltage level is at the middle of the vertical scale. (The whole graph can actually be shifted up and down using a knob called "position.") What is the trigger level currently set to? If the trigger level was changed to 2 mV, what would happen to the trace?

**P3** Referring to the chapter of your textbook on sound, which of the following would be a reasonable time base to use for an audio-frequency signal? 10 ns, 1  $\mu$  s, 1 ms, 1 s

**P4** Does the oscilloscope show you the signal's period, or its wavelength? Explain.

## Analysis

The format of the lab writeup can be informal. Just describe clearly what you observed and concluded.



A simplified diagram of the controls on a digital oscilloscope.

# 7 Electromagnetism

## Apparatus

oscilloscope (Tektronix TDS 1001B) ..... 1/group  
microphone (RS 33-1067) ..... for 6 groups  
microphone (Shure C606) ..... for 1 group  
various tuning forks, mounted on wooden boxes  
solenoid (Heath) ..... 1/group  
2-meter wire with banana plugs ..... 1/group  
magnet (stack of 6 Nd) ..... 1/group  
masking tape  
string

## Goals

- Learn to use an oscilloscope.
- Observe electric fields induced by changing magnetic fields.
- Build a generator.
- Discover Lenz's law.

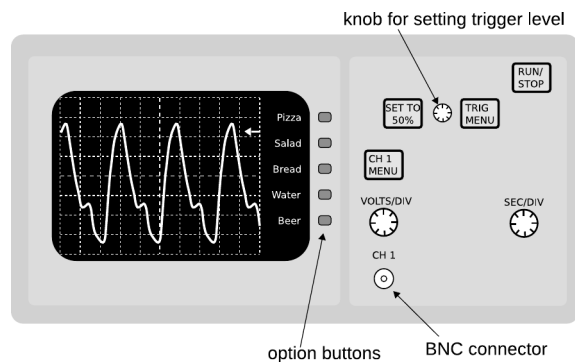
## Introduction

Physicists hate complication, and when physicist Michael Faraday was first learning physics in the early 19th century, an embarrassingly complex aspect of the science was the multiplicity of types of forces. Friction, normal forces, gravity, electric forces, magnetic forces, surface tension — the list went on and on. Today, 200 years later, ask a physicist to enumerate the fundamental forces of nature and the most likely response will be “four: gravity, electromagnetism, the strong nuclear force and the weak nuclear force.” Part of the simplification came from the study of matter at the atomic level, which showed that apparently unrelated forces such as friction, normal forces, and surface tension were all manifestations of electrical forces among atoms. The other big simplification came from Faraday's experimental work showing that electric and magnetic forces were intimately related in previously unexpected ways, so intimately related in fact that we now refer to the two sets of force-phenomena under a single term, “electromagnetism.”

Even before Faraday, Oersted had shown that there was at least some relationship between electric and

magnetic forces. An electrical current creates a magnetic field, and magnetic fields exert forces on an electrical current. In other words, electric forces are forces of charges acting on charges, and magnetic forces are forces of moving charges on moving charges. (Even the magnetic field of a bar magnet is due to currents, the currents created by the orbiting electrons in its atoms.)

Faraday took Oersted's work a step further, and showed that the relationship between electricity and magnetism was even deeper. He showed that a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. Faraday's work forms the basis for such technologies as the transformer, the electric guitar, the transformer, and generator, and the electric motor. It also led to the understanding of light as an electromagnetic wave.

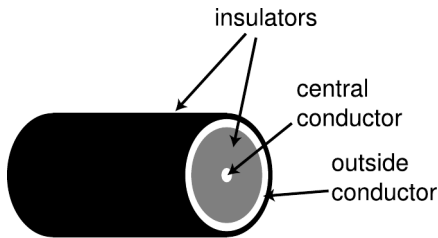


### The Oscilloscope

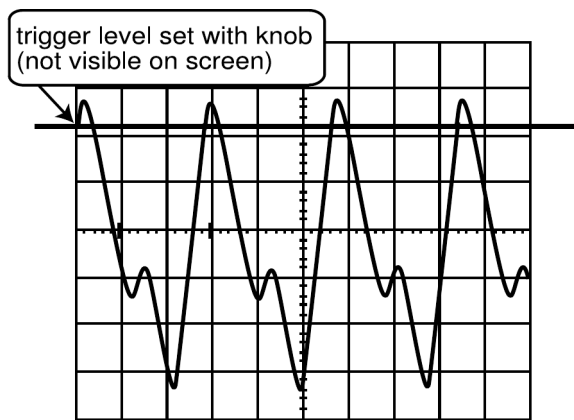
An oscilloscope graphs an electrical signal that varies as a function of time. The graph is drawn from left to right across the screen, being painted in real time as the input signal varies. The purpose of the oscilloscope in this lab is to measure electromagnetic induction, but to get familiar with the oscilloscope, we'll start out by using the signal from a microphone as an input, allowing you to see sound waves.

The input signal is supplied in the form of a voltage. The input connector on the front of the oscilloscope accepts a type of cable known as a BNC cable. A BNC cable is a specific example of coaxial cable (“coax”), which is also used in cable TV, radio, and computer networks. The electric current flows in one direction through the central conductor, and

returns in the opposite direction through the outside conductor, completing the circuit. The outside conductor is normally kept at ground, and also serves as shielding against radio interference. The advantage of coaxial cable is that it is capable of transmitting rapidly varying signals without distortion.



Most of the voltages we wish to measure are not big enough to use directly for the vertical deflection voltage, so the oscilloscope actually amplifies the input voltage, i.e., the small input voltage is used to control a much larger voltage generated internally. The amount of amplification is controlled with a knob on the front of the scope. For instance, setting the knob on 1 mV selects an amplification such that 1 mV at the input deflects the electron beam by one square of the 1-cm grid. Each 1-cm division is referred to as a “division.”



### The Time Base and Triggering

Since the  $X$  axis represents time, there also has to be a way to control the time scale, i.e., how fast the imaginary “penpoint” sweeps across the screen. For instance, setting the knob on 10 ms causes it to sweep across one square in 10 ms. This is known as the time base.

In the figure, suppose the time base is 10 ms. The scope has 10 divisions, so the total time required for the beam to sweep from left to right would be 100 ms. This is far too short a time to allow the user

to examine the graph. The oscilloscope has a built-in method of overcoming this problem, which works well for periodic (repeating) signals. The amount of time required for a periodic signal to perform its pattern once is called the period. With a periodic signal, all you really care about seeing what one period or a few periods in a row look like — once you’ve seen one, you’ve seen them all. The scope displays one screenful of the signal, and then keeps on overlaying more and more copies of the wave on top of the original one. Each trace is erased when the next one starts, but is being overwritten continually by later, identical copies of the wave form. You simply see one persistent trace.

How does the scope know when to start a new trace? If the time for one sweep across the screen just happened to be exactly equal to, say, four periods of the signal, there would be no problem. But this is unlikely to happen in real life — normally the second trace would start from a different point in the waveform, producing an offset copy of the wave. Thousands of traces per second would be superimposed on the screen, each shifted horizontally by a different amount, and you would only see a blurry band of light.

To make sure that each trace starts from the same point in the waveform, the scope has a triggering circuit. You use a knob to set a certain voltage level, the trigger level, at which you want to start each trace. The scope waits for the input to move across the trigger level, and then begins a trace. Once that trace is complete, it pauses until the input crosses the trigger level again. To make extra sure that it is really starting over again from the same point in the waveform, you can also specify whether you want to start on an increasing voltage or a decreasing voltage — otherwise there would always be at least two points in a period where the voltage crossed your trigger level.

## Setup

To start with, we’ll use a sine wave generator, which makes a voltage that varies sinusoidally with time. This gives you a convenient signal to work with while you get the scope working.

The figure on the preceding page is a simplified drawing of the front panel of a digital oscilloscope, showing only the most important controls you’ll need for this lab. When you turn on the oscilloscope, it will take a while to start up.

## Preliminaries:

Press DEFAULT SETUP.

Use the SEC/DIV knob to put the time base on something reasonable compared to the period of the signal you're looking at. The time base is displayed on the screen, e.g., 10 ms/div, or 1 s/div.

Use the VOLTS/DIV knob to put the voltage scale (Y axis) on a reasonable scale compared to the amplitude of the signal you're looking at.

The scope has two channels, i.e., it can accept input through two BNC connectors and display both or either. You'll only be using channel 1, which is the only one represented in the simplified drawing. By default, the oscilloscope draws graphs of both channels' inputs; to get rid of ch. 2, hold down the CH 2 MENU button (not shown in the diagram) for a couple of seconds. You also want to make sure that the scope is triggering on CH 1, rather than CH 2. To do that, press the TRIG MENU button, and use an option button to select CH 1 as the source. Set the triggering mode to normal, which is the mode in which the triggering works as I've described above. If the trigger level is set to a level that the signal never actually reaches, you can play with the knob that sets the trigger level until you get something. A quick and easy way to do this without trial and error is to use the SET TO 50% button, which automatically sets the trigger level to midway between the top and bottom peaks of the signal.

You want to select AC, not DC or GND, on the channel you're using. You are looking at a voltage that is alternating, creating an alternating current, "AC." The "DC" setting is only necessary when dealing with constant or very slowly varying voltages. The "GND" simply draws a graph using  $y = 0$ , which is only useful in certain situations, such as when you can't find the trace. To select AC, press the CH 1 MENU button, and select AC coupling.

Observe the effect of changing the voltage scale and time base on the scope. Try changing the frequency and amplitude on the sine wave generator.

You can freeze the display by pressing RUN/STOP, and then unfreeze it by pressing the button again.

## Preliminary Observations

Now try observing signals from the microphone.

Once you have your setup working, try measuring the period and frequency of the sound from a tuning fork, and make sure your result for the frequency is the same as what's written on the tuning fork.

## Qualitative Observations

In this lab you will use a permanent magnet to produce changing magnetic fields. This causes an electric field to be induced, which you will detect using a solenoid (spool of wire) connected to an oscilloscope. The electric field drives electrons around the solenoid, producing a current which is detected by the oscilloscope.

Note that although I've described the standard way of triggering a scope, when the time base is very long, triggering becomes unnecessary. These scopes are programmed so that when the time base is very long, they simply continuously display traces.

### A A constant magnetic field

Do you detect any signal on the oscilloscope when the magnet is simply placed at rest inside the solenoid? Try the most sensitive voltage scale.

### B A changing magnetic field

Do you detect any signal when you move the magnet or wiggle it inside the solenoid or near it? What happens if you change the speed at which you move the magnet?

### C Moving the solenoid

What happens if you hold the magnet still and move the solenoid?

The poles of the magnet are its flat faces. In later parts of the lab you will need to know which is north. Determine this now by hanging it from a string and seeing how it aligns itself with the Earth's field. The pole that points north is called the north pole of the magnet. The field pattern funnels into the body of the magnet through its south pole, and reemerges at its north pole.

### D A generator

Tape the magnet securely to the eraser end of a pencil so that its flat face (one of its two poles) is like the head of a hammer, and mark the north and south poles of the magnet for later reference. Spin the pencil near the solenoid and observe the induced signal.



You have built a generator. (I have unfortunately not had any luck lighting a lightbulb with the setup, due to the relatively high internal resistance of the solenoid.)

## Trying Out Your Understanding

### E Changing the speed of the generator

If you change the speed at which you spin the pencil, you will of course cause the induced signal to have a longer or shorter period. Does it also have any effect on the *amplitude* of the wave?

### F A solenoid with fewer loops

Use the two-meter cable to make a second solenoid with the same diameter but fewer loops. Compare the strength of the induced signals.

### G Dependence on distance

How does the signal picked up by your generator change with distance?

Try to explain what you have observed, and discuss your interpretations with your instructor.

## Lenz's Law

Lenz's law describes how the clockwise or counterclockwise direction of the induced electric field's whirlpool pattern relates to the changing magnetic field. The main result of this lab is a determination of how Lenz's law works. To focus your reasoning, here are four possible forms for Lenz's law:

1. The electric field forms a pattern that is clockwise when viewed along the direction of the  $B$  vector of the changing magnetic field.
2. The electric field forms a pattern that is counterclockwise when viewed along the direction of the  $B$  vector of the changing magnetic field.
3. The electric field forms a pattern that is clockwise when viewed along the direction of the  $\Delta B$  vector of the changing magnetic field.
4. The electric field forms a pattern that is counterclockwise when viewed along the direction of the  $\Delta B$  vector of the changing magnetic field.

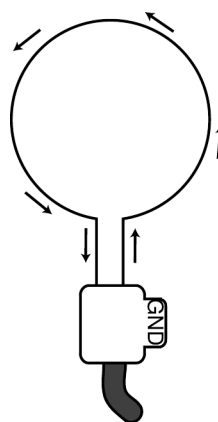
Your job is to figure out which is correct.

The most direct way to figure out Lenz's law is to make a tomahawk-chopping motion that ends up with the magnet in the solenoid, observing whether the pulse induced is positive or negative. What hap-

pens when you reverse the chopping motion, or when you reverse the north and south poles of the magnet? Try all four possible combinations and record your results.

To set up the scope, press DEFAULT SETUP. This should have the effect of setting the scope on DC coupling, which is what you want. (If it's on AC coupling, it tries to filter out any DC part of the input signals, which distorts the results.) To check that you're on DC coupling, you can do CH 1 MENU, and check that Coupling says DC. Set the triggering mode ("Mode") to Auto.

Make sure the scope is on DC coupling, not AC coupling, or your pulses will be distorted.



It can be tricky to make the connection between the polarity of the signal on the screen of the oscilloscope and the direction of the electric field pattern. The figure shows an example of how to interpret a positive pulse: the current must have flowed through the scope from the center conductor of the coax cable to its outer conductor (marked GND on the coax-to-banana converter).

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** In the sample oscilloscope trace shown on page 31, what is the period of the waveform? What is its frequency? The time base is 10 ms.

**P2** In the same example, again assume the time base is 10 ms/division. The voltage scale is 2 mV/division. Assume the zero voltage level is at the middle

of the vertical scale. (The whole graph can actually be shifted up and down using a knob called “position.”) What is the trigger level currently set to? If the trigger level was changed to 2 mV, what would happen to the trace?

**P3** Referring to the chapter of your textbook on sound, which of the following would be a reasonable time base to use for an audio-frequency signal? 10 ns,  $1\mu$  s, 1 ms, 1 s

**P4** Does the oscilloscope show you the signal’s period, or its wavelength? Explain. [Skip this question if you’re in Physics 222.]

**P5** The time-scale for all the signals is determined by the fact that you’re wiggling and waving the magnet by hand, so what’s a reasonable order of magnitude to choose for the time base on the oscilloscope? [Skip this question if you’re in Physics 222.]

## Self-Check

Determine which version of Lenz’s law is correct.



# 8 The Charge to Mass Ratio of the Electron

## Apparatus

vacuum tube with Helmholtz coils (Leybold) .....	1
Cenco 33034 HV supply .....	1
12-V DC power supplies (Thornton) .....	1
multimeters (Fluke or HP) .....	2
compass .....	1
ruler .....	1
banana-plug cables	

## Goal

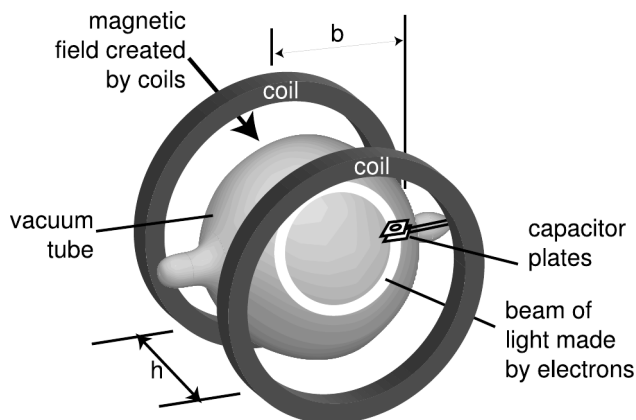
Measure the charge-to-mass ratio of the electron.

## Introduction

Why should you believe electrons exist? By the turn of the twentieth century, not all scientists believed in the literal reality of atoms, and few could imagine smaller objects from which the atoms themselves were constructed. Over two thousand years had elapsed since the Greeks first speculated that atoms existed based on philosophical arguments without experimental evidence. During the Middle Ages in Europe, “atomism” had been considered highly suspect, and possibly heretical. Finally by the Victorian era, enough evidence had accumulated from chemical experiments to make a persuasive case for atoms, but subatomic particles were not even discussed.

If it had taken two millennia to settle the question of atoms, it is remarkable that another, subatomic level of structure was brought to light over a period of only about five years, from 1895 to 1900. Most of the crucial work was carried out in a series of experiments by J.J. Thomson, who is therefore often considered the discoverer of the electron.

In this lab, you will carry out a variation on a crucial experiment by Thomson, in which he measured the ratio of the charge of the electron to its mass,  $q/m$ . The basic idea is to observe a beam of electrons in a region of space where there is an approximately uniform magnetic field,  $B$ . The electrons are emitted perpendicular to the field, and, it turns out, travel in a circle in a plane perpendicular to it. The force



of the magnetic field on the electrons is

$$F = qvB \quad , \quad (1)$$

directed towards the center of the circle. Their acceleration is

$$a = \frac{v^2}{r} \quad , \quad (2)$$

so using  $F = ma$ , we can write

$$qvB = \frac{mv^2}{r} \quad . \quad (3)$$

If the initial velocity of the electrons is provided by accelerating them through a voltage difference  $V$ , they have a kinetic energy equal to  $qV$ , so

$$\frac{1}{2}mv^2 = qV \quad . \quad (4)$$

From equations 3 and 4, you can determine  $q/m$ . Note that since the force of a magnetic field on a moving charged particle is always perpendicular to the direction of the particle’s motion, the magnetic field can never do any work on it, and the particle’s KE and speed are therefore constant.

You will be able to see where the electrons are going, because the vacuum tube is filled with a hydrogen gas at a low pressure. Most electrons travel large distances through the gas without ever colliding with a hydrogen atom, but a few do collide, and the atoms then give off blue light, which you can see. Although I will loosely refer to “seeing the beam,” you are really seeing the light from the collisions, not the beam of electrons itself. The manufacturer of the

tube has put in just enough gas to make the beam visible; more gas would make a brighter beam, but would cause it to spread out and become too broad to measure it precisely.

The field is supplied by an electromagnet consisting of two circular coils, each with 130 turns of wire (the same on all the tubes we have). The coils are placed on the same axis, with the vacuum tube at the center. A pair of coils arranged in this type of geometry are called Helmholtz coils. Such a setup provides a nearly uniform field in a large volume of space between the coils, and that space is more accessible than the inside of a solenoid.

### Safety

You will use the Cenco high-voltage supply to make a DC voltage of about 300 V. Two things automatically keep this from being very dangerous:

Several hundred DC volts are far less dangerous than a similar AC voltage. The household AC voltages of 110 and 220 V are more dangerous because AC is more readily conducted by body tissues.

The HV supply will blow a fuse if too much current flows.

Do the high voltage safety checklist, Appendix 7, tear it out, and turn it in at the beginning of lab. If you don't understand something, don't initial that point, and ask your instructor for clarification before you start the lab.

## Setup

Before beginning, make sure you do not have any computer disks near the apparatus, because the magnetic field could erase them.

Heater circuit: As with all vacuum tubes, the cathode is heated to make it release electrons more easily. There is a separate low-voltage power supply built into the high-voltage supply. It has a set of green plugs that, in different combinations, allow you to get various low voltage values. Use it to supply 6 V to the terminals marked "heater" on the vacuum tube. The tube should start to glow.

Electromagnet circuit: Connect the other Thornton power supply, in series with an ammeter, to the terminals marked "coil." The current from this power supply goes through both coils to make the magnetic

field. Verify that the magnet is working by using it to deflect a nearby compass.

High-voltage circuit: Leave the Cenco HV supply unplugged. It is really three HV circuits in one box. You'll be using the circuit that goes up to 500 V. Connect it to the terminals marked "anode." Ask your instructor to check your circuit. Now plug in the HV supply and turn up the voltage to 300 V. You should see the electron beam. If you don't see anything, try it with the lights dimmed.

## Observations

Make the necessary observations in order to find  $q/m$ , carrying out your plan to deal with the effects of the Earth's field. The high voltage is supposed to be 300 V, but to get an accurate measurement of what it really is you'll need to use a multimeter rather than the poorly calibrated meter on the front of the high voltage supply.

The beam can be measured accurately by using the glass rod inside the tube, which has a centimeter scale marked on it.

Be sure to compute  $q/m$  before you leave the lab. That way you'll know you didn't forget to measure something important, and that your result is reasonable compared to the currently accepted value.

There is a glass rod inside the vacuum tube with a centimeter scale on it, so you can measure the diameter  $d$  of the beam circle simply by looking at the place where the glowing beam hits the scale. This is much more accurate than holding a ruler up to the tube, because it eliminates the parallax error that would be caused by viewing the beam and the ruler along a line that wasn't perpendicular to the plane of the beam. However, the manufacturing process used in making these tubes (they're probably hand-blown by a glass blower) isn't very precise, and on many of the tubes you can easily tell by comparison with the a ruler that, e.g., the 10.0 cm point on the glass rod is not really 10.0 cm away from the hole from which the beam emerges. Past students have painstakingly determined the appropriate corrections,  $a$ , to add to the observed diameters by the following electrical method. If you look at your answer to prelab question P1, you'll see that the product  $Br$  is always a fixed quantity in this experiment. It therefore follows that  $Id$  is also supposed to be constant. They measured  $I$  and  $d$  at two different values of  $I$ , and determined the correction  $a$  that had to be added to their  $d$  values in order to make the two values of  $Id$  equal. The results are as follows:

<i>serial number</i>	<i>a</i> (cm)
98-16	0.0
9849	0.0
99-08	+0.15
99-10	-0.2
99-17	+0.2
99-56	+0.3
031427	-0.3

If your apparatus is one that hasn't already had its  $a$  determined, then you should do the necessary measurements to calibrate it.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

The week before you are to do the lab, briefly familiarize yourself visually with the apparatus.

Do the high voltage safety checklist, Appendix 7, tear it out, and turn it in at the beginning of lab. If you don't understand something, don't initial that point, and ask your instructor for clarification before you start the lab.

**P1** Derive an equation for  $q/m$  in terms of  $V$ ,  $r$  and  $B$ .

**P2** For an electromagnet consisting of a single circular loop of wire of radius  $b$ , the field at a point on its axis, at a distance  $z$  from the plane of the loop, is given by

$$B = \frac{2\pi k I b^2}{c^2(b^2 + z^2)^{3/2}} .$$

Starting from this equation, derive an equation for the magnetic field at the center of a pair of Helmholtz coils. Let the number of turns in each coil be  $N$  (in our case,  $N = 130$ ), let their radius be  $b$ , and let the distance between them be  $h$ . (In the actual experiment, the electrons are never exactly on the axis of the Helmholtz coils. In practice, the equation you will derive is sufficiently accurate as an approximation to the actual field experienced by the electrons.) If you have trouble with this derivation, see your instructor in his/her office hours.

**P3** Find the currently accepted value of  $q/m$  for the electron.

**P4** The electrons will be affected by the Earth's magnetic field, as well as the (larger) field of the

coils. Devise a plan to eliminate, correct for, or at least estimate the effect of the Earth's magnetic field on your final  $q/m$  value.

**P5** Of the three circuits involved in this experiment, which ones need to be hooked up with the right polarity, and for which ones is the polarity irrelevant?

**P6** What would you infer if you found the beam of electrons formed a helix rather than a circle?

## Analysis

Determine  $q/m$ , with error bars.

Answer the following questions:

Q1. Thomson started to become convinced during his experiments that the "cathode rays" observed coming from the cathodes of vacuum tubes were building blocks of atoms — what we now call electrons. He then carried out observations with cathodes made of a variety of metals, and found that  $q/m$  was the same in every case. How would that observation serve to test his hypothesis?

Q2. Why is it not possible to determine  $q$  and  $m$  themselves, rather than just their ratio, by observing electrons' motion in electric or magnetic fields?

Q3. Thomson found that the  $q/m$  of an electron was thousands of times larger than that of ions in electrolysis. Would this imply that the electrons had more charge? Less mass? Would there be no way to tell? Explain.



# 9 Relativity

## Apparatus

- magnetic balance ..... 1/group
- meter stick ..... 1/group
- multimeter (BK or PRO-100, not HP) .... 1/group
- laser ..... 1/group
- vernier calipers ..... 1/group
- photocopy paper, for use as a weight
- DC power supply (Mastech, 30 A)
- box of special cables
- scissors

## Goal

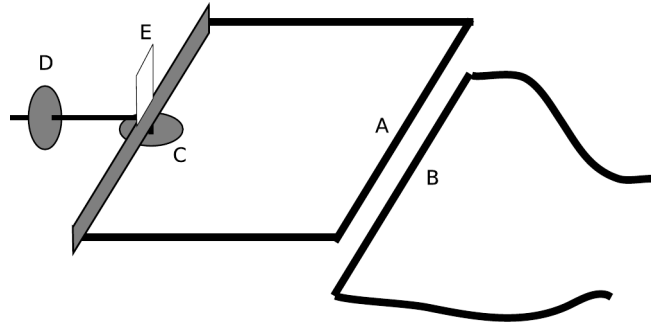
Measure the speed of light.

## Introduction

Oersted discovered that magnetism is an interaction of moving charges with moving charges, but it wasn't until almost a hundred years later that Einstein showed why such an interaction must exist: magnetism occurs as a direct result of his theory of relativity. Since magnetism is a purely relativistic effect, and relativistic effects depend on the speed of light, any measurement of a magnetic effect can be used to determine the speed of light.

## Setup

The idea is to set up opposite currents in two wires, A and B, one under the other, and use the repulsion between the currents to create an upward force on the top wire, A. The top wire is on the arm of a balance, which has a stable equilibrium because of the weight C hanging below it. You initially set up the balance with no current through the wires, adjusting the counterweight D so that the distance between the wires is as small as possible. What we care about is really the center-to-center distance (which we'll call  $R$ ), so even if the wires are almost touching, there's still a millimeter or two worth of distance between them. By shining a laser at the mirror, E, and observing the spot it makes on the wall, you can very accurately determine this particular position of the balance, and tell later on when you've reproduced it. If you put a current through the wires, it will raise



wire A. The torque made by the magnetic repulsion is now canceling the torque made by gravity directly on all the hardware, such as the masses C and D. This gravitational torque was zero before, but now you don't know what it is. The trick is to put a tiny weight on top of wire A, and adjust the current so that the balance returns to the position it originally had, as determined by the laser dot on the wall. You now know that the gravitational torque acting on the original apparatus (everything except for the weight) is back to zero, so the only torques acting are the torque of gravity on the staple and the magnetic torque. Since both these torques are applied at the same distance from the axis, the forces creating these torques must be equal as well. You can therefore infer the magnetic force that was acting.

For a weight, you can carefully and accurately cut a small rectangular piece out of a sheet of photocopy paper. In fall 2013, my students found that 500 sheets of SolCopy 20 lb paper were 2307.0 g. About 1/100 of a sheet seemed to be a good weight to use.

It's very important to get the wires A and B perfectly parallel. The result depends strongly on the small distance  $R$  between their centers, and if the wires aren't straight and parallel, you won't even have a well defined value of  $R$ .

The following technique allows  $R$  to be measured accurately. The idea is to compare the position of the laser spot on the wall when the balance is in its normal position, versus the position where the wires are touching. Using a small-angle approximation, you can then find the angle  $\theta_r$  by which the reflected beam moved. This is twice the angle  $\theta_m = \theta_r/2$  by which the mirror moved.<sup>1</sup> Once you

<sup>1</sup>To see this, imagine the following example that is unreal-



know the angle by which the moving arm of the apparatus moved, you can accurately find the air gap between the wires, and then add in twice the radius of the wires, which can be measured accurately with vernier calipers. For comparison, try to do as good a job as you can of measuring  $R$  directly by positioning the edges of the vernier calipers at the centers of the wires. If the two values of  $R$  don't agree, go back and figure out what went wrong; one possibility is that your wire is slightly bent and needs to be straightened.

You need to minimize the resistance of the apparatus, or else you won't be able to get enough current through it to cancel the weight of the staple. Most of the resistance is at the polished metal knife-edges that the moving part of the balance rests on. It may be necessary to clean the surfaces, or even to freshen them a little with a file to remove any layer of oxidation. Use the separate BK meter to measure the current — not the meter built into the power supply.

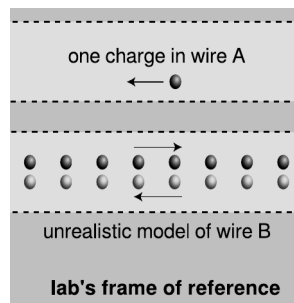
The power supply has some strange behavior that makes it not work unless you power it up in exactly the right way. It has four knobs, going from left to right: (1) current regulation, (2) over-voltage protection, (3) fine voltage control, (4) coarse voltage control. Before turning the power supply on, turn knobs 1 and 2 all the way up, and knobs 3 and 4 all the way down. Turn the power supply on. Now use knobs 3 and 4 to control how much current flows.

## Analysis

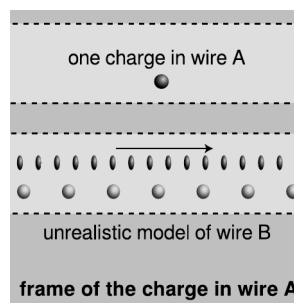
The first figure below shows a model that explains the repulsion felt by one of the charges in wire A due to all the charges in wire B. This is represented in the frame of the lab. For convenience of analysis, we give the model some unrealistic features: rather than having positively charged nuclei at rest and negatively charged electrons moving, we pretend that both are moving, in opposite directions. Since wire B has zero net density of charge everywhere, it creates no electric fields. (If you like, you can verify this during lab by putting tiny pieces of paper near the wires and verifying that they do not feel any static-electrical attraction.) Since there is no electric field, the force on the charge in wire A must be purely magnetic.

The second figure shows the same scene from the

istic but easy to figure out. Suppose that the incident beam is horizontal, and the mirror is initially vertical, so that the reflected beam is also horizontal. If the mirror is then tilted backward by 45 degrees, the reflected beam will be straight up,  $\theta_r = 90$  degrees.



point of view of the charge in wire A. This charge considers itself to be at rest, and it also sees the light-colored charges in B as being at rest. In this frame the dark-colored charges in B are the only ones moving, and they move with twice the speed they had in the lab frame. In this frame, the particle in A is at rest, so it can't feel any magnetic force. The force is now considered to be purely electric. This electric force exists because the dark charges are relativistically contracted, which makes them more dense than their light-colored neighbors, causing a nonzero net density of charge in wire B.



We've considered the force acting on a single charge in wire A. The actual force we observe in the experiment is the sum of all the forces acting on all such charges (of both signs). As in the slightly different example analyzed in section 23.3 of *Light and Matter*, this effect is proportional to the product of the speeds of the charges in the two wires, divided by  $c^2$ . Therefore the effect must be proportional to the product of the currents over  $c^2$ . In this experiment, the same current flows through wire A and then comes back through B in the opposite direction, so we conclude that the force must be proportional to  $I^2/c^2$ .

In the second frame, the force is purely electrical, and as shown in example 4 in section 22.3 of *Light and Matter*, the electric field of a charged wire falls off in proportion to  $1/R$ , where  $R$  is the distance from the wire. Electrical forces are also proportional to the Coulomb constant  $k$ .

The longer the wires, the more charges interact, so we must also have a proportionality to the length  $\ell$ .

Putting all these factors together, we find that the force is proportional to  $kI^2\ell/c^2R$ . We can easily verify that the units of this expression are newtons, so the only possible missing factor is something unitless. This unitless factor turns out to be 2 — essentially the same 2 found in example 14 in section 22.7. The result for the repulsive force between the two wires is

$$F = \frac{k}{c^2} \cdot \frac{2I^2\ell}{R} .$$

By solving this equation you can find  $c$ . Your final result is the speed of light, with error bars. Compare with the previously measured value of  $c$  and give a probabilistic interpretation, as in the examples in appendix 2.

In your writeup, give both the values of  $R$  (laser and eyeball). The laser technique is inherently better, so that's the value you should use in extracting  $c$ , but I want to see both values of  $R$  because some groups in the past have had a bigger discrepancy than I would have expected. If you have a large discrepancy, get my attention during lab and we can see whether it might be due to a bent wire, or some other cause.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

Do the laser safety checklist, Appendix 8, tear it out, and turn it in at the beginning of lab. If you don't understand something, don't initial that point, and ask your instructor for clarification before you start the lab.

**P1** Show that the equation for the force between the wires has units of newtons.

**P2** Do the algebra to solve for  $c$  in terms of the measured quantities.



# 10 Energy in Fields

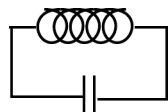
## Apparatus

Heath coils	1/group
0.01 $\mu\text{F}$ capacitors	1/group
PASCO PI-9587C sine-wave generator	1/group
oscilloscope	1/group

## Goal

Observe how the energy content of a field relates to the field strength.

## Introduction

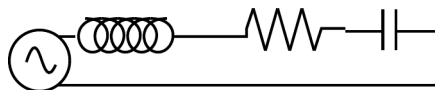


A simplified version of the circuit.

The basic idea of this lab is to observe a circuit like the one shown in the figure above, consisting of a capacitor and a coil of wire (inductor). Imagine that we first deposit positive and negative charges on the plates of the capacitor. If we imagined that the universe was purely mechanical, obeying Newton's laws of motion, we would expect that the attractive force between these charges would cause them to come back together and reestablish a stable equilibrium in which there was zero net charge everywhere in the circuit.

However, the capacitor in its initial, charged, state has an electric field between its plates, and this field possesses energy. This energy can't just go away, because energy is conserved. What really happens is that as charge starts to flow off of the capacitor plates, a current is established in the coil. This current creates a magnetic field in the space inside and around the coil. The electric energy doesn't just evaporate; it turns into magnetic energy. We end up with an oscillation in which the capacitor and the coil trade energy back and forth. Your goal is to monitor this energy exchange, and to use it to deduce a power-law relationship between each field and its energy.

The practical realization of the circuit involves some further complications, as shown in the second figure.



The actual circuit.

The wires are not superconductors, so the circuit has some nonzero resistance, and the oscillations would therefore gradually die out, as the electric and magnetic energies were converted to heat. The sine wave generator serves both to initiate the oscillations and to maintain them, replacing, in each cycle, the energy that was lost to heat.

Furthermore, the circuit has a resonant frequency at which it prefers to oscillate, and when the resistance is very small, the width of the resonance is very narrow. To make the resonance wider and less finicky, we intentionally insert a 10 k $\Omega$  resistor. The inductance of the coil is about 1 H, which gives a resonant frequency of about 1.5 kHz.

The actual circuit consists of the 1 H Heath coil, a 0.01  $\mu\text{F}$  capacitance supplied by the decade capacitor box, a 10 k $\Omega$  resistor, and the PASCO sine wave generator (using the GND and LO  $\Omega$  terminals).

## Observations

Let  $E$  be the magnitude of the electric field between the capacitor plates, and let  $\tilde{E}$  be the maximum value of this quantity. It is then convenient to define  $x = E/\tilde{E}$ , a unitless quantity ranging from  $-1$  to  $1$ . Similarly, let  $y = B/\tilde{B}$  for the corresponding magnetic quantities. The electric field is proportional to the voltage difference across the capacitor plates, which is something we can measure directly using the oscilloscope:

$$x = \frac{E}{\tilde{E}} = \frac{V_C}{\tilde{V}_C}$$

Magnetic fields are created by moving charges, i.e., by currents. Unfortunately, an oscilloscope doesn't measure current, so there's no equally direct way to get a handle on the magnetic field. However, all the current that goes through the coil must also go through the resistor, and Ohm's law relates the current through the resistor to the voltage drop across it. This voltage drop is something we can measure

with the oscilloscope, so we have

$$y = \frac{B}{\bar{B}} = \frac{I}{\bar{I}} = \frac{V_R}{\bar{V}_R}$$

To measure  $x$  and  $y$ , you need to connect channels 1 and 2 of the oscilloscope across the resistor and the capacitor. Since both channels of the scope are grounded on one side (the side with the ground tab on the banana-to-bnc connector), you need to make sure that their grounded sides both go to the piece of wire between the resistor and the capacitor. Furthermore, one output of the sine wave generator is normally grounded, which would mess everything up: two different points in the circuit would be grounded, which would mean that there would be a short across some of the circuit elements. To avoid this, loosen the banana plug connectors on the sine wave generator, and swing away the piece of metal that normally connects one of the output plugs to the ground.

Tune the sine wave generator's frequency to resonance, and take the data you'll need in order to determine  $x$  and  $y$  at a whole bunch of different places over one cycle.

Some of the features of the digital oscilloscopes can make the measurements a lot easier. Doing `Acquire>Average` tells the scope to average together a series of up to 128 measurements in order to reduce the amount of noise. Doing `CH 1 MENU>Volts/Div>Fine` allows you to scale the display arbitrarily. Rather than reading voltages by eye from the scope's x-y grid, you can make the scope give you a measuring cursor. Do `Cursor>Type>Time`. Use the top left knob to move the cursor to different times. Doing `Source>CH 1` and `Source>CH 2` gives you the voltage measurement for each channel. (Always use Cursor 1, never Cursor 2.)

The quality of the results can depend a lot on the quality of the connections. If the display on the scope changes noticeably when you wiggle the wires, you have a problem.

## Analysis

Plot  $y$  versus  $x$  on a piece of graph paper. Let's assume that the energy in a field depends on the field's strength raised to some power  $p$ . Conservation of energy then gives

$$|x|^p + |y|^p = 1 \quad .$$

Use your graph to determine  $p$ , and interpret your result.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Sketch what your graph would look like for  $p = 0.1$ ,  $p = 1$ ,  $p = 2$ , and  $p = 10$ . (You should be able to do  $p = 1$  and  $p = 2$  without any computations. For  $p = 0.1$  and  $p = 10$ , you can either run some numbers on your calculator or use your mathematical knowledge to sketch what they would turn out like.)

# 11 RC Circuits

## Apparatus

oscilloscope  
 Pasco PI-8127 function generator (in lab benches in 415)  
 unknown capacitor  
 known capacitors,  $0.05 \mu\text{F}$   
 resistors of various values

Note: It is also possible to do this lab using the Pasco PI-9587C function generators.

## Goals

Observe the exponential curve of a discharging capacitor.

Determine the capacitance of an unknown capacitor.

## Introduction

God bless the struggling high school math teacher, but some of them seem to have a talent for making interesting and useful ideas seem dull and useless. On certain topics such as the exponential function,  $e^x$ , the percentage of students who figure out from their teacher's explanation what it really means and why they should care approaches zero. That's a shame, because there are so many cases where it's useful. The graphs show just a few of the important situations in which this function shows up.

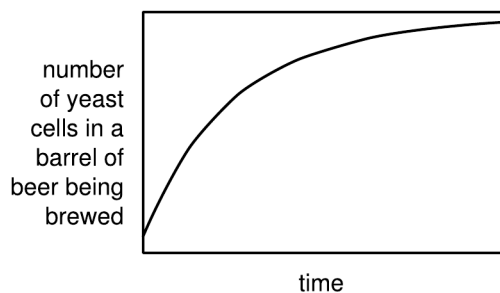
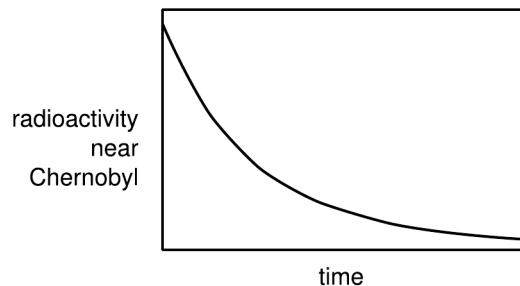
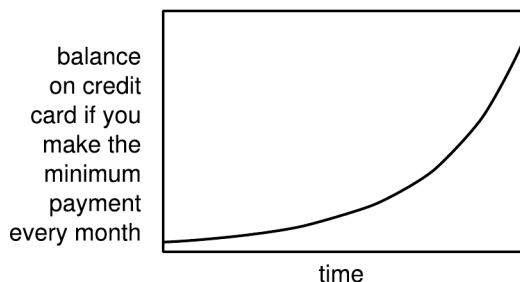
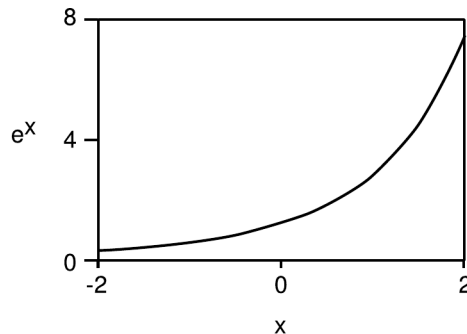
The credit card example is of the form

$$y = ae^{t/k} \quad ,$$

while the Chernobyl graph is like

$$y = ae^{-t/k} \quad ,$$

In both cases,  $e$  is the constant  $2.718\dots$ , and  $k$  is a positive constant with units of time, referred to as the time constant. The first type of equation is referred to as exponential growth, and the second as exponential decay. The significance of  $k$  is that it tells you how long it takes for  $y$  to change by a factor of  $e$ . For instance, an 18% interest rate on your credit card converts to  $k = 6.0$  years. That means that if your credit card balance is \$1000 in



1996, by 2002 it will be \$2718, assuming you never really start paying down the principal.

An important fact about the exponential function is that it never actually becomes zero — it only gets closer and closer to zero. For instance, the radioactivity near Chernobyl will never ever become exactly zero. After a while it will just get too small to pose

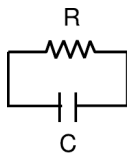
any health risk, and at some later time it will get too small to measure with practical measuring devices.

Why is the exponential function so ubiquitous? Because it occurs whenever a variable's rate of change is proportional to the variable itself. In the credit card and Chernobyl examples,

$$\begin{aligned} & \text{(rate of increase of credit card debt)} \\ & \quad \propto \text{(present credit card debt)} \\ & \text{(rate of decrease of the number of radioactive atoms)} \\ & \quad \propto \text{(present number of radioactive atoms)} \end{aligned}$$

For the credit card, the proportionality occurs because your interest payment is proportional to how much you currently owe. In the case of radioactive decay, there is a proportionality because fewer remaining atoms means fewer atoms available to decay and release radioactive particles. This line of thought leads to an explanation of what's so special about the constant  $e$ . If the rate of increase of a variable  $y$  is proportional to  $y$ , then the time constant  $k$  equals one over the proportionality constant, and this is true only if the base of the exponential is  $e$ , not 10 or some other number.

Exponential growth or decay can occur in circuits containing resistors and capacitors. Resistors and capacitors are the most common, inexpensive, and simple electrical components. If you open up a cell phone or a stereo, the vast majority of the parts you see inside are resistors and capacitors. Indeed, many useful circuits, known as RC circuits, can be built out of nothing but resistors and capacitors. In this lab, you will study the exponential decay of the simplest possible RC circuit, shown below, consisting of one resistor and one capacitor in series.



Suppose we initially charge up the capacitor, making an excess of positive charge on one plate and an excess of negative on the other. Since a capacitor behaves like  $V = Q/C$ , this creates a voltage difference across the capacitor, and by Kirchoff's loop rule there must be a voltage drop of equal magnitude across the resistor. By Ohm's law, a current  $I = V/R = Q/RC$  will flow through the resistor,

and we have therefore established a proportionality,

$$\begin{aligned} & \text{(rate of decrease of charge on capacitor)} \\ & \quad \propto \text{(present charge on capacitor)} \end{aligned}$$

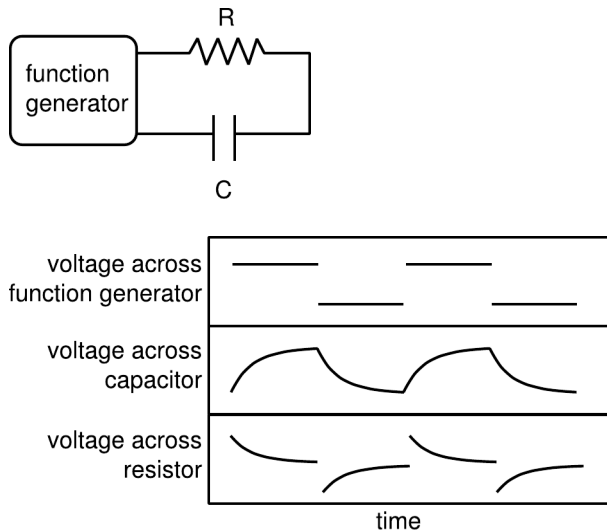
It follows that the charge on the capacitor will decay exponentially. Furthermore, since the proportionality constant is  $1/RC$ , we find that the time constant of the decay equals the product of  $R$  and  $C$ . (It may not be immediately obvious that ohms times farads equals seconds, but it does.)

Note that even if we put the charge on the capacitor very suddenly, the discharging process still occurs at the same rate, characterized by  $RC$ . Thus RC circuits can be used to filter out rapidly varying electrical signals while accepting more slowly varying ones. A classic example occurs in stereo speakers. If you pull the front panel off of the wooden box that we refer to as "a speaker," you will find that there are actually *two* speakers inside, a small one for reproducing high frequencies and a large one for the low notes. The small one, called the tweeter, not only cannot produce low frequencies but would actually be damaged by attempting to accept them. It therefore has a capacitor wired in series with its own resistance, forming an RC circuit that filters out the low frequencies while permitting the highs to go through. This is known as a high-pass filter. A slightly different arrangement of resistors and inductors is used to make a low-pass filter to protect the other speaker, the woofer, from high frequencies.

## Observations

In typical filtering applications, the RC time constant is of the same order of magnitude as the period of a sound vibration, say  $\sim 1$  ms. It is therefore necessary to observe the changing voltages with an oscilloscope rather than a multimeter. The oscilloscope needs a repetitive signal, and it is not possible for you to insert and remove a battery in the circuit hundreds of times a second, so you will use a function generator to produce a voltage that becomes positive and negative in a repetitive pattern. Such a wave pattern is known as a square wave. The mathematical discussion above referred to the exponential decay of the charge on the capacitor, but an oscilloscope actually measures voltage, not charge. As shown in the graphs below, the resulting voltage patterns simply look like a chain of exponential curves strung together.

Make sure that the yellow or red "VAR" knob, on



the front of the knob that selects the time scale, is clicked into place, not in the range where it moves freely — otherwise the times on the scope are not calibrated.

### A Preliminary observations

Pick a resistor and capacitor with a combined RC time constant of  $\sim 1$  ms. Make sure the resistor is at least  $\sim 10\text{k}\Omega$ , so that the internal resistance of the function generator is negligible compared to the resistance you supply.

Note that the capacitance values printed on the sides of capacitors often violate the normal SI conventions about prefixes. If just a number is given on the capacitor with no units, the implied units are microfarads, mF. Units of nF are avoided by the manufacturers in favor of fractional microfarads, e.g., instead of 1 nF, they would use “0.001,” meaning 0.001  $\mu\text{F}$ . For picofarads, a capital P is used, “PF,” instead of the standard SI “pF.”

Use the oscilloscope to observe what happens to the voltages across the resistor and capacitor as the function generator’s voltage flips back and forth. Note that the oscilloscope is simply a fancy voltmeter, so you connect it to the circuit the same way you would a voltmeter, in parallel with the component you’re interested in. Make sure the scope is set on DC, not AC, by doing CH 1>Coupling>DC.<sup>1</sup> A complication is added by the fact that the scope and the function generator are fussy about having the grounded sides of their circuits connected to each other. The banana-to-BNC converter that goes on

<sup>1</sup>AC coupling filters the input to remove any DC component. However, this has the effect of distorting non-sinusoidal waveforms such as the ones we’re using in this lab.

the input of the scope has a small tab on one side marked “GND.” This side of the scope’s circuit must be connected to the grounded terminal black terminal of the function generator. This means that when you want to switch from measuring the capacitor’s voltage to measuring the resistor’s, you will need to rearrange the circuit a little.

If the trace on the oscilloscope does not look like the one shown above, it may be because the function generator is flip-flopping too rapidly or too slowly. The function generator’s frequency has no effect on the RC time constant, which is just a property of the resistor and the capacitor.

With the Tektronix TDS1001B scopes, I have observed a problem in which internal interference occurs in the scope when the time base is set to 1 ms or shorter. This interference looks like a periodic spike superimposed on the signal. It becomes a problem if it makes triggering not work right. One possible solution is to use the run/stop button on the scope to get a frozen image of a single trace, so you don’t need steady triggering.

If you think you have a working setup, observe the effect of temporarily placing a second capacitor in parallel with the first capacitor. If your setup is working, the exponential decay on the scope should become more gradual because you have increased RC. If you don’t see any effect, it probably means you’re measuring behavior coming from the internal  $R$  and  $C$  of the function generator and the scope.

Use the scope to determine the RC time constant, and check that it is correct. Rather than reading times and voltages by eye from the scope’s x-y grid, you can make the scope give you a measuring cursor. Do Cursor>Type>Time, and Source>CH 1 . Use the top left knob to move the cursor to different times.

### B Unknown capacitor

Build a similar circuit using your unknown capacitor plus a known resistor. Use the unknown capacitor with the same number as your group number. Take the data you will need in order to determine the RC time constant, and thus the unknown capacitance.

As a check on your result, obtain a known capacitor with a value similar to the one you have determined for your unknown, and see if you get nearly the same curve on the scope if you replace the unknown capacitor with the new one.



## **Prelab**

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Plan how you will determine the capacitance and what data you will need to take.

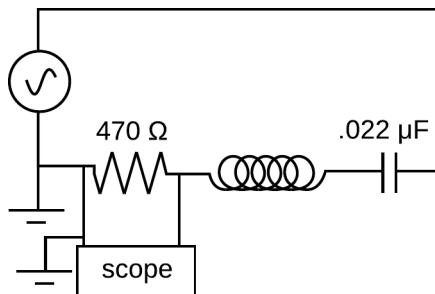
## **Analysis**

Determine the capacitance, with error analysis (appendices 2 and 3).

# 12 AC Circuits

## Apparatus

- Heath coils ..... 2/group
- 0.022  $\mu\text{F}$  capacitor ..... 2/group
- 0.05  $\mu\text{F}$  capacitor
- 470  $\Omega$  resistor
- Pasco PI-8127 function generator (in lab benches in 415)
- oscilloscope
- banana to BNC converters
- alligator clips



Observing the response of an LRC circuit to a driving voltage.

## Goals

Observe the resonant behavior of an LRC circuit.

Predict and observe the behavior of capacitances and inductances in parallel and series.

Observe phase relationships in capacitors and inductors.

Predict and observe the complex impedance of a capacitor.

*Observe:* Use the setup shown above to observe the current as a function of the driving frequency. Although the oscilloscope is a voltmeter, not an ammeter, by using it to measure  $V_R$ , we are getting a measure of  $I_R$  as well, by Ohm's law. Note the grounds, which have to coincide.

For safety, put the function generator's voltage at zero while setting the circuit up, then always use it after that on the lowest practicable setting. The voltages  $V_C$  and  $V_L$  can be large, even when the function generator's voltage is small. When operating near resonance, reduce the voltage as much as possible. It should be possible to do the whole lab without ever exceeding  $\sim 0.3$  V on the function generator's output.

## Preliminary

For use later in the lab, summarize what you know about how resistances, capacitances, and inductances combine in parallel and series. An easy way to do this is to use the fact that the corresponding *impedances* combine like resistances.

	series	parallel
resistances		
capacitances		
inductances		

### A Resonance

*Predict:* The Heath coils are not intended to be used as inductors, and are not labeled with inductance values, but we expect them to have  $L \sim 1$  H (*Fields and Circuits*, ch. 13, problem 10). Make a rough estimate of the resonant frequency of this series LRC circuit:

Determine the actual resonant frequency, and compare with your prediction. (Afterward, turn off the voltage for safety when changing the circuit for the next part.)

### B Effect of $C$ on the resonant frequency

*Predict:* Predict the new resonant frequency when  $C$  is changed to 0.05  $\mu\text{F}$ . Use ratios, not a plug-in.

*Observe:* Check your prediction. (Afterward, turn off the voltage.)

### C Inductances in series and parallel

Switch back to the default value of  $C$  from part A. Predict and observe the cases where the Heath coil is replaced by *two* Heath coils (1) in series, and (2) in parallel. (Turn off the voltage when making changes to the circuit.)

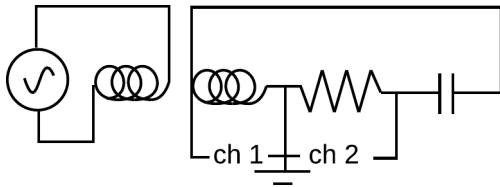
## D Capacitances in series

*Predict:* Predict the resonant frequency when the capacitance in the original circuit is provided by two  $0.022 \mu\text{F}$  capacitors in *series*.

*Observe:* Check your prediction. (Afterward, turn off the voltage.)

## E Phase relationship between voltage and current for an inductor

At some point it becomes inconvenient to take detailed measurements on this circuit because of the grounding of both the function generator's output and the scope's inputs. For this reason, let's drive it this way:



Driving the LRC circuit inductively, so that it is isolated from the function generator's ground.

The varying magnetic field made by the first coil induces a curly electric field, which is felt as a voltage on the second coil. This is a type of transformer, used in this case for electrical isolation. Now we can take any measurements we like on the LRC circuit, as long as the grounded sides of the scope's two channels are connected to the same point.

But because of this constraint imposed by the grounds of the scope's inputs, we are forced to set things up in such a way that the signs in our measurements of  $V_L$  and  $V_R$  are inconsistent. For example, if the electric field is to the right, then channel 1 will read positive, but channel 2 will read negative. To compensate for this, we will tell the scope to negate channel 1. Press CH 1 MENU and do Invert On.

*Observe* the phase relationship between  $V_L$  (channel 1) and  $I_L$  (which is the same as  $I_R$  and therefore has the same phase as  $V_R$ , measured on channel 2).

*Explain* this phase relationship.

## F Measurement of a complex impedance

*Predict* the complex impedance of the capacitor at this frequency, including both the magnitude and the argument.

For safety, turn the voltage on the function generator to zero before going on. Rearrange the connections to the scope so you can measure  $V_C$  and  $V_R$ . Think carefully about grounds. It may be necessary to rearrange the order of the series circuit.

*Measure* the amplitudes and phases of these voltages, and use them to find

$$Z_C = \frac{\tilde{V}_C}{\tilde{I}} = \frac{\tilde{V}_C}{\tilde{V}_R} \cdot R.$$

## Troubleshooting

In part E, touching the housings of the coils together may produce a zero signal. Leave some air.

If the function generator won't turn on, check whether the AC power cable is firmly inserted in the connector for the DC power supply.

Some capacitors are labeled in unclear ways, or the ink has faded. If necessary, use a multimeter to check their values.

With the Tektronix TDS1001B scopes, I have observed a problem in which internal interference occurs in the scope when the time base is set to 1 ms or shorter. This interference looks like a periodic spike superimposed on the signal. It becomes a problem if it makes triggering not work right. One possible solution is to use the run/stop button on the scope to get a frozen image of a single trace, so you don't need steady triggering.

We have sometimes observed signals that have strange waveforms rather than sine waves. This problem was fixed by pressing the default setup on the scope. It must result from some unfortunate interaction between the scope and the circuit.

## Additional notes for the instructor

An alternative technique for dealing with grounding in this lab is to connect all the grounded inputs and outputs to one point in the circuit, and then use the arithmetic functions on the scope to display the difference between inputs as necessary.

The Heath coil has a DC resistance of about 62 ohms, and the signal generator may have an output impedance of several hundred ohms. All the parts of this lab are constructed so as to be essentially insensitive to this (except that the  $Q$  of the circuit is affected).

# 13 Faraday's Law

## Apparatus

Pasco PI-8127 function generator (in lab benches in 415; see note below)  
solenoid (Heath) 1/group plus a few more  
oscilloscope ..... 1/group  
100 Ω resistor ..... 1/group  
secondary coils (see note below)  
palm-sized pieces of iron or steel  
masking tape  
rulers

Notes: It is also probably possible to do this lab using the Pasco PI-9587C function generators, but I haven't tested it.

We have a variety of coils that can be used as secondary coils. The text below describes using a loop made out of a 4-meter piece of wire. We have a bunch of these made up using white wire with banana plugs on the ends. Although these can be made to work, the signal is rather weak because of the small number of turns. We have a set of small rectangular coils with various numbers of turns, PASCO SF-8617. I've successfully used the 3200-turn coil, which produces a big signal. The others have are also probably usable. I have also made a couple of hand-wrapped coils for use in this lab.

## Goals

Observe electric fields induced by changing magnetic fields.

Test Faraday's law.

## Introduction

Physicists hate complication, and when physicist Michael Faraday was first learning physics in the early 19th century, an embarrassingly complex aspect of the science was the multiplicity of types of forces. Friction, normal forces, gravity, electric forces, magnetic forces, surface tension — the list went on and on. Today, 200 years later, ask a physicist to enumerate the fundamental forces of nature and the most likely response will be “four: gravity, electromagnetism, the strong nuclear force and the weak nuclear force.” Part of the simplification came from

the study of matter at the atomic level, which showed that apparently unrelated forces such as friction, normal forces, and surface tension were all manifestations of electrical forces among atoms. The other big simplification came from Faraday's experimental work showing that electric and magnetic forces were intimately related in previously unexpected ways, so intimately related in fact that we now refer to the two sets of force-phenomena under a single term, “electromagnetism.”

Even before Faraday, Oersted had shown that there was at least some relationship between electric and magnetic forces. An electrical current creates a magnetic field, and magnetic fields exert forces on an electrical current. In other words, electric forces are forces of charges acting on charges, and magnetic forces are forces of moving charges on moving charges. (Even the magnetic field of a bar magnet is due to currents, the currents created by the orbiting electrons in its atoms.)

Faraday took Oersted's work a step further, and showed that the relationship between electricity and magnetism was even deeper. He showed that a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. Faraday's law,

$$\int \mathbf{E} \cdot \ell = -d\Phi_B/dt$$

relates the integral of the electric field around a closed loop to the rate of change of the magnetic flux through the loop. It forms the basis for such technologies as the transformer, the electric guitar, the amplifier, and generator, and the electric motor.

## Observations

### A Qualitative Observations

To observe Faraday's law in action you will first need to produce a varying magnetic field. You can do this by using a function generator to produce a current in a solenoid that that varies like a sine wave as a function of time. The solenoid's magnetic field will thus also vary sinusoidally.

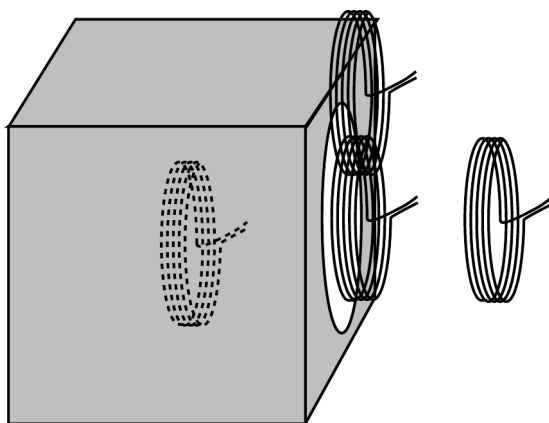
The emf in Faraday's law can be observed around a loop of wire positioned inside or close to the solenoid. To make the emf larger and easier to see on an os-

cilloscope, you will use 5-10 loops, which multiplies the flux by that number of loops.

The only remaining complication is that the rate of change of the magnetic flux,  $d\Phi_B/dt$ , is determined by the rate of change of the magnetic field, which relates to the rate of change of the current through the solenoid,  $dI/dt$ . The oscilloscope, however, measures voltage, not current. You might think that you could simply observe the voltage being supplied to the solenoid and divide by the solenoid's 62-ohm resistance to find the current through the solenoid. This will not work, however, because Faraday's law produces not only an emf in the loops of wire but also an emf in the solenoid that produced the magnetic field in the first place. The current in the solenoid is being driven not just by the emf from the function generator but also by this "self-induced" emf. Even though the solenoid is just a long piece of wire, it does not obey Ohm's law under these conditions. To get around this difficulty, you can insert the 100  $\Omega$  resistor in the circuit in series with the function generator and the solenoid. The resistor does obey Ohm's law, so by using the scope to observe the voltage drop across it you can infer the current flowing through it, which is the same as the current flowing through the solenoid.

Create the solenoid circuit, and hook up one channel of the scope to observe the voltage drop across the resistor. A sine wave with a frequency on the order of 1 kHz will work.

Wind the 2-m wire into circular loops small enough to fit inside the solenoid, and hook it up to the other channel of the scope.



As always, you need to watch out for ground loops. The output of the function generator has one of its terminals grounded, so that ground and the grounded side of the scope's input have to be at the same place in the circuit.

The signals tend to be fairly noisy. You can clean them up a little by having the scope average over a series of traces. To turn on averaging, do `Acquire>Average>128`. To turn it back off, press `Sample`.

First try putting the loops at the mouth of the solenoid, and observe the emf induced in them. Observe what happens when you flip the loops over. You will observe that the two sine waves on the scope are out of phase with each other. Sketch the phase relationship in your notebook, and make sure you understand in terms of Faraday's law why it is the way it is, i.e., why the induced emf has the greatest value at a certain point, why it is zero at a certain point, etc.

Observe the induced emf at with the loops at several other positions such as those shown in the figure. Make sure you understand in the resulting variations of the strength of the emf in terms of Faraday's law.

## B A Metal Detector

Obtain one of the spare solenoids so that you have two of them. Substitute it for the loops of wire, so that you can observe the emf induced in the second solenoid by the first solenoid. If you put the two solenoids close together with their mouths a few cm apart and then insert a piece of iron or steel between them, you should be able to see a small increase in the induced emf. The iron distorts the magnetic field pattern produced by the first solenoid, channeling more of the field lines through the second solenoid.

## C Quantitative Observations

This part of the lab is a quantitative test of Faraday's law. Going back to the setup for part A, measure the amplitude (peak-to-peak height) of the voltage across the resistor. Check against your prediction from prelab question 1.

There is a feature of the scope that seems to come on by default and changes your voltages by a factor of 10. Since this occurs for both channels, it ends up not affecting your results if you express them as a ratio between the amplitudes measured on the two channels. However, if you want to turn this off, you can. Press `CH 1 Menu`, and you will see something that says `Probe 10X Voltage`. This means that all your voltage measurements will be off by this factor. Push the button and then set this to `Attenuation 1X` rather than `10X`. Do this for channel 2 as well.

Another thing that can cause some confusion is that the function generator will probably say "0.00A rms," which will make you think that there is a blown fuse or an open circuit. Actually, the current that flows in

the solenoid circuit is simply so small that it rounds off to 0.00 on this readout.

Choose a position for the loops of wire that you think will make it as easy as possible to calculate  $d\Phi_B/dt$  accurately based on knowledge of the variation of the current in the solenoid as a function of time. Put the loops in that position, and measure the amplitude of the induced emf. Repeat these measurements with a frequency that is different by a factor of two.

## Self-Check

Before leaving, analyze your results from part C and make sure you get reasonable agreement with Faraday's law.

## Analysis

Describe your observations in parts A and B and interpret them in terms of Faraday's law.

Compare your observations in part C quantitatively with Faraday's law. The solenoid isn't very long, so the approximate expression for the interior field of a long solenoid isn't very accurate here. To correct for that, multiply the expression for the field by the correction factor  $\zeta = (\cos \theta_1 - \cos \theta_2)/2$ , (*Fields and Circuits*, ch. 11, problem 13), where  $\theta_1$  and  $\theta_2$  are angles between the axis and the lines connecting the point of interest to the edges of the solenoid's mouths.

This analysis is horrible to do and to read if you do it all numerically. Let  $V$  be the voltage across the resistor and  $\mathcal{E}$  the emf measured on the secondary coil. It's nice to work with the unitless ratio  $\mathcal{E}/V$ . For your theoretical result, you should be able to express this ratio symbolically in terms of the following symbols:

$k$  = Columb constant

$c$  = speed of light

$N_1$  = number of turns in the primary coil

$N_2$  = number of turns in the secondary coil

$f$  = frequency

$A$  = area of the secondary coil

$\ell$  = length of the primary coil

$R$  = resistance of the resistor

$\zeta$  = correction factor for the magnetic field.

Check that the units work. Only plug in numbers at the end.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Find the theoretical equation for  $\mathcal{E}/V$ .



# 14 Polarization

## Apparatus

laser .....	1/group
calcite crystal (flattest available) .....	1/group
polarizing films .....	2/group
Na gas discharge tube .....	1/group
photovoltaic cell and collimator .....	1/group

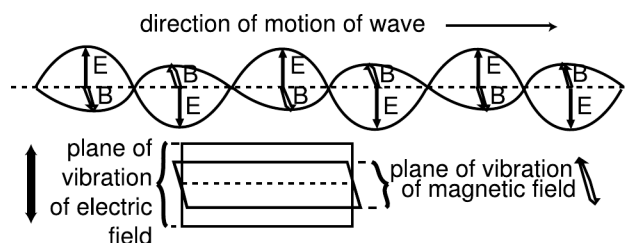
## Goals

Make qualitative observations about the polarization of light.

Test quantitatively the hypothesis that polarization relates to the direction of the field vectors in an electromagnetic wave.

## Introduction

It's common knowledge that there's more to light than meets the eye: everyone has heard of infrared and ultraviolet light, which are visible to some other animals but not to us. Another invisible feature of the wave nature of light is far less well known. Electromagnetic waves are transverse, i.e., the electric and magnetic field vectors vibrate in directions perpendicular to the direction of motion of the wave. Two electromagnetic waves with the same wavelength can therefore be physically distinguishable, if their electric and magnetic fields are twisted around in different directions. Waves that differ in this way are said to have different polarizations.



An electromagnetic wave has electric and magnetic field vectors that vibrate in the directions perpendicular to its direction of motion. The wave's direction of polarization is defined as the line along which the electric field lies.

Maybe we polarization-blind humans are missing out on something. Some fish, insects, and crustaceans

can detect polarization. Most sources of visible light (such as the sun or a light bulb) are unpolarized. An unpolarized beam of light contains a random mixture of waves with many different directions of polarization, all of them changing from moment to moment, and from point to point within the beam.

## Qualitative Observations

Before doing anything else, turn on your gas discharge tube, so it will be warmed up when you are ready to do part E.

### A Double refraction in calcite

Place a calcite crystal on this page. You will see two images of the print through the crystal.

To understand why this happens, try shining the laser beam on a piece of paper and then inserting the calcite crystal in the beam. If you rotate the crystal around in different directions, you should be able to get two distinct spots to show up on the paper. (This may take a little trial and error, partly because the effect depends on the correct orientation of the crystal, but also because the crystals are not perfect, and it can be hard to find a nice smooth spot through which to shine the beam.)

In the refraction lab, you've already seen how a beam of light can be bent as it passes through the interface between two media. The present situation is similar because the laser beam passes in through one face of the crystal and then emerges from a parallel face at the back. You have already seen that in this type of situation, when the beam emerges again, its direction is bent back parallel to its original direction, but the beam is offset a little bit. What is different here is that the same laser beam splits up into two parts, which bumped off course by different amounts.

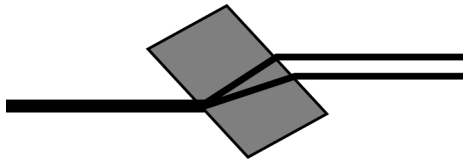
What's happening is that calcite, unlike most substances, has a different index of refraction depending on the polarization of the light. Light travels at a different speed through calcite depending on how the electric and magnetic fields are oriented compared to the crystal. The atoms inside the crystal are packed in a three-dimensional pattern sort of like a stack of oranges or cannonballs. This packing arrangement has a special axis of symmetry, and light polarized along that axis moves at one speed, while light polarized perpendicular to that axis moves at a different



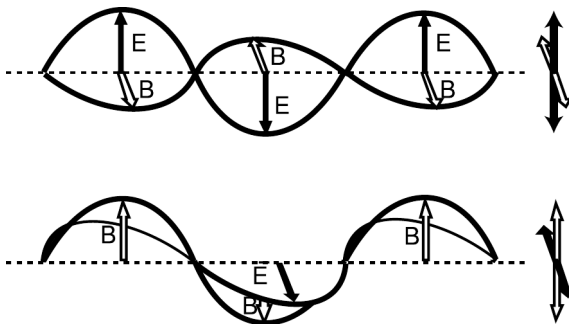
speed.

It makes sense that if the original laser beam was a random mixture of all possible directions of polarization, then each part would be refracted by a different amount. What is a little more surprising is that two separated beams emerge, with nothing in between. The incoming light was composed of light with every possible direction of polarization. You would therefore expect that the part of the incoming light polarized at, say,  $45^\circ$  compared to the crystal's axis would be refracted by an intermediate amount, but that doesn't happen. This surprising observation, and all other polarization phenomena, can be understood based on the vector nature of electric and magnetic fields, and the purpose of this lab is to lead you through a series of observations to help you understand what's really going on.

### B A polarized beam entering the calcite



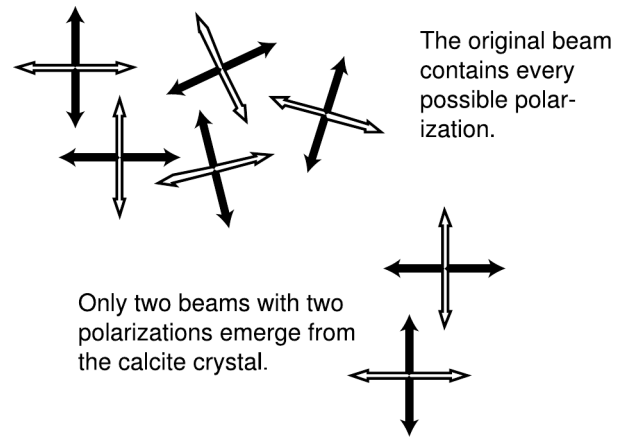
A single laser beam entering a calcite crystal breaks up into two parts, which are refracted by different amounts.



The calcite splits the wave into two parts, polarized in perpendicular directions compared to each other.

We need not be restricted to speculation about what was happening to the part of the light that entered the calcite crystal polarized at a  $45^\circ$  angle. You can use a polarizing film, often referred to informally as a "Polaroid," to change unpolarized light into a beam of only one specific polarization. In this part of the lab, you will use a polarizing film to produce a beam of light polarized at a  $45^\circ$  angle to the crystal's internal axis.

If you simply look through the film, it doesn't look like anything special — everything just looks dimmer, like looking through sunglasses. The light reach-



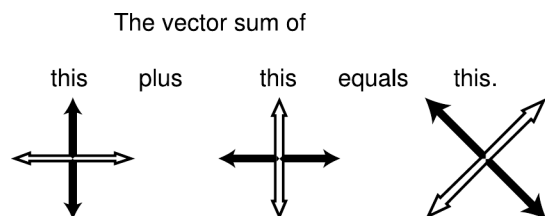
ing your eye is polarized, but your eye can't tell that. If you looked at the film under a microscope, you'd see a pattern of stripes, which select only one direction of polarization of the light that passes through.

Now try interposing the film between the laser and the crystal. The beam reaching the crystal is now polarized along some specific direction. If you rotate the film, you change beam's direction of polarization. If you try various orientations, you will be able to find one that makes one of the spots disappear, and another orientation of the film, at a  $90^\circ$  angle compared to the first, that makes the other spot go away. When you hold the film in one of these directions, you are sending a beam into the crystal that is either purely polarized along the crystal's axis or purely polarized at  $90^\circ$  to the axis.

By now you have already seen what happens if the film is at an intermediate angle such as  $45^\circ$ . Two spots appear on the paper in the same places produced by an unpolarized source of light, not just a single spot at the midpoint. This shows that the crystal is not just throwing away the parts of the light that are out of alignment with its axis. What is happening instead is that the crystal will accept a beam of light with any polarization whatsoever, and split it into two beams polarized at  $0$  and  $90^\circ$  compared to the crystal's axis.

This behavior actually makes sense in terms of the wave theory of light. Light waves are supposed to obey the principle of superposition, which says that waves that pass through each other add on to each other. A light wave is made of electric and magnetic fields, which are vectors, so it is vector addition we're talking about in this case. A vector at a  $45^\circ$  angle can be produced by adding two perpendicular vectors of equal length. The crystal therefore cannot respond any differently to  $45^\circ$  polarized light than it would to a 50-50 mixture of light with 0-

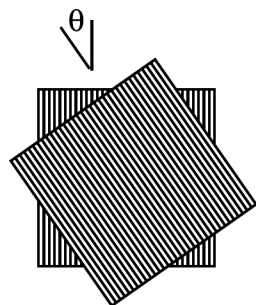
degree and 90-degree polarization.



The principle of superposition implies that if the  $0^\circ$  and  $90^\circ$  polarizations produce two different spots, then the two waves superimposed must produce those two spots, not a single spot at an intermediate location.

### C Two polarizing films

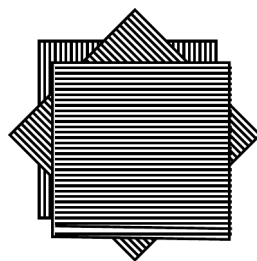
So far I've just described the polarizing film as a device for producing polarized light. But one can apply to the polarizing film the same logic of superposition and vector addition that worked with the calcite crystal. It would not make sense for the film simply to throw away any waves that were not perfectly aligned with it, because a field oriented on a slant can be analyzed into two vector components, at  $0$  and  $90^\circ$  with respect to the film. Even if one component is entirely absorbed, the other component should still be transmitted.



Based on these considerations, now think about what will happen if you look through two polarizing films at an angle to each other, as shown in the figure above. *Do not look into the laser beam!* Just look around the room. What will happen as you change the angle  $\theta$ ?

### D Three polarizing films

Now suppose you start with two films at a  $90^\circ$  angle to each other, and then sandwich a third film between them at a  $45^\circ$  angle, as shown in the two figures above. Make a prediction about what will happen, and discuss your prediction with your instructor before you make the actual observation.



## Quantitative Observations

### E Intensity of light passing through two polarizing films

In this part of the lab, you will make numerical measurements of the transmission of initially unpolarized light transmitted through two polarizing films at an angle  $\theta$  to each other. To measure the intensity of the light that gets through, you will use a photovoltaic cell, which is a device that converts light energy into an electric current. The ones we're using are of a type known as a silicon photodiode.

You will use an ammeter to measure the current flowing from the photocell when light is shining on it. This is known as the "short-circuit current," because the ammeter ideally has zero resistance, so it acts like a short. Normally when you create a short through an ammeter, it blows the fuse in the meter, but here there is about  $40 \text{ k}\Omega$  of internal resistance in the silicon, which is a semiconductor. A photovoltaic cell is a complicated nonlinear device, but I've found empirically that under the conditions we're using in this experiment, the current is proportional to the power of the light striking the cell: twice as much light results in twice the current.<sup>1</sup>

This measurement requires a source of light that is unpolarized, constant in intensity, has a wavelength that the polaroids work with, and comes from a specific direction so it can't get to the photocell without going through the polaroids. The ambient light in the room is nearly unpolarized, but varies randomly as people walk in front of the light fixtures, etc. An incandescent lightbulb doesn't work, because it puts out a huge amount of infrared light, which the silicon cell measures but the polaroids can't work with. A laser beam is constant in intensity, but as I was creating this lab I found to my surprise that the

<sup>1</sup>It's also possible to use the same cell with a voltmeter across it, in which case we'd be measuring the "open-circuit voltage;" but the open-circuit voltage varies in a much more nonlinear way with the intensity of the light. When rooftop photovoltaic cells are used to generate power, the resistance of the load is neither zero nor infinity, and is chosen to maximize the efficiency.

light from the laser I tried was partially polarized, with a polarization that varied over time. A more suitable source of light is the sodium gas discharge tube, which makes a nearly monochromatic, unpolarized yellow light. Make sure you have allowed it to warm up for at least 15-20 minutes before using it; before it warms up, it makes a reddish light, and the polaroids do not work very well on that color.

Make measurements of the relative intensity of light transmitted through the two polarizing films, using a variety of angles  $\theta$ . Don't assume that the notches on the plastic housing of the polarizing films are a good indication of the orientation of the films themselves.

## Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

**P1** Given the angle  $\theta$  between the polarizing films, predict the ratio  $|\mathbf{E}'|/|\mathbf{E}|$  of the transmitted electric field to the incident electric field.

**P2** Based on your answer to P1, predict the ratio  $P'/P$  of the transmitted power to the incident power.

**P3** Sketch a graph of your answer to P2. Superimposed on the same graph, show a qualitative prediction of how it would change if the polaroids were not 100% perfect at filtering out one component of the field.

## Analysis

Discuss your qualitative results in terms of superposition and vector addition.

Graph your results from part E, and superimpose a theoretical curve for comparison. Discuss how your results compare with theory. Since your measurements of light intensity are relative, just scale the points so that their maximum matches that of the experimental data. (You might think of comparing the intensity transmitted through the two polaroids with the intensity that you get with no polaroids in the way at all. This doesn't really work, however, because in addition to acting as polarizers, the polaroids simply absorb a certain percentage of the light, just as any transparent material would.)

# Appendix 1: Format of Lab Writeups

Lab reports must be three pages or less, not counting your raw data. The format should be as follows:

## Title

**Raw data** — *Keep actual observations separate from what you later did with them.*

These are the results of the measurements you take down during the lab, hence they come first. Write your raw data directly in your lab book; don't write them on scratch paper and recopy them later. Don't use pencil. The point is to separate facts from opinions, observations from inferences.

**Procedure** — *Did you have to create your own methods for getting some of the raw data?*

Do not copy down the procedure from the manual. In this section, you only need to explain any methods you had to come up with on your own, or cases where the methods suggested in the handout didn't work and you had to do something different. Don't write anything here unless you think I will really care and want to change how we do the lab in the future. In most cases this section can be totally blank. Do not discuss how you did your calculations here, just how you got your raw data.

**Abstract** — *What did you find out? Why is it important?*

The "abstract" of a scientific paper is a *short* paragraph at the top that summarizes the experiment's results in a few sentences.

Many of our labs are comparisons of theory and experiment. The abstract for such a lab needs to say whether you think the experiment was consistent with theory, or not consistent with theory. If your results deviated from the ideal equations, don't be afraid to say so. After all, this is real life, and many of the equations we learn are only approximations, or are only valid in certain circumstances. However, (1) if you simply mess up, it is your responsibility to realize it in lab and do it again, right; (2) you will never get exact agreement with theory, because measurements are not perfectly exact — the important issue is whether your results agree with theory to roughly within the error bars.

The abstract is not a statement of what you hoped to find out. It's a statement of what you *did* find out. It's like the brief statement at the beginning of a debate: "The U.S. should have free trade with China." It's not this: "In this debate, we will discuss

whether the U.S. should have free trade with China."

If this is a lab that has just one important numerical result (or maybe two or three of them), put them in your abstract, with error bars where appropriate. There should normally be no more than two to four numbers here. Do not recapitulate your raw data here — this is for your final results.

If you're presenting a final result with error bars, make sure that the number of significant figures is consistent with your error bars. For example, if you write a result as  $323.54 \pm 6$  m/s, that's wrong. Your error bars say that you could be off by 6 in the ones' place, so the 5 in the tenths' place and the four in the hundredths' place are completely meaningless.

If you're presenting a number in scientific notation, with error bars, don't do it like this

$$1.234 \times 10^{-89} \text{ m/s} \pm 3 \times 10^{-92} \text{ m/s} \quad ,$$

do it like this

$$(1.234 \pm 0.003) \times 10^{-89} \text{ m/s} \quad ,$$

so that we can see easily which digit of the result the error bars apply to.

**Calculations and Reasoning** — *Convince me of what you claimed in your abstract.*

Often this section consists of nothing more than the calculations that you started during lab. If those calculations are clear enough to understand, and there is nothing else of interest to explain, then it is not necessary to write up a separate narrative of your analysis here. If you have a long series of similar calculations, you may just show one as a sample. If your prelab involved deriving equations that you will need, repeat them here without the derivation.

In some labs, you will need to go into some detail here by giving logical arguments to convince me that the statements you made in the abstract follow logically from your data. Continuing the debate metaphor, if your abstract said the U.S. should have free trade with China, this is the rest of the debate, where you convince me, based on data and logic, that we should have free trade.



## Appendix 2: Basic Error Analysis

### No measurement is perfectly exact.

One of the most common misconceptions about science is that science is “exact.” It is always a struggle to get beginning science students to believe that no measurement is perfectly correct. They tend to think that if a measurement is a little off from the “true” result, it must be because of a mistake — if a pro had done it, it would have been right on the mark. Not true!

What scientists can do is to estimate just how far off they might be. This type of estimate is called an error bar, and is expressed with the  $\pm$  symbol, read “plus or minus.” For instance, if I measure my dog’s weight to be  $52 \pm 2$  pounds, I am saying that my best estimate of the weight is 52 pounds, and I think I could be off by roughly 2 pounds either way. The term “error bar” comes from the conventional way of representing this range of uncertainty of a measurement on a graph, but the term is also used when no graph is involved.

Some very good scientific work results in measurements that nevertheless have large error bars. For instance, the best measurement of the age of the universe is now  $15 \pm 5$  billion years. That may not seem like wonderful precision, but the people who did the measurement knew what they were doing. It’s just that the only available techniques for determining the age of the universe are inherently poor.

Even when the techniques for measurement are very precise, there are still error bars. For instance, electrons act like little magnets, and the strength of a very weak magnet such as an individual electron is customarily measured in units called Bohr magnetons. Even though the magnetic strength of an electron is one of the most precisely measured quantities ever, the best experimental value still has error bars:  $1.0011596524 \pm 0.0000000002$  Bohr magnetons.

There are several reasons why it is important in scientific work to come up with a numerical estimate of your error bars. If the point of your experiment is to test whether the result comes out as predicted by a theory, you know there will always be some disagreement, even if the theory is absolutely right. You need to know whether the measurement is reasonably consistent with the theory, or whether the discrepancy is too great to be explained by the lim-

itations of the measuring devices.

Another important reason for stating results with error bars is that other people may use your measurement for purposes you could not have anticipated. If they are to use your result intelligently, they need to have some idea of how accurate it was.

### Error bars are not absolute limits.

Error bars are not absolute limits. The true value may lie outside the error bars. If I got a better scale I might find that the dog’s weight is  $51.3 \pm 0.1$  pounds, inside my original error bars, but it’s also possible that the better result would be  $48.7 \pm 0.1$  pounds. Since there’s always some chance of being off by a somewhat more than your error bars, or even a lot more than your error bars, there is no point in being extremely conservative in an effort to make absolutely sure the true value lies within your stated range. When a scientist states a measurement with error bars, she is not saying “If the true value is outside this range, I deserve to be drummed out of the profession.” If that was the case, then every scientist would give ridiculously inflated error bars to avoid having her career ended by one fluke out of hundreds of published results. What scientists are communicating to each other with error bars is a typical amount by which they might be off, not an upper limit.

The important thing is therefore to define error bars in a standard way, so that different people’s statements can be compared on the same footing. By convention, it is usually assumed that people estimate their error bars so that about two times out of three, their range will include the true value (or the results of a later, more accurate measurement with an improved technique).

### Random and systematic errors.

Suppose you measure the length of a sofa with a tape measure as well as you can, reading it off to the nearest millimeter. If you repeat the measurement again, you will get a different answer. (This is assuming that you don’t allow yourself to be psychologically biased to repeat your previous answer, and that 1 mm is about the limit of how well you can see.) If you kept on repeating the measurement,

you might get a list of values that looked like this:

203.1 cm	203.4	202.8	203.3	203.2
203.4	203.1	202.9	202.9	203.1

Variations of this type are called random errors, because the result is different every time you do the measurement.

The effects of random errors can be minimized by averaging together many measurements. Some of the measurements included in the average are too high, and some are too low, so the average tends to be better than any individual measurement. The more measurements you average in, the more precise the average is. The average of the above measurements is 203.1 cm. Averaging together many measurements cannot completely eliminate the random errors, but it can reduce them.

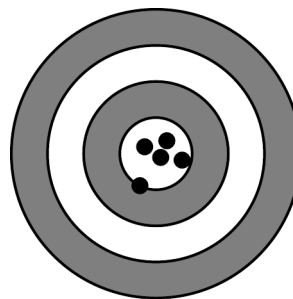
On the other hand, what if the tape measure was a little bit stretched out, so that your measurements always tended to come out too low by 0.3 cm? That would be an example of a systematic error. Since the systematic error is the same every time, averaging didn't help us to get rid of it. You probably had no easy way of finding out exactly the amount of stretching, so you just had to suspect that there might a systematic error due to stretching of the tape measure.

Some scientific writers make a distinction between the terms “accuracy” and “precision.” A precise measurement is one with small random errors, while an accurate measurement is one that is actually close to the true result, having both small random errors and small systematic errors. Personally, I find the distinction is made more clearly with the more memorable terms “random error” and “systematic error.”

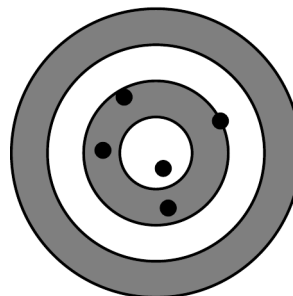
The  $\pm$  sign used with error bars normally implies that random errors are being referred to, since random errors could be either positive or negative, whereas systematic errors would always be in the same direction.

## The goal of error analysis

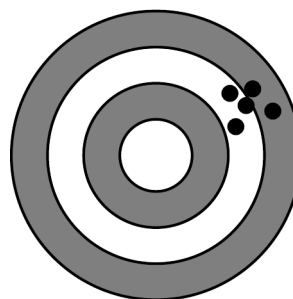
Very seldom does the final result of an experiment come directly off of a clock, ruler, gauge or meter. It is much more common to have raw data consisting of direct measurements, and then calculations based on the raw data that lead to a final result. As an example, if you want to measure your car's gas mileage, your raw data would be the number of gallons of gas consumed and the number of miles you went. You would then do a calculation, dividing



small random errors,  
small systematic error



large random errors,  
small systematic error



small random errors,  
large systematic error

miles by gallons, to get your final result. When you communicate your result to someone else, they are completely uninterested in how accurately you measured the number of miles and how accurately you measured the gallons. They simply want to know how accurate your final result was. Was it  $22 \pm 2$  mi/gal, or  $22.137 \pm 0.002$  mi/gal?

Of course the accuracy of the final result is ultimately based on and limited by the accuracy of your raw data. If you are off by 0.2 gallons in your measurement of the amount of gasoline, then that amount of error will have an effect on your final result. We say that the errors in the raw data “propagate” through the calculations. When you are requested to do “error analysis” in a lab writeup, that means that you

are to use the techniques explained below to determine the error bars on your final result. There are two sets of techniques you'll need to learn:

techniques for finding the accuracy of your raw data

techniques for using the error bars on your raw data to infer error bars on your final result

## Estimating random errors in raw data

We now examine three possible techniques for estimating random errors in your original measurements, illustrating them with the measurement of the length of the sofa.

### Method #1: Guess

If you're measuring the length of the sofa with a metric tape measure, then you can probably make a reasonable guess as to the precision of your measurements. Since the smallest division on the tape measure is one millimeter, and one millimeter is also near the limit of your ability to see, you know you won't be doing better than  $\pm 1$  mm, or 0.1 cm. Making allowances for errors in getting tape measure straight and so on, we might estimate our random errors to be a couple of millimeters.

Guessing is fine sometimes, but there are at least two ways that it can get you in trouble. One is that students sometimes have too much faith in a measuring device just because it looks fancy. They think that a digital balance must be perfectly accurate, since unlike a low-tech balance with sliding weights on it, it comes up with its result without any involvement by the user. That is incorrect. No measurement is perfectly accurate, and if the digital balance only displays an answer that goes down to tenths of a gram, then there is no way the random errors are any smaller than about a tenth of a gram.

Another way to mess up is to try to guess the error bars on a piece of raw data when you really don't have enough information to make an intelligent estimate. For instance, if you are measuring the range of a rifle, you might shoot it and measure how far the bullet went to the nearest centimeter, concluding that your random errors were only  $\pm 1$  cm. In reality, however, its range might vary randomly by fifty meters, depending on all kinds of random factors you don't know about. In this type of situation, you're better off using some other method of estimating your random errors.

### Method #2: Repeated Measurements and the Two-Thirds Rule

If you take repeated measurements of the same thing, then the amount of variation among the numbers can tell you how big the random errors were. This approach has an advantage over guessing your random errors, since it automatically takes into account all the sources of random error, even ones you didn't know were present.

Roughly speaking, the measurements of the length of the sofa were mostly within a few mm of the average, so that's about how big the random errors were. But let's make sure we are stating our error bars according to the convention that the true result will fall within our range of errors about two times out of three. Of course we don't know the "true" result, but if we sort out our list of measurements in order, we can get a pretty reasonable estimate of our error bars by taking half the range covered by the middle two thirds of the list. Sorting out our list of ten measurements of the sofa, we have

202.8 cm	202.9	202.9	203.1	203.1
203.1	203.2	203.3	203.4	203.4

Two thirds of ten is about 6, and the range covered by the middle six measurements is 203.3 cm - 202.9 cm, or 0.4 cm. Half that is 0.2 cm, so we'd estimate our error bars as  $\pm 0.2$  cm. The average of the measurements is 203.1 cm, so your result would be stated as  $203.1 \pm 0.2$  cm.

One common mistake when estimating random errors by repeated measurements is to round off all your measurements so that they all come out the same, and then conclude that the error bars were zero. For instance, if we'd done some overenthusiastic rounding of our measurements on the sofa, rounding them all off to the nearest cm, every single number on the list would have been 203 cm. That wouldn't mean that our random errors were zero! The same can happen with digital instruments that automatically round off for you. A digital balance might give results rounded off to the nearest tenth of a gram, and you may find that by putting the same object on the balance again and again, you always get the same answer. That doesn't mean it's perfectly precise. Its precision is no better than about  $\pm 0.1$  g.

### Method #3: Repeated Measurements and the Standard Deviation

The most widely accepted method for measuring error bars is called the standard deviation. Here's how the method works, using the sofa example again.



(1) Take the average of the measurements.

$$\text{average} = 203.1 \text{ cm}$$

(2) Find the difference, or “deviation,” of each measurement from the average.

$$\begin{array}{ccccc} -0.3 \text{ cm} & -0.2 & -0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.3 & 0.3 \end{array}$$

(3) Take the square of each deviation.

$$\begin{array}{ccccc} 0.09 \text{ cm}^2 & 0.04 & 0.04 & 0.00 & 0.00 \\ 0.00 & 0.01 & 0.01 & 0.09 & 0.09 \end{array}$$

(4) Average together all the squared deviations.

$$\text{average} = 0.04 \text{ cm}^2$$

(5) Take the square root. This is the standard deviation.

$$\text{standard deviation} = 0.2 \text{ cm}$$

If we’re using the symbol  $x$  for the length of the couch, then the result for the length of the couch would be stated as  $x = 203.1 \pm 0.2 \text{ cm}$ , or  $x = 203.1 \text{ cm}$  and  $\sigma_x = 0.2 \text{ cm}$ . Since the Greek letter sigma ( $\sigma$ ) is used as a symbol for the standard deviation, a standard deviation is often referred to as “a sigma.”

Step (3) may seem somewhat mysterious. Why not just skip it? Well, if you just went straight from step (2) to step (4), taking a plain old average of the deviations, you would find that the average is zero! The positive and negative deviations always cancel out exactly. Of course, you could just take absolute values instead of squaring the deviations. The main advantage of doing it the way I’ve outlined above are that it is a standard method, so people will know how you got the answer. (Another advantage is that the standard deviation as I’ve described it has certain nice mathematical properties.)

A common mistake when using the standard deviation technique is to take too few measurements. For instance, someone might take only two measurements of the length of the sofa, and get 203.4 cm and 203.4 cm. They would then infer a standard deviation of zero, which would be unrealistically small because the two measurements happened to come out the same.

In the following material, I’ll use the term “standard deviation” as a synonym for “error bar,” but that does not imply that you must always use the standard deviation method rather than the guessing method or the 2/3 rule.

There is a utility on the class’s web page for calculating standard deviations.

## Probability of deviations

You can see that although 0.2 cm is a good figure for the typical size of the deviations of the measurements of the length of the sofa from the average, some of the deviations are bigger and some are smaller. Experience has shown that the following probability estimates tend to hold true for how frequently deviations of various sizes occur:

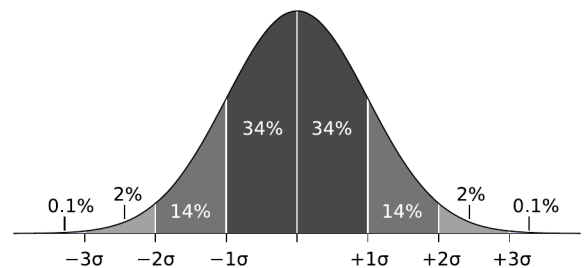
> 1 standard deviation about 1 times out of 3

> 2 standard deviations about 1 time out of 20

> 3 standard deviations about 1 in 500

> 4 standard deviations about 1 in 16,000

> 5 standard deviations about 1 in 1,700,000



The probability of various sizes of deviations, shown graphically. Areas under the bell curve correspond to probabilities. For example, the probability that the measurement will deviate from the truth by less than one standard deviation ( $\pm 1\sigma$ ) is about  $34 \times 2 = 68\%$ , or about 2 out of 3. (J. Kemp, P. Strandmark, Wikipedia.)

### Example: How significant?

In 1999, astronomers Webb et al. claimed to have found evidence that the strength of electrical forces in the ancient universe, soon after the big bang, was slightly weaker than it is today. If correct, this would be the first example ever discovered in which the laws of physics changed over time. The difference was very small,  $5.7 \pm 1.0$  parts per million, but still highly statistically significant. Dividing, we get  $(5.7 - 0)/1.0 = 5.7$  for the number of standard deviations by which their measurement was different from the expected result of zero. Looking at the table above, we see that if the true value really was zero, the chances of this happening would be less than one in a million. In general, five standard deviations (“five sigma”) is considered the gold standard for statistical significance.

This is an example of how we test a hypothesis statistically, find a probability, and interpret the probability. The probability we find is the probability that our results

would differ this much from the hypothesis, if the hypothesis was true. It's not the probability that the hypothesis is true or false, nor is it the probability that our experiment is right or wrong.

However, there is a twist to this story that shows how statistics always have to be taken with a grain of salt. In 2004, Chand et al. redid the measurement by a more precise technique, and found that the change was  $0.6 \pm 0.6$  parts per million. This is only one standard deviation away from the expected value of 0, which should be interpreted as being statistically consistent with zero. If you measure something, and you think you know what the result is supposed to be theoretically, then one standard deviation is the amount you typically *expect* to be off by — that's why it's called the "standard" deviation. Moreover, the Chand result is wildly statistically inconsistent with the Webb result (see the example on page 69), which means that one experiment or the other is a mistake. Most likely Webb et al. underestimated their random errors, or perhaps there were systematic errors in their experiment that they didn't realize were there.

## Precision of an average

We decided that the standard deviation of our measurements of the length of the couch was 0.2 cm, i.e., the precision of each individual measurement was about 0.2 cm. But I told you that the average, 203.1 cm, was more precise than any individual measurement. How precise is the average? The answer is that the standard deviation of the average equals

$$\frac{\text{standard deviation of one measurement}}{\sqrt{\text{number of measurements}}}$$

(An example on page 68 gives the reasoning that leads to the square root.) That means that you can theoretically measure anything to any desired precision, simply by averaging together enough measurements. In reality, no matter how small you make your random error, you can't get rid of systematic errors by averaging, so after a while it becomes pointless to take any more measurements.



# Appendix 3: Propagation of Errors

## Propagation of the error from a single variable

In the previous appendix we looked at techniques for estimating the random errors of raw data, but now we need to know how to evaluate the effects of those random errors on a final result calculated from the raw data. For instance, suppose you are given a cube made of some unknown material, and you are asked to determine its density. Density is defined as  $\rho = m/v$  ( $\rho$  is the Greek letter “rho”), and the volume of a cube with edges of length  $b$  is  $v = b^3$ , so the formula

$$\rho = m/b^3$$

will give you the density if you measure the cube’s mass and the length of its sides. Suppose you measure the mass very accurately as  $m = 1.658 \pm 0.003$  g, but you know  $b = 0.85 \pm 0.06$  cm with only two digits of precision. Your best value for  $\rho$  is  $1.658 \text{ g}/(0.85 \text{ cm})^3 = 2.7 \text{ g/cm}^3$ .

How can you figure out how precise this value for  $\rho$  is? We’ve already made sure not to keep more than two significant figures for  $\rho$ , since the less accurate piece of raw data had only two significant figures. We expect the last significant figure to be somewhat uncertain, but we don’t yet know how uncertain. A simple method for this type of situation is simply to change the raw data by one sigma, recalculate the result, and see how much of a change occurred. In this example, we add 0.06 cm to  $b$  for comparison.

$$\begin{aligned} b = 0.85 \text{ cm} & \text{ gave } \rho = 2.7 \text{ g/cm}^3 \\ b = 0.91 \text{ cm} & \text{ gives } \rho = 2.2 \text{ g/cm}^3 \end{aligned}$$

The resulting change in the density was  $0.5 \text{ g/cm}^3$ , so that is our estimate for how much it could have been off by:

$$\rho = 2.7 \pm 0.5 \text{ g/cm}^3$$

## Propagation of the error from several variables

What about the more general case in which no one piece of raw data is clearly the main source of error? For instance, suppose we get a more accurate measurement of the edge of the cube,  $b = 0.851 \pm 0.001$  cm. In percentage terms, the accuracies of  $m$  and

$b$  are roughly comparable, so both can cause significant errors in the density. The following more general method can be applied in such cases:

(1) Change one of the raw measurements, say  $m$ , by one standard deviation, and see by how much the final result,  $\rho$ , changes. Use the symbol  $Q_m$  for the absolute value of that change.

$$\begin{aligned} m = 1.658 \text{ g} & \text{ gave } \rho = 2.690 \text{ g/cm}^3 \\ m = 1.661 \text{ g} & \text{ gives } \rho = 2.695 \text{ g/cm}^3 \end{aligned}$$

$$Q_m = \text{change in } \rho = 0.005 \text{ g/cm}^3$$

(2) Repeat step (1) for the other raw measurements.

$$\begin{aligned} b = 0.851 \text{ cm} & \text{ gave } \rho = 2.690 \text{ g/cm}^3 \\ b = 0.852 \text{ cm} & \text{ gives } \rho = 2.681 \text{ g/cm}^3 \end{aligned}$$

$$Q_b = \text{change in } \rho = 0.009 \text{ g/cm}^3$$

(3) The error bars on  $\rho$  are given by the formula

$$\sigma_\rho = \sqrt{Q_m^2 + Q_b^2},$$

yielding  $\sigma_\rho = 0.01 \text{ g/cm}^3$ . Intuitively, the idea here is that if our result could be off by an amount  $Q_m$  because of an error in  $m$ , and by  $Q_b$  because of  $b$ , then if the two errors were in the same direction, we might be off by roughly  $|Q_m| + |Q_b|$ . However, it’s equally likely that the two errors would be in opposite directions, and at least partially cancel. The expression  $\sqrt{Q_m^2 + Q_b^2}$  gives an answer that’s smaller than  $Q_m + Q_b$ , representing the fact that the cancellation might happen.

The final result is  $\rho = 2.69 \pm 0.01 \text{ g/cm}^3$ .

### Example: An average

On page 66 I claimed that averaging a bunch of measurements reduces the error bars by the square root of the number of measurements. We can now see that this is a special case of propagation of errors.

For example, suppose Alice measures the circumference  $c$  of a guinea pig’s waist to be 10 cm. Using the guess method, she estimates that her error bars are about  $\pm 1$  cm (worse than the normal normal  $\sim 1$  mm error bars for a tape measure, because the guinea pig was squirming). Bob then measures the same thing, and gets 12 cm. The average is computed as

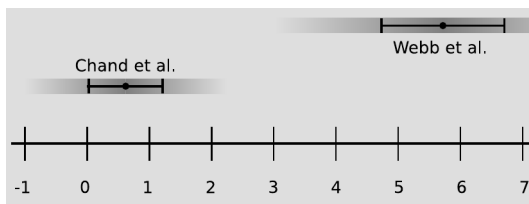
$$c = \frac{A + B}{2},$$

where  $A$  is Alice’s measurement, and  $B$  is Bob’s, giving 11 cm. If Alice had been off by one standard deviation (1 cm), it would have changed the average by 0.5

cm, so we have  $Q_A = 0.5$  cm, and likewise  $Q_B = 0.5$  cm. Combining these, we find  $\sigma_c = \sqrt{Q_A^2 + Q_B^2} = 0.7$  cm, which is simply  $(1.0 \text{ cm})/\sqrt{2}$ . The final result is  $c = (11.0 \pm 0.7)$  cm. (This violates the usual rule for significant figures, which is that the final result should have no more sig figs than the least precise piece of data that went into the calculation. That's okay, because the sig fig rules are just a quick and dirty way of doing propagation of errors. We've done real propagation of errors in this example, and it turns out that the error is in the first decimal place, so the 0 in that place is entitled to hold its head high as a real sig fig, albeit a relatively imprecise one with an uncertainty of  $\pm 7$ .)

*Example: The difference between two measurements*

In the example on page 65, we saw that two groups of scientists measured the same thing, and the results were  $W = 5.7 \pm 1.0$  for Webb et al. and  $C = 0.6 \pm 0.6$  for Chand et al. It's of interest to know whether the difference between their two results is small enough to be explained by random errors, or so big that it couldn't possibly have happened by chance, indicating that someone messed up. The figure shows each group's results, with error bars, on the number line. We see that the two sets of error bars don't overlap with one another, but error bars are not absolute limits, so it's perfectly possible to have non-overlapping error bars by chance, but the gap between the error bars is very large compared to the error bars themselves, so it looks implausible that the results could be statistically consistent with one another. I've tried to suggest this visually with the shading underneath the data-points.



To get a sharper statistical test, we can calculate the difference  $d$  between the two results,

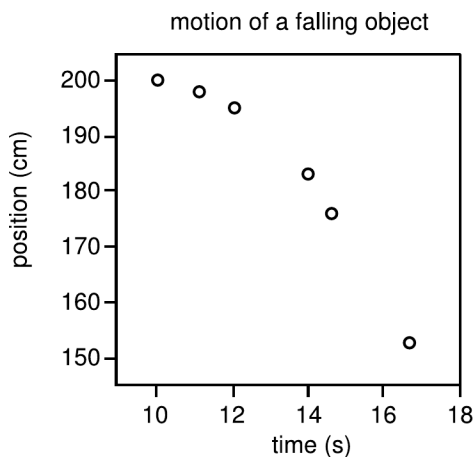
$$d = W - C$$

which is 5.1. Since the operation is simply the subtraction of the two numbers, an error in either input just causes an error in the output that is of the same size. Therefore we have  $Q_W = 1.0$  and  $Q_C = 0.6$ , resulting in  $\sigma_d = \sqrt{Q_W^2 + Q_C^2} = 1.2$ . We find that the difference between the two results is  $d = 5.1 \pm 1.2$ , which differs from zero by  $5.1/1.2 \approx 4$  standard deviations. Looking at the table on page 65, we see that the chances that  $d$  would be this big by chance are extremely small, less than about one in ten thousand. We can conclude to a high level of statistical confidence that the two groups' measurements are inconsistent with one another, and that one group is simply wrong.

# Appendix 4: Graphing

## Review of Graphing

Many of your analyses will involve making graphs. A graph can be an efficient way of presenting data visually, assuming you include all the information needed by the reader to interpret it. That means labeling the axes and indicating the units in parentheses, as in the example. A title is also helpful. Make sure that distances along the axes correctly represent the differences in the quantity being plotted. In the example, it would not have been correct to space the points evenly in the horizontal direction, because they were not actually measured at equally spaced points in time.



## Graphing on a Computer

Making graphs by hand in your lab notebook is fine, but in some cases you may find it saves you time to do graphs on a computer. For computer graphing, I recommend LibreOffice, which is free, open-source software. It's installed on the computers in rooms 416 and 418. Because LibreOffice is free, you can download it and put it on your own computer at home without paying money. If you already know Excel, it's very similar — you almost can't tell it's a different program.

Here's a brief rundown on using LibreOffice:

On Windows, go to the Start menu and choose All Programs, LibreOffice, and LibreOffice Calc. On Linux, do Applications, Office, OpenOffice, Spreadsheet.

Type in your x values in the first column, and your y values in the second column. For scientific notation, do, e.g., 5.2e-7 to represent  $5.2 \times 10^{-7}$ .

Select those two columns using the mouse.

From the Insert menu, do Object:Chart.

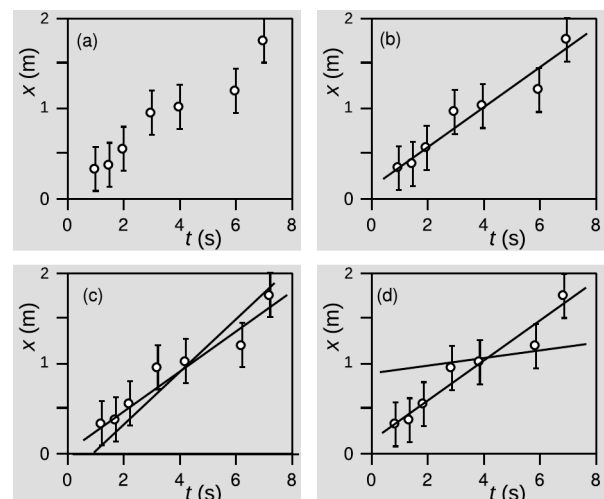
When it offers you various styles of graphs to choose from, choose the icon that shows a scatter plot, with dots on it (XY Chart).

Adjust the scales so the actual data on the plot is as big as possible, eliminating wasted space. To do this, double-click on the graph so that it's surrounded by a gray border. Then do Format, Axis, X Axis or Y Axis, Scale.

If you want error bars on your graph you can either draw them in by hand or put them in a separate column of your spreadsheet and doing Insert, Y Error Bars, Cell Range. Under Parameters, check "Same value for both." Click on the icon, and then use the mouse in the spreadsheet to select the cells containing the error bars.

## Fitting a Straight Line to a Graph by Hand

Often in this course you will end up graphing some data points, fitting a straight line through them with a ruler, and extracting the slope.



In this example, panel (a) shows the data, with error bars on each data point. Panel (b) shows a best fit, drawn by eye with a ruler. The slope of this best fit line is 100 cm/s. Note that the slope should be extracted from the line itself, not from two data points. The line is more reliable than any pair of individual data points.

In panel (c), a “worst believable fit” line has been drawn, which is as different in slope as possible from the best fit, while still pretty much staying consistent the data (going through or close to most of the error bars). Its slope is 60 cm/s. We can therefore estimate that the precision of our slope is +40 cm/s.

There is a tendency when drawing a “worst believable fit” line to draw instead an “unbelievably crazy fit” line, as in panel (d). The line in panel (d), with a very small slope, is just not believable compared to the data — it is several standard deviations away from most of the data points.

## Fitting a Straight Line to a Graph on a Computer

It’s also possible to fit a straight line to a graph using computer software such as LibreOffice.

To do this, first double-click on the graph so that a gray border shows up around it. Then right-click on a data-point, and a menu pops up. Choose Insert Trend Line.<sup>1</sup> choose Linear, and check the box for Show equation.

How accurate is your slope? A method for getting error bars on the slope is to artificially change one of your data points to reflect your estimate of how much it could have been off, and then redo the fit and find the new slope. The change in the slope tells you the error in the slope that results from the error in this data-point. You can then repeat this for the other points and proceed as in appendix 3.

An alternative method is to use the LINEST function that is available in many spreadsheet programs. For a description, see [tinyurl.com/ya7wmdft](http://tinyurl.com/ya7wmdft). Create the following formula in one cell of your spreadsheet: =Linest(y-values,x-value, True,True). Then, in excel, you need to press alt+ctrl+enter. In google sheets, press enter. A table with two columns and five rows will appear. The first number in the first column is the slope of the graph, and the second

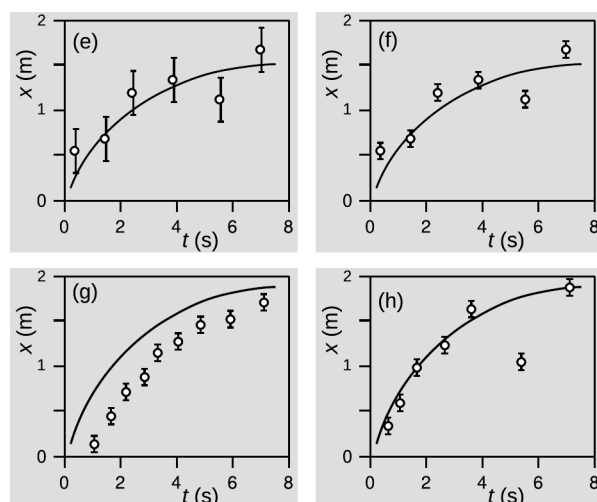
<sup>1</sup>“Trend line” is scientifically illiterate terminology that originates from Microsoft Office, which LibreOffice slavishly copies. If you don’t want to come off as an ignoramus, call it a “fit” or “line of best fit.”

number in the first column is the error in the slope.

In some cases, such as the absolute zero lab and the photoelectric effect lab, it’s very hard to tell how accurate your raw data are *a priori*; in these labs, you can use the typical amount of deviation of the points from the line as an estimate of their accuracy.

## Comparing Theory and Experiment

Figures (e) through (h) are examples of how we would compare theory and experiment on a graph. The convention is that theory is a line and experiment is points; this is because the theory is usually a prediction in the form of an equation, which can in principle be evaluated at infinitely many points, filling in all the gaps. One way to accomplish this with computer software is to graph both theory and experiment as points, but then print out the graph and draw a smooth curve through the theoretical points by hand.



The point here is usually to compare theory and experiment, and arrive at a yes/no answer as to whether they agree. In (e), the theoretical curve goes through the error bars on four out of six of the data points. This is about what we expect statistically, since the probability of being within one standard deviation of the truth is about 2/3 for a standard bell curve. Given these data, we would conclude that theory and experiment agreed.

In graph (f), the points are exactly the same as in (e), but the conclusion is the opposite. The error bars are smaller, too small to explain the observed discrepancies between theory and experiment. The theoretical curve only goes through the error bars on two of the six points, and this is quite a bit less than

we would expect statistically.

Graph (g) also shows disagreement between theory and experiment, but now we have a clear systematic error. In (h), the fifth data point looks like a mistake. Ideally you would notice during lab that something had gone wrong, and go back and check whether you could reproduce the result.





# Appendix 5: Finding Power Laws from Data

For many people, it is hard to imagine how scientists originally came up with all the equations that can now be found in textbooks. This appendix explains one method for finding equations to describe data from an experiment.

## Linear and nonlinear relationships

When two variables  $x$  and  $y$  are related by an equation of the form

$$y = cx \quad ,$$

where  $c$  is a constant (does not depend on  $x$  or  $y$ ), we say that a linear relationship exists between  $x$  and  $y$ . As an example, a harp has many strings of different lengths which are all of the same thickness and made of the same material. If the mass of a string is  $m$  and its length is  $L$ , then the equation

$$m = cL$$

will hold, where  $c$  is the mass per unit length, with units of kg/m. Many quantities in the physical world are instead related in a nonlinear fashion, i.e., the relationship does not fit the above definition of linearity. For instance, the mass of a steel ball bearing is related to its diameter by an equation of the form

$$m = cd^3 \quad ,$$

where  $c$  is the mass per unit volume, or density, of steel. Doubling the diameter does not double the mass, it increases it by a factor of eight.

## Power laws

Both examples above are of the general mathematical form

$$y = cx^p \quad ,$$

which is known as a power law. In the case of a linear relationship,  $p = 1$ . Consider the (made-up) experimental data shown in the table.

	$h$ =height of rodent at the shoulder (cm)	$f$ =food eaten per day (g)
shrew	1	3
rat	10	300
capybara	100	30,000

It's fairly easy to figure out what's going on just by staring at the numbers a little. Every time you increase the height of the animal by a factor of 10, its food consumption goes up by a factor of 100. This implies that  $f$  must be proportional to the square of  $h$ , or, displaying the proportionality constant  $k = 3$  explicitly,

$$f = 3h^2 \quad .$$

## Use of logarithms

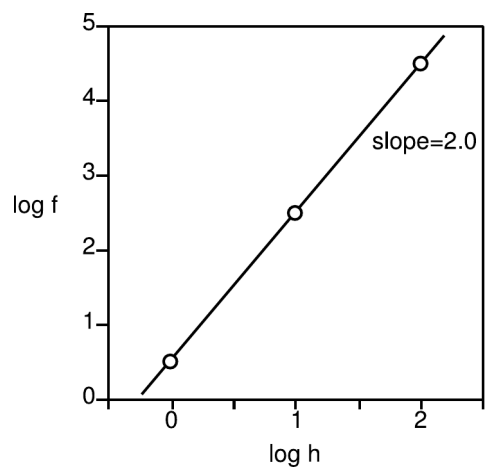
Now we have found  $c = 3$  and  $p = 2$  by inspection, but that would be much more difficult to do if these weren't all round numbers. A more generally applicable method to use when you suspect a power-law relationship is to take logarithms of both variables. It doesn't matter at all what base you use, as long as you use the same base for both variables. Since the data above were increasing by powers of 10, we'll use logarithms to the base 10, but personally I usually just use natural logs for this kind of thing.

	$\log_{10} h$	$\log_{10} f$
shrew	0	0.48
rat	1	2.48
capybara	2	4.48

This is a big improvement, because differences are so much simpler to work mentally with than ratios. The difference between each successive value of  $h$  is 1, while  $f$  increases by 2 units each time. The fact that the logs of the  $f$ 's increase twice as quickly is the same as saying that  $f$  is proportional to the square of  $h$ .

## Log-log plots

Even better, the logarithms can be interpreted visually using a graph, as shown on the next page. The slope of this type of log-log graph gives the power  $p$ . Although it is also possible to extract the proportionality constant,  $c$ , from such a graph, the proportionality constant is usually much less interesting than  $p$ . For instance, we would suspect that if  $p = 2$  for rodents, then it might also equal 2 for frogs or ants. Also,  $p$  would be the same regardless of what units we used to measure the variables. The constant  $c$ , however, would be different if we used different units, and would also probably be different for other types of animals.



# Appendix 6: Using a Multimeter

The most convenient instrument for measuring currents and voltage differences is called a digital multimeter (DMM), or simply a multimeter. “Digital” means that it shows the thing being measured on a calculator-style LCD display. “Multimeter” means that it can measure current, voltage, or resistance, depending on how you have it set up. Since we have many different types of multimeters, these instructions only cover the standard rules and methods that apply to all such meters. You may need to check with your instructor regarding a few of the particulars for the meter you have available.

## Measuring current

When using a meter to measure current, the meter must be in series with the circuit, so that every electron going by is forced to go through the meter and contribute to a current in the meter. Many multimeters have more than one scale for measuring a given thing. For instance, a meter may have a milliamp scale and an amp scale. One is used for measuring small currents and the other for large currents. You may not be sure in advance what scale is appropriate, but that’s not big problem — once everything is hooked up, you can try different scales and see what’s appropriate. Use the switch or buttons on the front to select one of the current scales. The connections to the meter should be made at the “common” socket (“COM”) and at the socket labeled “A” for Amperes.

## Measuring voltage

For a voltage measurement, use the switch or buttons on the front to select one of the voltage scales. (If you forget, and hook up the meter while the switch is still on a current scale, you may blow a fuse.) You always measure voltage differences with a meter. One wire connects the meter to one point in the circuit, and the other connects the meter to another point in a circuit. The meter measures the difference in voltage between those two points. For example, to measure the voltage across a resistor, you must put the meter in parallel with the resistor. The connections to the meter should be made at the “common” socket (“COM”) and at the socket labeled “V” for Volts.

## Blowing a fuse is not a big deal.

If you hook up your multimeter incorrectly, it is possible to blow a fuse inside. This is especially likely to happen if you set up the meter to measure current (meaning it has a small internal resistance) but hook it up in parallel with a resistor, creating a large voltage difference across it. Blowing a fuse is not a big problem, but it can be frustrating if you don’t realize what’s happened. If your meter suddenly stops working, you should check the fuse.



# Appendix 7: High Voltage Safety Checklist

Name: \_\_\_\_\_

\_\_\_\_\_ Never work with high voltages by yourself.

\_\_\_\_\_ Do not leave HV wires exposed - make sure there is insulation.

\_\_\_\_\_ Turn the high-voltage supply off while working on the circuit.

\_\_\_\_\_ When the voltage is on, avoid using both hands at once to touch the apparatus. Keep one hand in your pocket while using the other to touch the apparatus. That way, it is unlikely that you will get a shock across your chest.

\_\_\_\_\_ It is possible for an electric current to cause your hand to clench involuntarily. If you observe this happening to your partner, do not try to pry their hand away, because you could become incapacitated as well — simply turn off the switch or pull the plug out of the wall.









# Appendix ??: Comment Codes for Lab Writeups

## A. General

- a1. Don't write numbers without units. (25% off)
- a2. If something is wrong, cross it out. Don't make me guess which version to grade.
- a3. Your writeup is too long. The length limit is 3 pages, not including raw data.
- a4. If your writeup includes printouts, staple them in sideways with a single staple.
- a5. See appendix 1 for the format of lab writeups.
- a6. Don't state speculation as a firm conclusion.
- a7. Leave more space for me to write comments.
- a8. Cut unnecessary words. Use active voice. Write in a simple, direct style.
- a9. Don't write walls of text. Use paragraph breaks.
- a10. Cut any sentence that doesn't carry information.
- a11. This paragraph needs a topic sentence.
- a12. Express this as an equation.
- a13. Don't present details unless you've already made it clear why we would care. Don't write slavishly in chronological order.
- a14. The first sentence of any piece of writing must make an implicit promise that the remainder will interest the reader.

## B. Raw data

- b1. Don't mix raw data with calculations. (25% off)
- b2. Write raw data in pen, directly in the notebook.
- b3. This isn't raw data. This is a summary or copy.

## C. Procedure

- c1. Don't repeat the lab manual.
- c2. Don't write anything about your procedure unless it's something truly original that you think I would be interested in knowing about, or I wouldn't be able to understand your writeup without it.

## D. Abstract – see appendix 1

- d1. Your abstract is too long.
- d2. Don't recap raw data in your abstract.
- d3. Don't describe calculations in your abstract.
- d4. The only numbers that should be in your abstract are important final results that support your conclusion or that constitute the purpose of the lab.
- d5. Your abstract needs to include numerical results that support your conclusions.
- d6. Give error bars in your abstract.
- d7. Where is your abstract?
- d8. Your abstract is for results. This isn't a result of your experiment.
- d9. This isn't important enough to go in your abstract.
- d10. What was the point of the lab, and why would anyone care?
- d11. Don't just give results. Interpret them.
- d12. We knew this before you did the lab.
- d13. This lab was a quantitative test. Repeating it qualitatively isn't interesting.
- d14. This lab is a comparison of theory and experiment. Did they agree, or not?
- d15. Your results don't support your conclusions. Write about what really happened, not what you wanted to happen.
- d16. One observation can never prove a general rule.

## E. Error analysis – see appendices 2 and 3

- e1. A standard deviation only measures error if it comes from numbers that were supposed to be the same, e.g., repeated measurements of the same thing.
- e2. In propagation of errors, don't do both high and low. See appendix 3.
- e3. In propagation of errors, only change one variable at a time. See appendix 3.
- e4. Don't round severely when calculating Q's. Your Q's are just measuring your rounding errors.
- e5. A Q is the amount by which the output of the calculation changes, not its inputs.
- e6. A Q is a change in the result, not the

result itself.

e7. Use your error bars in forming your conclusions. Otherwise what was the point of calculating them?

e8. Give a probabilistic interpretation, as in the examples at the end of appendix 2.

e9. You're interpreting this probability incorrectly. It's the probability that your results would have differed this much from the hypothesis, if the hypothesis were true.

e10. % errors are useless. Teachers have you do them if you don't know about real error analysis.

e11. If random errors are included in your propagation of errors, listing them here verbally is pointless.

e12. Don't speculate about systematic errors without investigating them. Estimate their possible size. Would they produce an effect in the right direction?

#### **G. Graphing – see appendix 4**

g1. Label the axes to show what variables are being graphed and what their units are, e.g., x (km).

g2. Your graph should be bigger.

g3. If graphing by hand, do it on graph paper.

g4. Choose an appropriate scale for your graph, so that the data are not squished down. Don't just accept the default from the software if it's wrong. See app. 4 for how to do this using Libre Office.

g5. "Dot to dot" style is wrong in a scien-

tific graph.

g6. The independent variable (the one you control directly) goes on the x axis, and the dependent variable on the y. Or: cause on x, effect on y.

g7. On a scientific graph, use dots to show data, a line or curve for theory or a fit to the data.

g8. "Trend line" is scientifically illiterate. It's called a line of best fit.

#### **S. Calculations and sig figs**

s1. *Think* about the sizes of numbers and whether they make sense. This number doesn't make sense.

s2. Where did this number come from?

s3. This number has too many sig figs (e.g., more than the number of sig figs in the raw data).

s4. Don't round off severely for sig figs at intermediate steps. Rounding errors can accumulate.

s5. You're wasting your time by writing down many non-significant figures.

s6. Your result has too many sig figs. The error bars show that you don't have this much precision.

s7. The Calculations and Reasoning section usually just consists of the calculations you've already written. You don't need to write a separate narrative.

s8. Put your calculator in scientific notation mode.

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